Securitization of Mortality Risks in Life Annuities

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Objectives:

Introduce mortality-based securities

The Swiss Re mortality bond (December 2003)

The other side of the “mortality tail”-- longevity risk.

Mortality risk bond

Uncertainty in mortality forecasts.

Potential expansion of the individual annuity market.
Securitization: Insurance linked bond

\[ B_t + D_t = 1000C \]
Fixed bond cash flow

Securitization of Mortality Risks in Life Annuities
Floating for fixed swaps

- Each floating cash flow can be swapped for fixed.
- Special purpose company - Replaced by swap dealer

\[ P = x \sum_{k=1}^{T} d(0, k). \]

\[ y \sum_{k=1}^{T} d(0, k) = \sum_{k=1}^{T} E^*[D_t]d(0, k). \]
\[ x + y = B_t + D_t \]
Swiss Re’s Mortality Bond

- Issued December 2003, matures January 1 2007, a three year deal.
- No coupons at risk
- Priced to sell at par with a coupon of LIBOR + 1.35%.
- Principal at risk.
Swiss Re’s Mortality Bond

- $q = \text{weighted average of annual general population mortality in US, UK, France, Italy, and Switzerland.}$

- $q_0 = 2002 \text{ level,}$

- $q_1 = 2003 + i \text{ level, and } q = \max(q_1, q_2, q_3)$

- $Maturity \ value = \begin{cases} 400,000,000 & \text{if } q \leq 1.3q_0 \\ 400,000,000 \frac{1.5q_0 - q}{0.2q_0} & \text{if } 1.3q_0 < q \leq 1.5q_0 \\ 0 & \text{if } q > 1.5q_0 \end{cases}$
Wang Transform Pricing
The distribution implied by annuity prices

Given a distribution with cdf $F(t)$, a “distorted” distribution $F^*(t)$ is determined by $\lambda$ according to the equation

$$F^*(t) = g_\lambda(F(t)) = \Phi(\Phi^{-1}(F(t)) - \lambda)$$  \hspace{1cm} (1)

The parameter $\lambda$ is called the market price of risk, reflecting the level of systematic risk.
\[ F^*(t) = g_\lambda(F)(t) = \Phi[\Phi^{-1}(t_{q65}) - \lambda] \]  

For the distribution function \( F(t) = t_{q65} \), we use the 1996 IAM 2000 Basic Table for a male life age sixty–five and, separately, for a female life age sixty–five.
Securitization of Mortality Risk—Pricing of Mortality Bonds based on the Wang Transform

- The Wang Transform

\[ \left[ \Phi \right] \begin{bmatrix} \lambda \\ \Phi \end{bmatrix} = \left[ q \right] \begin{bmatrix} F_g \\ F_t \end{bmatrix} \]

Age  
65 70 75 80 85 90 95 100 105 110
lx  
0 100000 200000 300000 400000 500000 600000 700000 800000 900000 1000000

Female (65)

1995 US Buck Experience
1995 Market Mortality based on the Wang Transform

Securitization of Mortality Risks in Life Annuities
Transform pricing

\[ 12l_x a_{65}^{(12)} = \sum_{t=1/12}^{\infty} E^*[l_{x+t}]d(0, t) \]

\[ V = Fd(0, T) + \sum_{t=1}^{T} E^*[D_t]d(0, t) \]
Annual benefit to insurer

\[ B_t = \begin{cases} 
1000C & \text{if } \ell_{x+t} > X_t + C \\
1000(\ell_{x+t} - X_t) & \text{if } X_t < \ell_{x+t} \leq X_t + C \\
0 & \text{if } \ell_{x+t} \leq X_t 
\end{cases} \]
Coupon to investors

\[
D_t = \begin{cases} 
0 & \text{if } \ell_{x+t} > C + X_t \\
1000C - B_t & \text{if } X_t < \ell_{x+t} \leq C + X_t \\
1000C & \text{if } \ell_{x+t} \leq X_t 
\end{cases}
\]

\[
= \begin{cases} 
0 & \text{if } \ell_{x+t} > C + X_t \\
1000(C + X_t - \ell_{x+t}) & \text{if } X_t < \ell_{x+t} \leq C + X_t \\
1000C & \text{if } \ell_{x+t} \leq X_t 
\end{cases}
\]
Investors’ annual coupon

$E^*[D_t]$ is calculated as follows.

\[
\frac{1}{1000} D_t = \begin{cases} 
0 & \text{if } \ell_{x+t} > C + X_t \\
C + X_t - \ell_{x+t} & \text{if } X_t < \ell_{x+t} \leq C + X_t \\
C & \text{if } \ell_{x+t} \leq X_t 
\end{cases}
\]

\[
= C - \max(\ell_{x+t} - X_t, 0) + \max(\ell_{x+t} - X_t - C, 0) \\
= C - (\ell_{x+t} - X_t)_+ + (\ell_{x+t} - X_t - C)_+
\]
Strike levels

<table>
<thead>
<tr>
<th>Age Range</th>
<th>Change of Force of Mortality</th>
</tr>
</thead>
<tbody>
<tr>
<td>65–74</td>
<td>-0.0070</td>
</tr>
<tr>
<td>75–84</td>
<td>-0.0093</td>
</tr>
<tr>
<td>85–94</td>
<td>-0.0103</td>
</tr>
</tbody>
</table>

\[ X_t = \begin{cases} 
\ell_x t p_x e^{0.0070t} & \text{for } t = 1, \ldots, 10 \\
\ell_x t p_x e^{0.07} e^{0.0093(t-10)} & \text{for } t = 11, \ldots, 20 \\
\ell_x t p_x e^{0.163} e^{0.0103(t-20)} & \text{for } t = 21, \ldots, 30 
\end{cases} \]
### Pricing of Mortality Bonds

<table>
<thead>
<tr>
<th></th>
<th>Male (65)</th>
<th>Female (65)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market price of risk (λ)</td>
<td>0.1792</td>
<td>0.2312</td>
</tr>
<tr>
<td>Number of annuitants</td>
<td>10,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Annuity annual payout per person</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>Total premium from annuitants</td>
<td>99,650,768</td>
<td>107,232,089</td>
</tr>
<tr>
<td>Improvement level age 65 - 74</td>
<td>-0.0070</td>
<td>-0.0070</td>
</tr>
<tr>
<td>Improvement level age 75 - 84</td>
<td>-0.0093</td>
<td>-0.0093</td>
</tr>
<tr>
<td>Improvement level age 85 - 94</td>
<td>-0.0103</td>
<td>-0.0103</td>
</tr>
<tr>
<td>Face value of straight bond</td>
<td>10,000,000</td>
<td>10,000,000</td>
</tr>
<tr>
<td>Face value of mortality bond</td>
<td>10,000,000</td>
<td>10,000,000</td>
</tr>
<tr>
<td>Coupon rate of straight bond and mortality bond</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Annual aggregate cash flow out of SPC (1000C)</td>
<td>700,000</td>
<td>700,000</td>
</tr>
<tr>
<td>Straight bond price</td>
<td>10,000,000</td>
<td>10,000,000</td>
</tr>
<tr>
<td>Mortality bond price</td>
<td>9,988,507</td>
<td>9,955,663</td>
</tr>
<tr>
<td>Reinsurance premium</td>
<td>11,493</td>
<td>44,337</td>
</tr>
</tbody>
</table>

**TABLE 2.** The survival distribution underlying the 1996 immediate annuity market based on the 1996 US Annuity 2000 Basic Mortality Table, the Wang transform, the average immediate annuity market quotes in August 1996 and the US Treasury interest rates on August 15, 1996.
Mortality projections

- Trend is improvement (lower q)

- Optimistic: Life expectancy will increase to 200 years.

- Pessimistic: The environment in which mortality improved is changing, so the future is uncertain.

- Data situation in the US: poor, but may improve.
Final comments

- Securitization may be a viable tool for managing longevity risks through mortality linked bonds or mortality swaps.

- A market for mortality-based securities will develop if the prices and contracting features make the securities attractive to potential buyers and sellers.

- Development of a market will be facilitated by better collection and dissemination of data and research in mortality dynamics.