Securitization of Mortality Risks in Life Annuities

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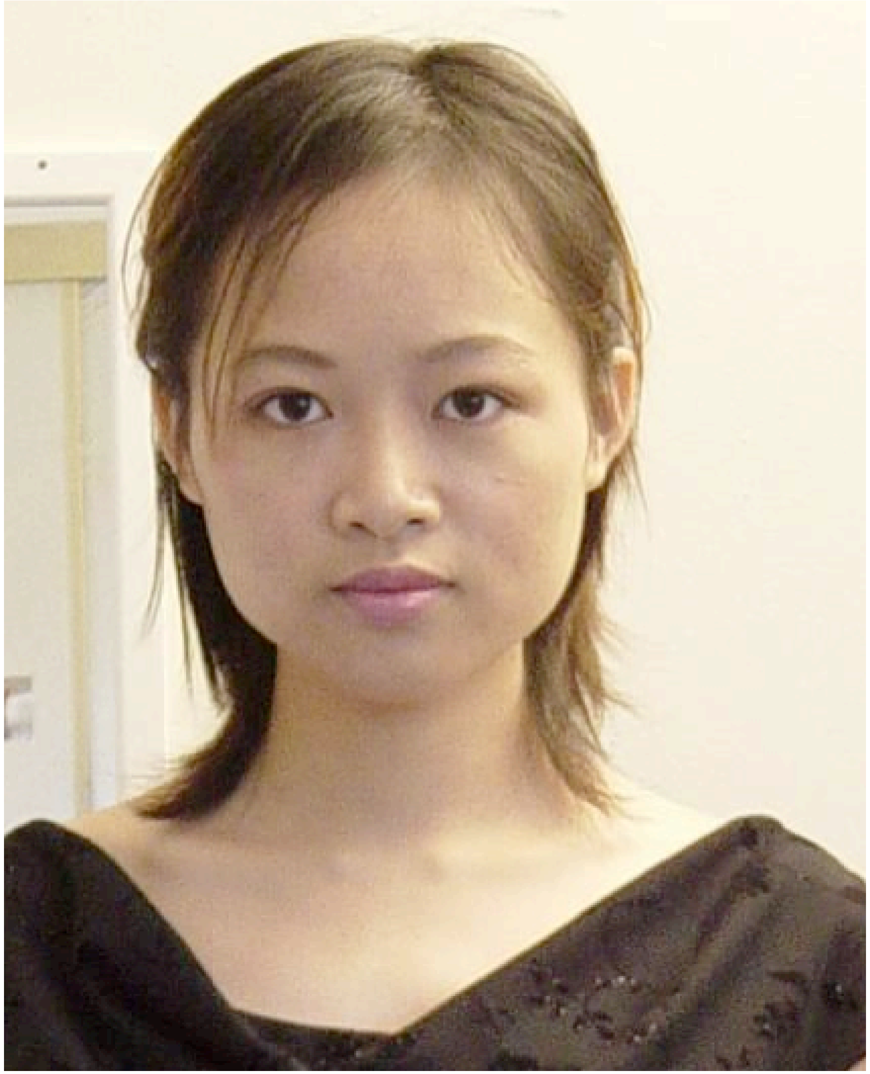

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Objectives:

Introduce mortality-based securities

The Swiss Re mortality bond (December 2003)

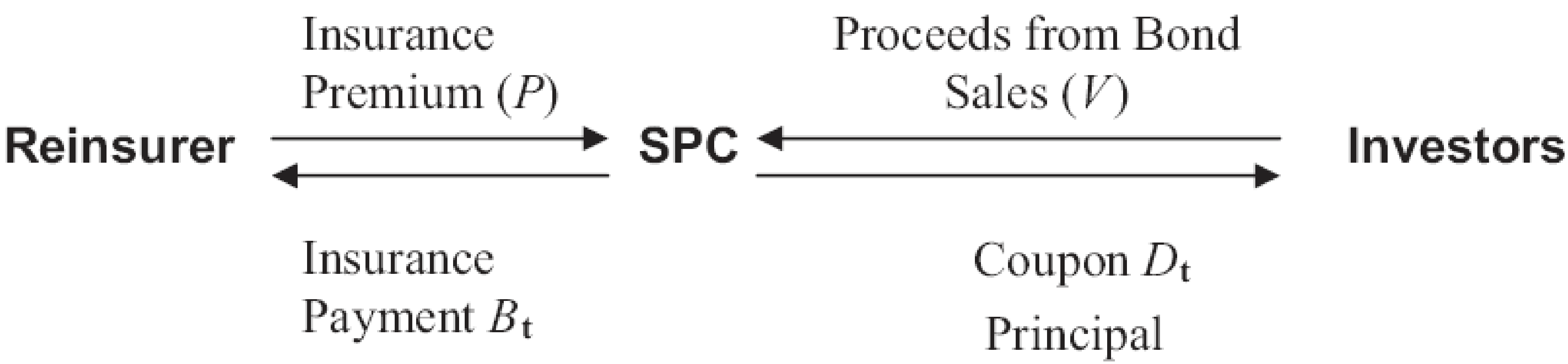
The other side of the "mortality tail"-- longevity risk.

Mortality risk bond

Uncertainty in mortality forecasts.

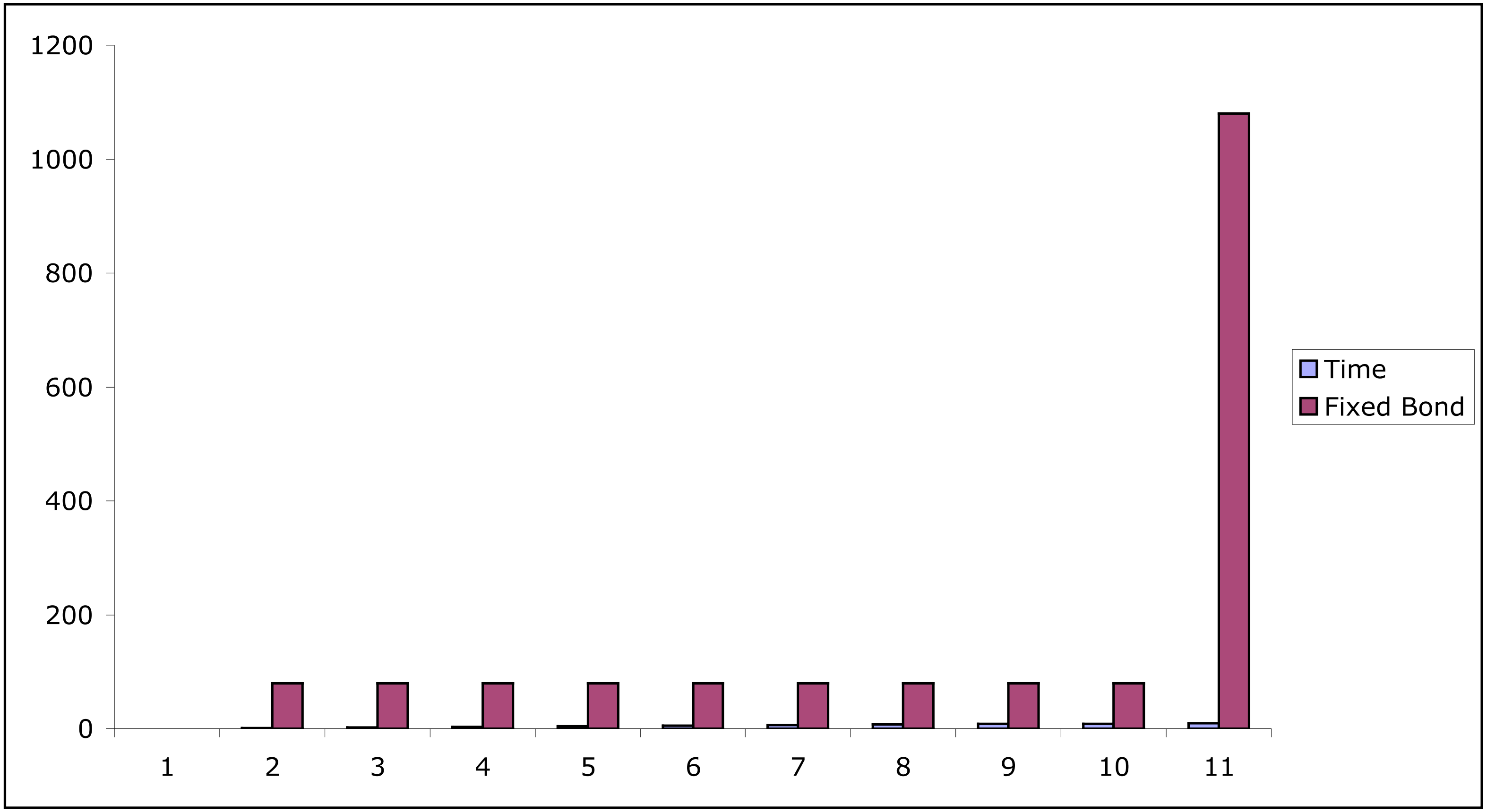
Potential expansion of the individual annuity market.

Securitization: Insurance linked bond

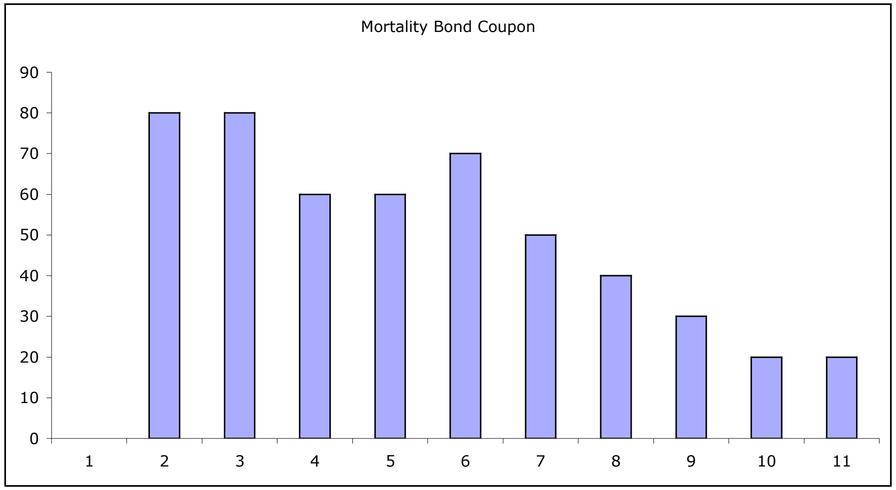


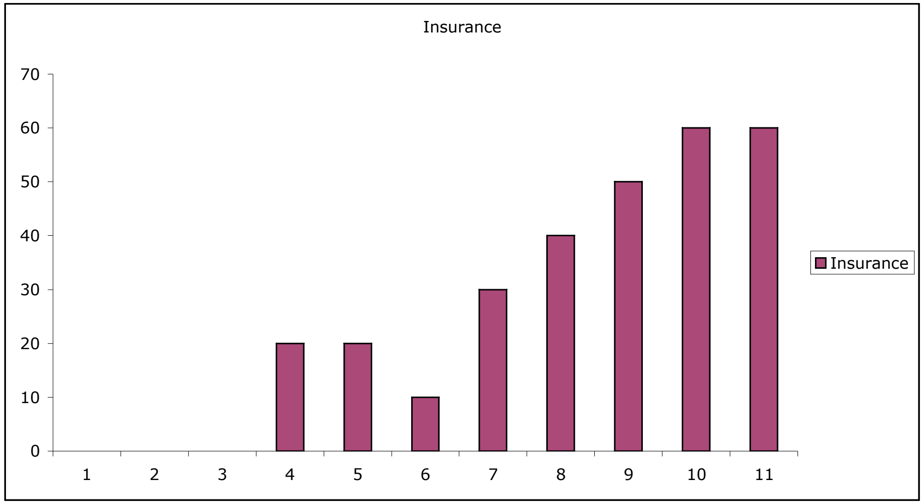
$$B_t + D_t = 1000C$$

Fixed bond cash flow



Bond Coupons and Insurance Benefits



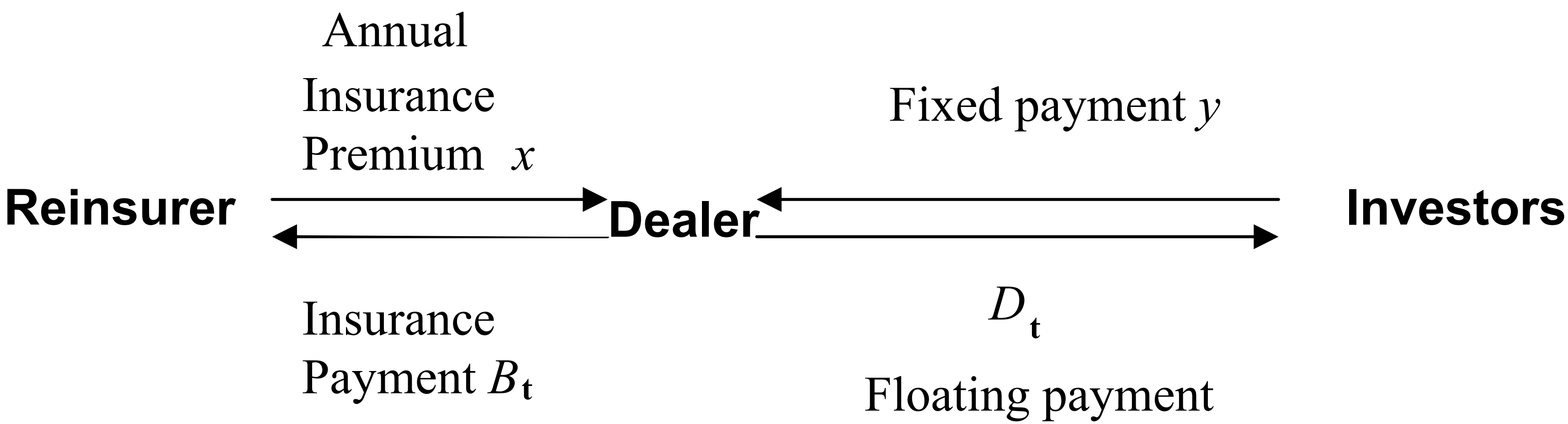


Floating for fixed swaps

- Each floating cash flow can be swapped for fixed.
- Special purpose company - Replaced by swap dealer

$$P = x \sum_{k=1}^{T} d(0,k).$$

$$y \sum_{k=1}^{T} d(0,k) = \sum_{k=1}^{T} \mathrm{E}^*[D_t] d(0,k).$$


Annual
Insurance
Premium x
Reinsurer
Dealer
Investors
Fixed payment y
Insurance
Payment B_t
D_t
Floating payment

$$x + y = B_t + D_t$$

Swiss Re's Mortality Bond

- Issued December 2003, matures January 1 2007, a three year deal.
- No coupons at risk
- Priced to sell at par with a coupon of $\mathrm{LIBOR} + 1.35\%$.
- Principal at risk.

Swiss Re's Mortality Bond

- q = weighted average of annual general population mortality in US, UK, France, Italy, and Switzerland.
- $q_0 = 2002$ level,
- $q_1 = 2003 + i$ level, and $q = \max(q_1, q_2, q_3)$
- $$\text{Maturity value} = \begin{cases} 400,000,000 & \text{if } q \le 1.3q_0 \\ 400,000,000\frac{1.5q_0 - q}{0.2q_0} & \text{if } 1.3q_0 < q \le 1.5q_0 \\ 0 & \text{if } q > 1.5q_0 \end{cases}$$

Wang Transform Pricing
The distribution implied by annuity prices

Given a distribution with cdf $F(t)$, a "distorted" distribution $F^*(t)$ is determined by λ according to the equation

$$F^*(t) = g_\lambda(F)(t) = \Phi(\Phi^{-1}(F(t)) - \lambda) \tag{1}$$

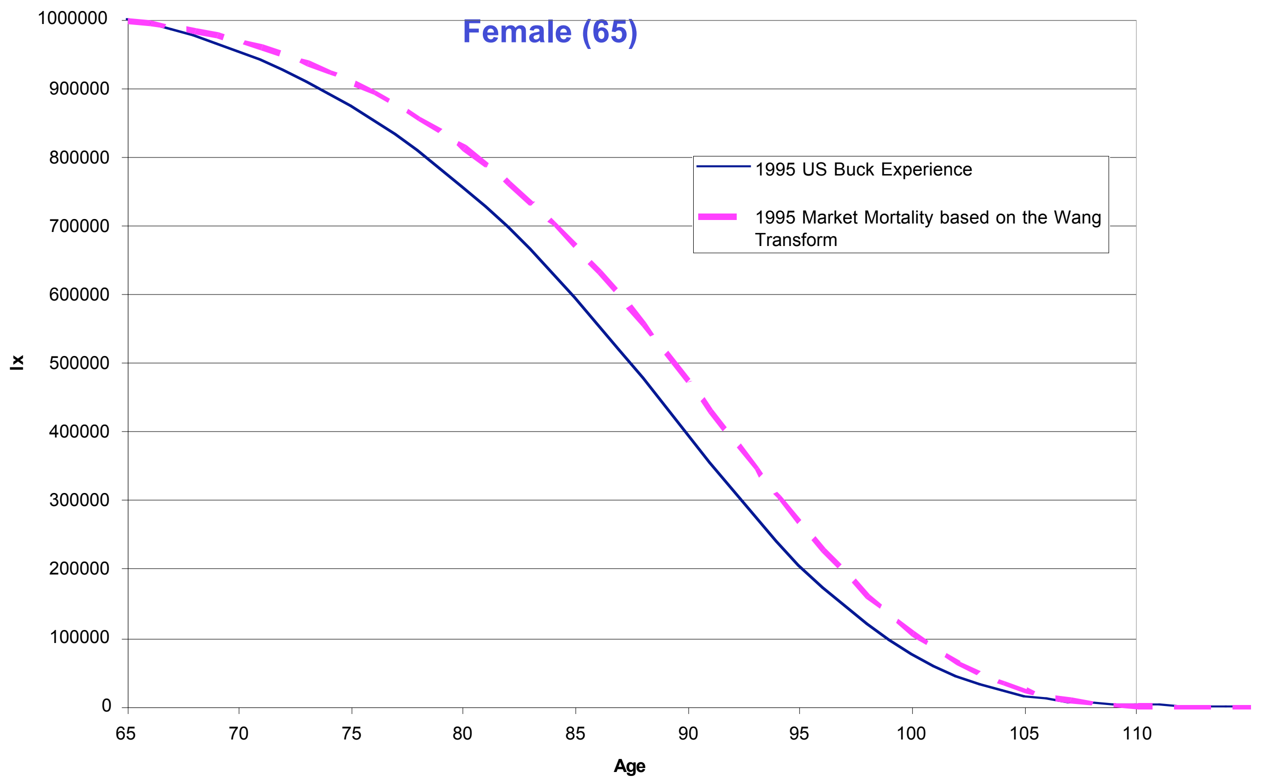
The parameter λ is called the market price of risk, reflecting the level of systematic risk.

$$F^*(t) = g_\lambda(F)(t) = \Phi[\Phi^{-1}({}_tq_{65}) - \lambda] \tag{1}$$

For the distribution function $F(t) = {}_tq_{65}$, we use the 1996 IAM 2000 Basic Table for a male life age sixty–five and, separately, for a female life age sixty–five.

Securitization of Mortality Risk—Pricing of Mortality Bonds based on the Wang Transform

➢ The Wang Transform



Transform pricing

$$12 l_x a_{65}^{(12)} = \sum_{t=1/12}^{\infty} \mathrm{E}^*[l_{x+t}] d(0,t)$$

$$V = F d(0,T) + \sum_{t=1}^{T} \mathrm{E}^*[D_t] d(0,t)$$

Annual benefit to insurer

$$B_t = \begin{cases} 1000C & \text{if } \ell_{x+t} > X_t + C \\ 1000(\ell_{x+t} - X_t) & \text{if } X_t < \ell_{x+t} \leq X_t + C \\ 0 & \text{if } \ell_{x+t} \leq X_t \end{cases}$$

Coupon to investors

$$D_t = \begin{cases} 0 & \text{if } \ell_{x+t} > C + X_t \\ 1000C - B_t & \text{if } X_t < \ell_{x+t} \leq C + X_t \\ 1000C & \text{if } \ell_{x+t} \leq X_t \end{cases}$$

$$= \begin{cases} 0 & \text{if } \ell_{x+t} > C + X_t \\ 1000(C + X_t - \ell_{x+t}) & \text{if } X_t < \ell_{x+t} \leq C + X_t \\ 1000C & \text{if } \ell_{x+t} \leq X_t \end{cases}$$

Investors' annual coupon

$\mathrm{E}^*[D_t]$ is calculated as follows.

$$\frac{1}{1000}D_t = \begin{cases} 0 & \text{if } \ell_{x+t} > C + X_t \\ C + X_t - \ell_{x+t} & \text{if } X_t < \ell_{x+t} \leq C + X_t \\ C & \text{if } \ell_{x+t} \leq X_t \end{cases}$$

$$= C - \max(\ell_{x+t} - X_t, 0) + \max(\ell_{x+t} - X_t - C, 0)$$
$$= C - (\ell_{x+t} - X_t)_+ + (\ell_{x+t} - X_t - C)_+$$

Strike levels

Age Range	Change of Force of Mortality
65–74	-0.0070
75–84	-0.0093
85–94	-0.0103

$$X_t = \begin{cases} \ell_x \, {}_tp_x \, e^{0.0070t} & \text{for } t = 1, \ldots, 10 \\ \ell_x \, {}_tp_x \, e^{0.07} e^{0.0093(t-10)} & \text{for } t = 11, \ldots, 20 \\ \ell_x \, {}_tp_x \, e^{0.163} e^{0.0103(t-20)} & \text{for } t = 21, \ldots, 30 \end{cases}$$

Pricing of Mortality Bonds

	Male (65)	Female (65)
Market price of risk (λ)	0.1792	0.2312
Number of annuitants	10,000	10,000
Annuity annual payout per person	1,000	1,000
Total premium from annuitants	99,650,768	107,232,089
Improvement level age 65 - 74	-0.0070	-0.0070
Improvement level age 75 - 84	-0.0093	-0.0093
Improvement level age 85 - 94	-0.0103	-0.0103
Face value of straight bond	10,000,000	10,000,000
Face value of mortality bond	10,000,000	10,000,000
Coupon rate of straight bond and mortality bond	0.07	0.07
Annual aggregate cash flow out of SPC ($1000C$)	700,000	700,000
Straight bond price	10,000,000	10,000,000
Mortality bond price	9,988,507	9,955,663
Reinsurance premium	11,493	44,337

TABLE 2. The survival distribution underlying the 1996 immediate annuity market based on the 1996 US Annuity 2000 Basic Mortality Table, the Wang transform, the average immediate annuity market quotes in August 1996 and the US Treasury interest rates on August 15, 1996.

Mortality projections

- Trend is improvement (lower q)

- Optimistic: Life expectancy will increase to 200 years.

- Pessimistic: The environment in which mortality improved is changing, so the future is uncertain.

- Data situation in the US: poor, but may improve.

Final comments

- Securitization may be a viable tool for managing longevity risks through mortality linked bonds or mortality swaps.
- A market for mortality-based securities will develop if the prices and contracting features make the securities attractive to potential buyers and sellers.
- Development of a market will be facilitated by better collection and dissemination of data and research in mortality dynamics.