ASSET ALLOCATION GIVEN NON-MARKET WEALTH AND ROLLOVER RISK

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Consider the standard portfolio problem. Let's say I am investing for my family's well being after my retirement, no intermediate consumption.

Invest in risky stocks and risk-free bonds

But now suppose we add some additional features:

1. My aunt is very rich and I will inherit her money at the end of the planning horizon.
2. I have my investment in a fund that converts at a certain age into an annuity.

But, as life would have it, there are some complications:

(Life is like a box of chocolates: you never know what you're going to get.)
Forrest Gump, University of Alabama

What if my rich aunt isn't so rich after all?

And what if she never really liked me anyway?

And what if my wife is secretly off in Las Vegas running up big gambling debts?

And …

And … what if my annuity is in a fund with a company that now seems ‘’questionable?’’

AIG

[Disclaimer: The trademark above is fictional. Any resemblance to real-life trademarks is purely coincidental.]

Point is: the rich aunt and annuity rollover affect my investment behavior. If they are each risky, it complicates matters even further.

Outline of Presentation

1. The Portfolio-Choice Problem

2. Non-Market Wealth

bequests to or from others, health-care expenses, labor income bonus, Social Security income

3. Riskiness of NMW (additive)

unexpected "surprises" to the above

divorce, cure your disease, win the lottery, rich aunt outlives you, pension reform

4. Rollover Risk (multiplicative)

pension rate not guaranteed, inflation, exchange-rate conversion, tax-rule changes

5. Combined Effect of Non-Market Wealth Risk and Rollover Risk

6. A Little Bit of Theory

7. Numerical Analysis

8. Conclusions

The Portfolio Problem

Dynamic Asset allocation over n periods
(Merton 1969, Cox & Huang 1989)

Our focus: stock/bond mixture in the (marketable) investment portfolio

Assume preferences satisfy CRRA, but for global wealth
See Meyer (2005) for an interesting analysis

Classic result of Merton:
If preferences satisfy CRRA and the market follows a GBM, the optimal strategy is to maintain a constant proportion of stock in the portfolio.

Thus: rebalance every period back to this proportion.

Non-market wealth and rollover risk

Merton (2001), Bodie (2001), Shiller (2003)

We know a great deal about investing to maximize the distribution of future wealth, but we ignore the purpose of such wealth.

Do not account for:
Annuity risks, inequality risks, education expense costs, medical costs, human capital changes, wars, global influences, etc.

Focus of Merton/Bodie/Shiller is the development of new financial tools.

Our focus is on the implications of these risks in our current (incomplete) market setting with the ''old tools'' but with new strategies – such as current "lifestyle" investment strategies.

The Theory in plain English

<u>Effect of fixed non-market wealth</u>

(Similar to Bodie, Merton & Samuelson (1992))

Let z denote the net-asset value of NMW at time n.

If $z > 0$, we receive a net cash inflow at the end of the investment horizon.

Note: This is no different than being given a zero-coupon bond at the outset.

<u>Result</u>: Adjust your market portfolio to include fewer bonds. This makes your behavior appear less risk averse. Under CRRA your behavior also appears to be IRRA. When stocks are up, you need to buy more bonds than a CRRA investor to keep balanced.

(We will say that *<u>derived utility</u>* is less r.a. and IRRA)

If z < 0, the effects are reversed:

Derived utility is more risk averse
Behavior appears to be DRRA

Let wealth at time n be given as: $x(\tilde{R}) + z$

We can think of the market return $\tilde{R}$ as indicating the state of nature.

We wish to $\max_x Eu(x(\tilde{R}) + z)$, where $u(w) = \frac{1}{1-\gamma} w^{1-\gamma}$ or $u(w) = \ln w$

Define $v(x) \equiv u(x + z) = \frac{1}{1-\gamma}(w + z)^{1-\gamma}$

(Derived utility looks like HARA)

Risk in NMW

Suppose first that z = 0.

CRRA implies that a zero-mean additive noise term $\tilde{\varepsilon}$ will induce more risk-averse behavior (Eeckhoudt/Kimball 1992).

Also, behavior becomes DRRA (our result).

These are pure risk affects.

Let wealth at time n be given as: $x(\tilde{R}) + z + \tilde{\varepsilon}$

What if $z \neq 0$?

For $z \leq 0$ both effects (negative-constant effect and the pure-risk effect) reinforce each other: we behave in a more risk-averse manner. Also, we exhibit DRRA.

But for $z > 0$, the positive constant effect leads to decreased risk aversion, but the pure-risk effect leads to increased risk aversion.

Similarly, we cannot determine IRRA or DRRA or neither.

As expected, for $z > 0$, the total effect depends upon which of these two effects is stronger. [Microeconomics 101]

Rollover Risk

We model rollover risk as a multiplicative random variable $\tilde{y}$, $E\tilde{y} = 1$

[Think of $\tilde{y}$ as the unanticipated component of rollover risk]

$$\text{Wealth} = x(\tilde{R})\tilde{y} + z$$

For z = 0, general case (not just CRRA) analyzed in F/S/S (2005)

As is well known, under CRRA the stock proportion is unaffected by $\tilde{y}$.

[Although $\tilde{y}$ does not affect the portfolio, it does affect welfare: as if poorer]

For z < 0, derived utility possibly more risk averse with $\tilde{y}$ [e.g. if γ>1]

For z > 0, derived utility possibly less risk averse with $\tilde{y}$ [e.g. if z < (γ-1)x]

Combined Effects

z = 0

NMW risk → ↑ risk aversion, DRRA
Rollover risk → no change in risk aversion

z < 0 ↑ risk aversion, DRRA
NMW risk → ↑ risk aversion, DRRA
Rollover risk → ↑ risk aversion, DRRA*

z > 0 ↓ risk aversion
NMW risk → ↑ risk aversion, DRRA
Rollover risk → ↓ risk aversion*

Total Effects = ??

* under certain conditions

Expected Return on Market	10%	Horizon,	n	5 years
Risk-free Rate	5%	Coefficient of Relative Risk aversion,	γ	2
Volatility of Market Return, σ_m	20%	Expected Non-market Wealth,	z_0	-30, 0, 30
		Investible wealth,	x_0	100
		Volatility of Non-market wealth	σ_ε	0, 0.2
		Standard deviation of Rollover Rate	σ_y	0, 0.3

1. We assume that the market return follows a discrete binomial process, with a mean return of 10 % over each year. The volatility, of the underlying continuous process is 20%.

2. The risk-free rate of interest is 5% on a discrete, annual basis.

3. In the right hand columns we show the investor characteristics. The horizon, when non-market wealth is realised is 5 years. The coefficient of relative risk aversion is $\gamma = 2$.

For this illustration we consider just the initial allocations.

<u>z = 0</u>	<u>% stock in portfolio</u>
No risk	56%
Rollover risk ($\tilde{y}$)	56%

Under CRRA, ROR does not affect portfolio choice.

For this illustration we consider just the initial allocations.

$\underline{z = 0}$		<u>% stock in portfolio</u>
No risk		56%
NMW risk ($\tilde{\varepsilon}$)	↓	39%

As expected, NMW since risk increases risk aversion

For this illustration we consider just the initial allocations.

z = 0		% stock in portfolio	
No risk		56%	
Rollover risk ($\tilde{y}$)		56%	
NMW risk ($\tilde{\varepsilon}$)	↓	39%	
Both risks	↓↓	28%	(more dramatic than without ROR)

Rollover risk has two effects:

Direct effect: no change in risk aversion

Indirect effect: strengthen $\tilde{\varepsilon}$ effect ($\tilde{y} \Rightarrow$ as if poorer under DARA)

Here again we consider just the initial allocations.

z = 30		% stock in portfolio	
No risk		72%	(higher due to positive NMW)
Rollover risk ($\tilde{y}$)	↑	73%	
NMW risk ($\tilde{\varepsilon}$)	↓	58%	
Both risks	↓↓	56%	(NOT a simple sum of the 2 effects)

Rollover risk has two effects:

- Direct effect: decrease risk aversion
- Indirect effect: strengthen $\tilde{\varepsilon}$ effect ($\tilde{y} \Rightarrow$ as if poorer under DARA)

The Theory in less-plain English

$$\max_x Eu(x(\tilde{R})\tilde{y} + z + \tilde{\varepsilon}) \quad \textbf{s.t.} \quad E[\phi(\tilde{R}) \cdot x(\tilde{R})] = x_0$$

Define derived utility $\qquad v(x) \equiv Eu(x\tilde{y} + z + \tilde{\varepsilon})$

$$v'(x) \equiv E[u'(x\tilde{y} + z + \tilde{\varepsilon})\tilde{y}]$$

FOC: $\qquad v'(x(R)) \equiv \lambda\phi(R) \quad \forall R \qquad$ [λ = MU of initial wealth]

Differentiate FOC w.r.t. *R*: $\quad v''(x(R))x'(R) = \lambda\phi'(R)$

Replacing λ in the FOC yields $\; x' = \dfrac{-\phi'/\phi}{-v''/v'} \qquad$ (m.r.a. → single crossing)

Rearranging, we obtain

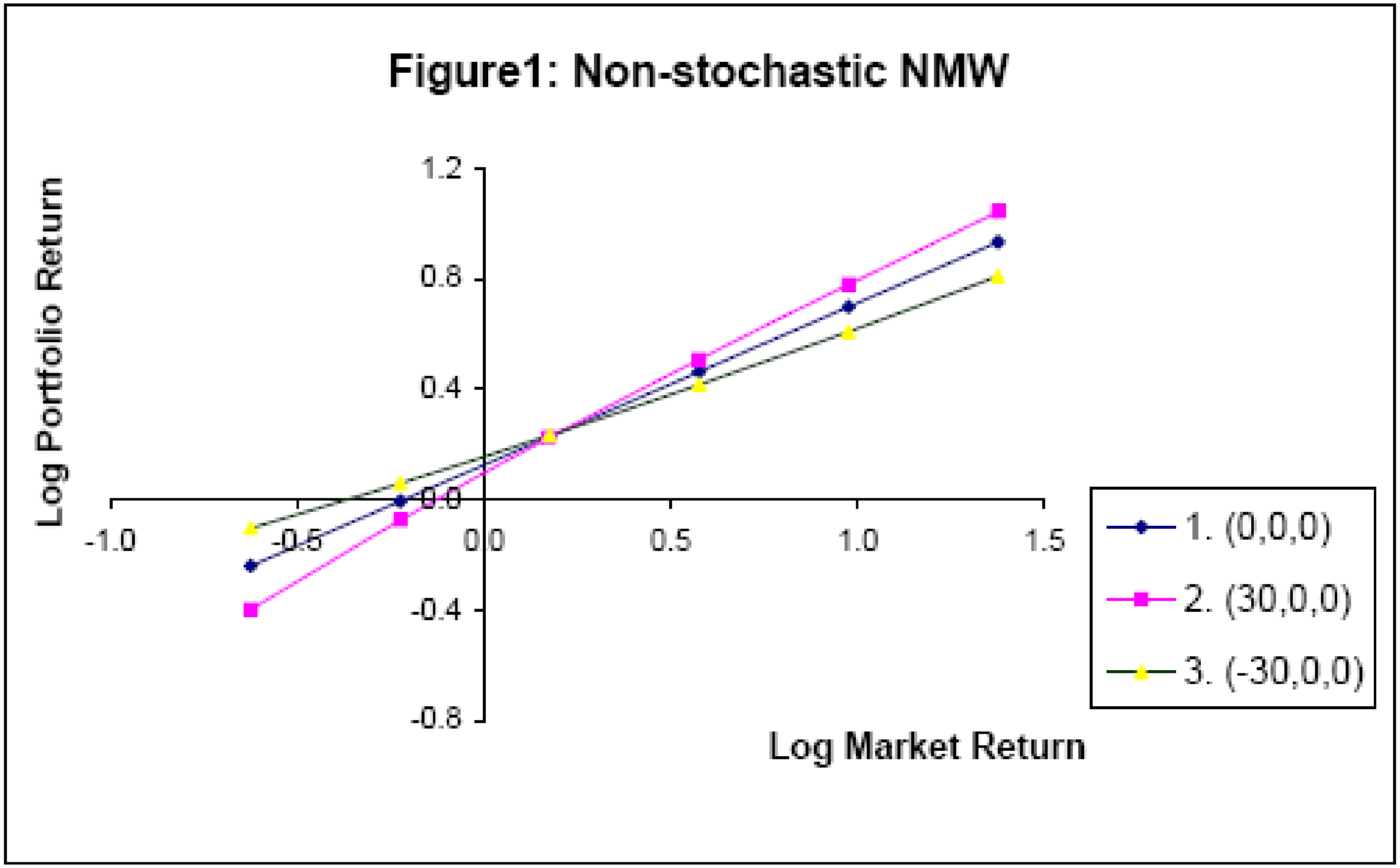
$$a_v^{-1}(x) \equiv -\frac{v'(x)}{x \cdot v''(x)} = -\frac{x' \cdot \phi}{x \cdot \phi'}$$

We can now calculate the elasticity of demand for contingent claims:

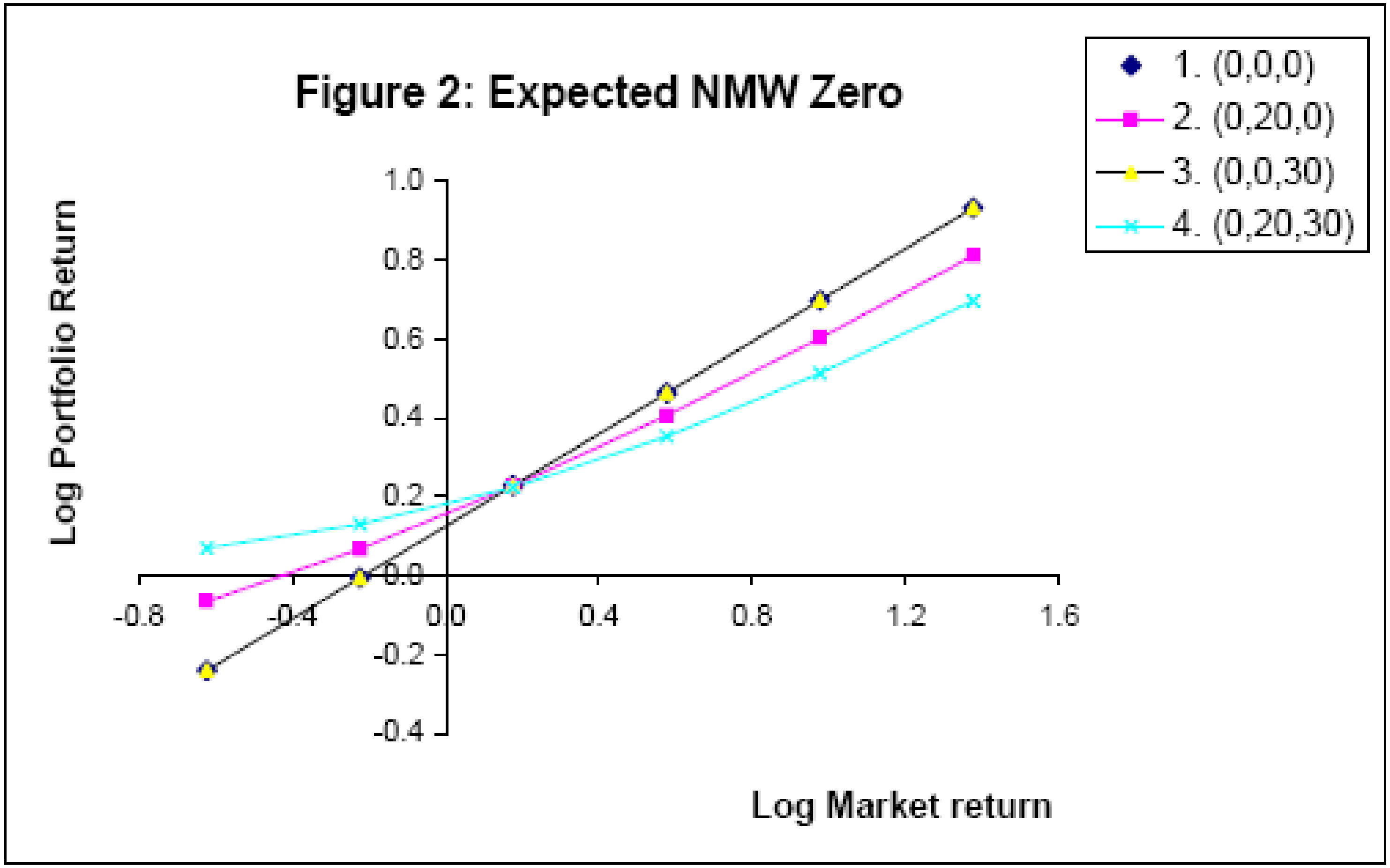
$$\frac{\partial \ln x}{\partial \ln R} = \left[\frac{\partial \ln x}{\partial \ln \phi}\right]\left[\frac{\partial \ln \phi}{\partial \ln R}\right] = a_v^{-1}(x(R)) \cdot \eta(\phi(R))$$

GBM $\Rightarrow$ elasticity of pricing kernel is constant.

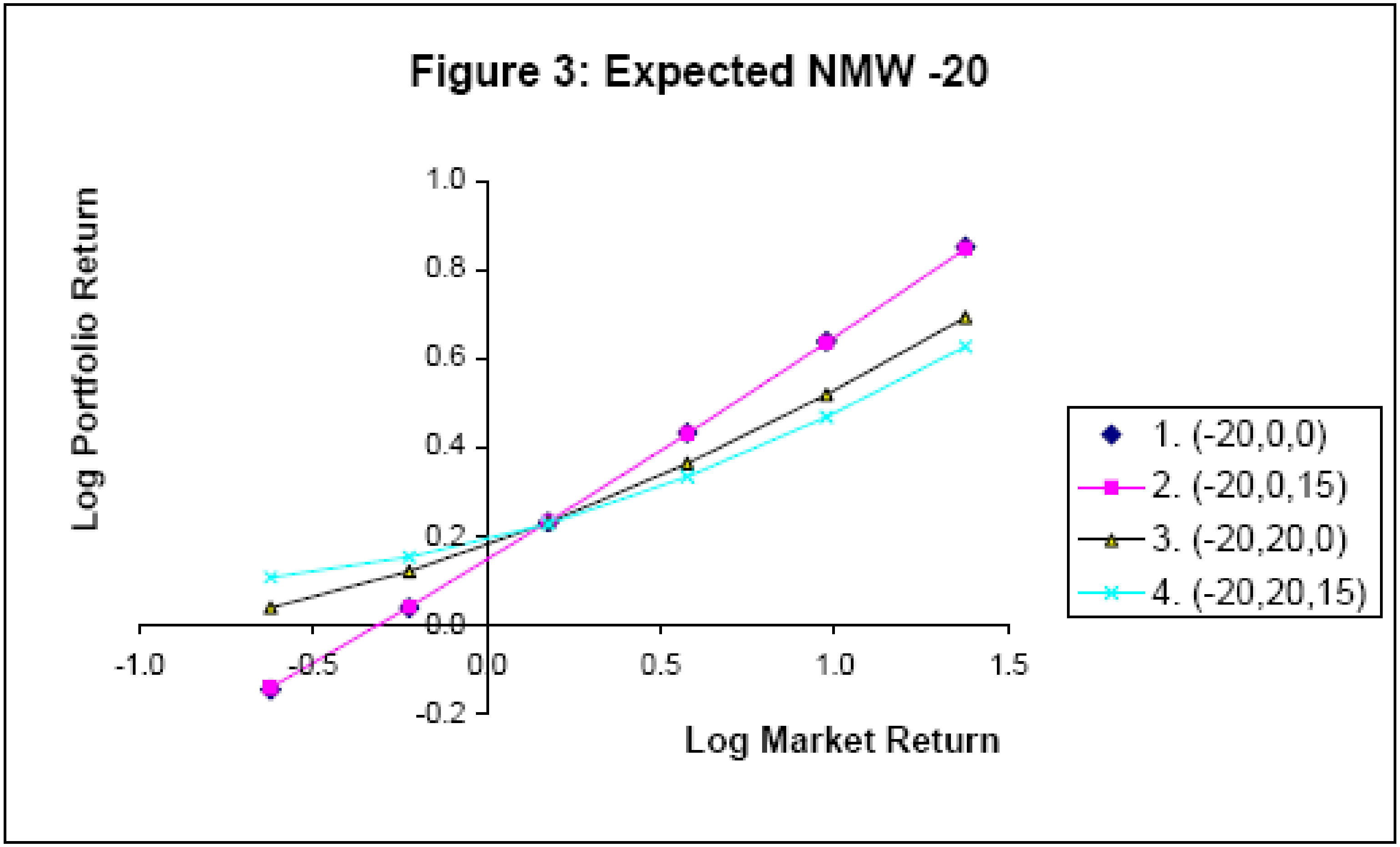
Thus, RRA of v determines demand for contingent claims.



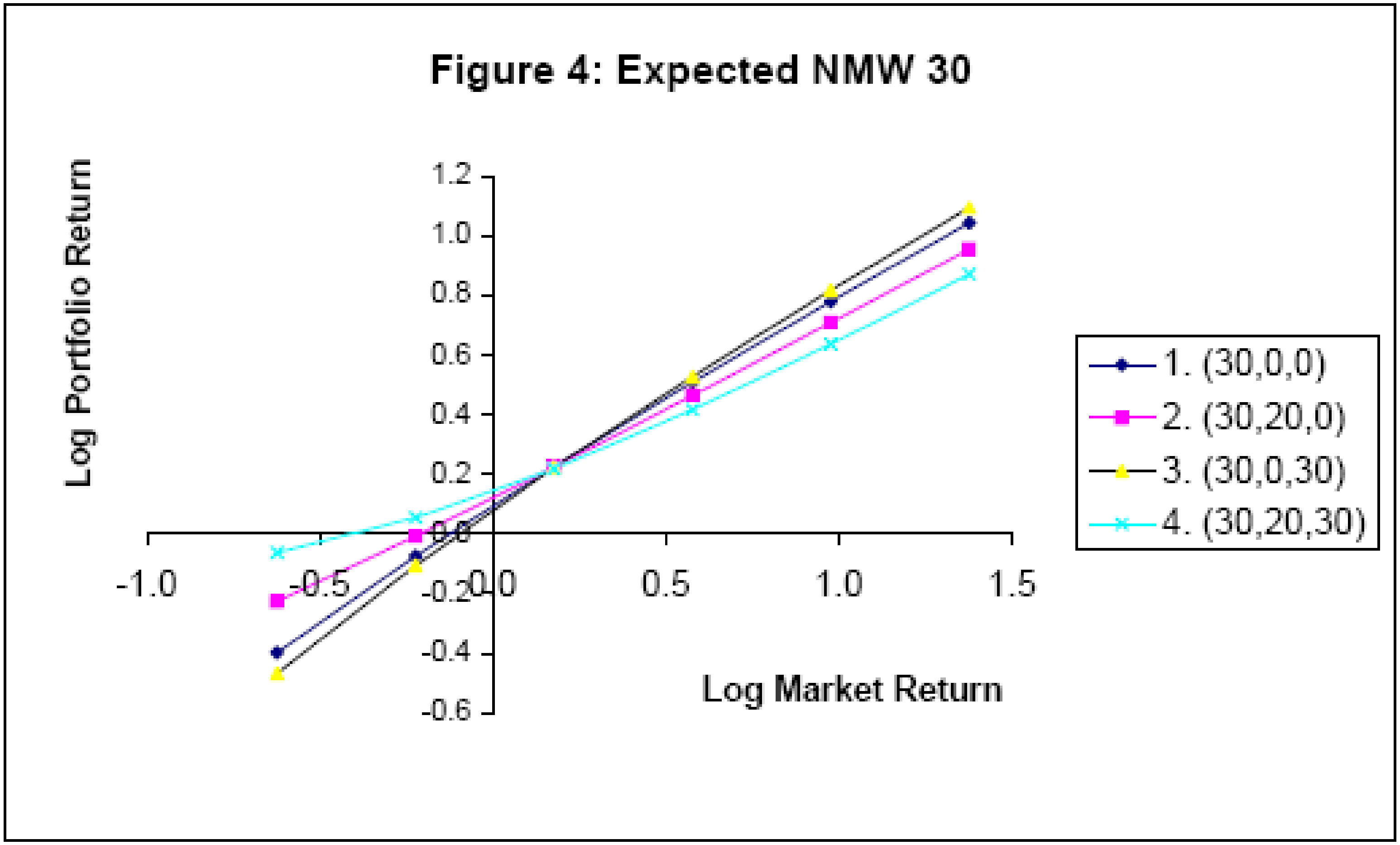
Effects of non-random NMW $\quad$ $1,2,3=(z,\sigma_{\varepsilon},\sigma_{y})$



z = 0   pure risk effects



z = -20   pure risk effects

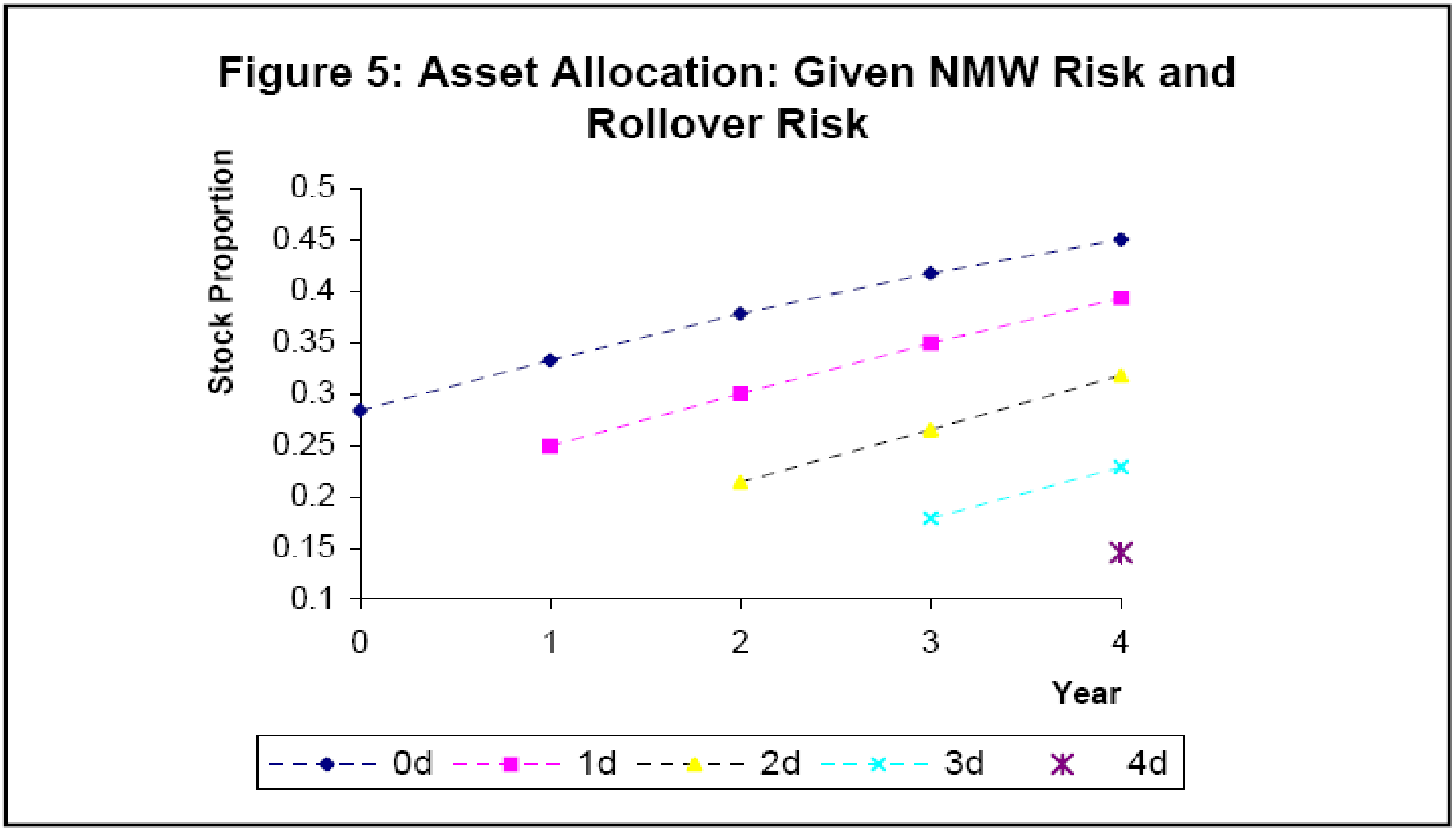


z = 30   pure risk effects

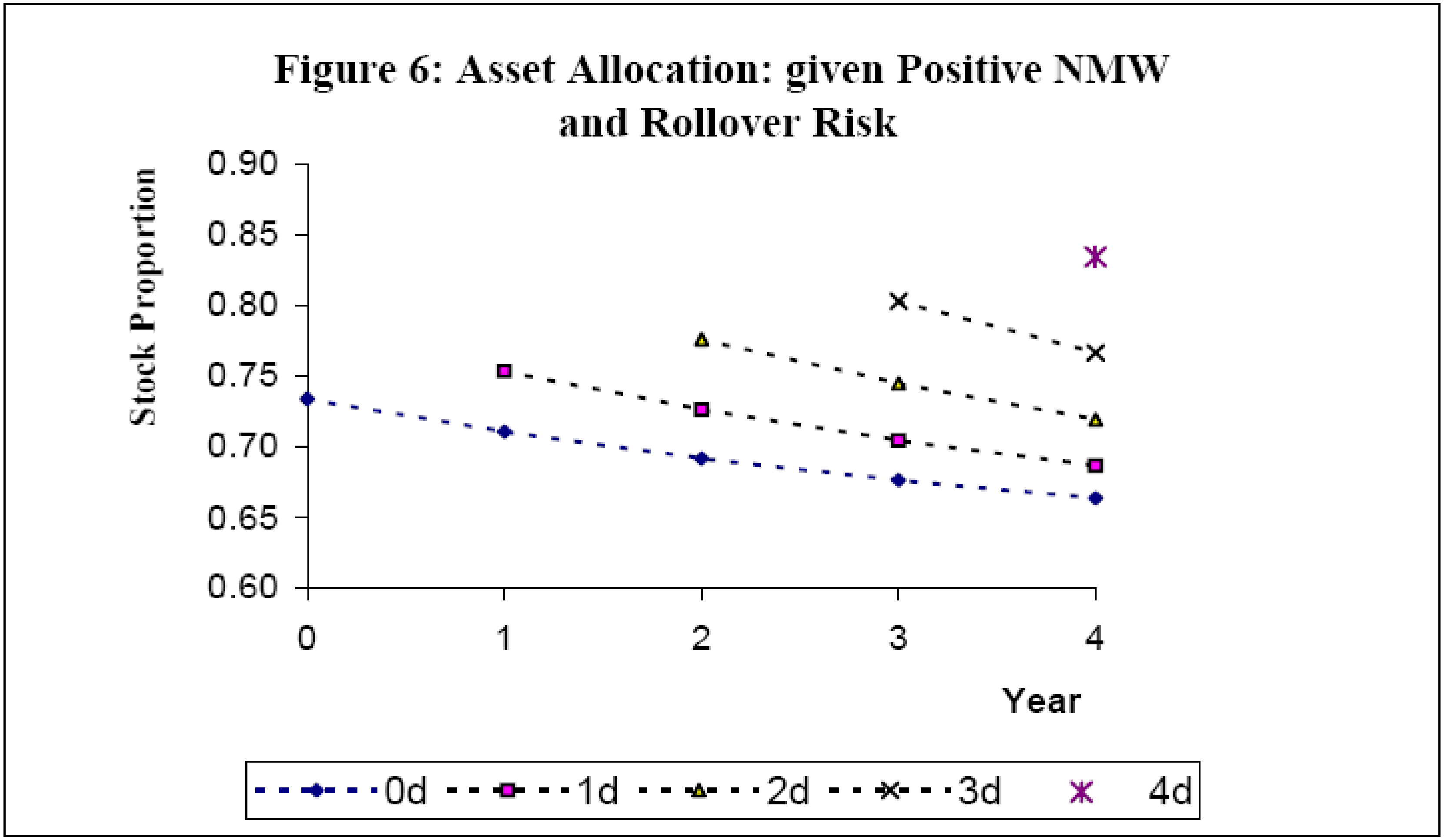
Stock Proportions: Year 0 and Year 4

					Year 4: state					
Case	z_0	σ_ε	σ_y	Year 0	0	1	2	3	4	Derived Utility
1	0	0	0	56	56	56	56	56	56	CRRA
2	0	0	0.3	56	56	56	56	56	56	CRRA
3	0	0.2	0	39	49	46	41	36	29	DRRA
4	0	0.2	0.3	28	45	39	32	23	15	DRRA
5	-20	0	0	49	52	51	49	48	46	DRRA
6	-20	0	0.15	47	52	51	49	47	45	DRRA
3	-20	0.2	0	30	43	38	33	27	20	DRRA
4	-20	0.2	0.15	23	39	33	26	18	11	DRRA
1	30	0	0	72	65	67	70	75	81	IRRA
2	30	0	0.3	73	66	67	72	77	83	IRRA
3	30	0.2	0	58	61	61	59	57	54	DRRA
4	30	0.2	0.3	56	60	59	57	53	47	DRRA

Year 4 state = number of ‘downticks’



Dynamic allocation strategy $z=0$



Dynamic allocation strategy $\varepsilon = 0$

CONCLUDING REMARKS

1. We model reaction to combined (additive) NMW risk and (multiplicative) rollover risk. Interaction of risks can be complex.

2. Interactive effect in case of $z > 0$ allows that risk aversion might be increasing or decreasing, or might switch. Particularly interesting case can yield U-shaped RRA for derived utility (Ait-Sahalia & Lo, 2000)

3. Correlations matter. Make real-world analyses more complicated.
(We study only the pure-risk effects here.)

4. We start with CRRA as basic preferences. We then examine observed market behavior (which is not CRRA). What of the dual question: if we observe CRRA, whence basic preferences?