# THE UTILITY PREMIUM

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Three papers:

Eeckhoudt & Schlesinger in Toulouse, summer 2004. Looking at the potential uses for the utility premium of Friedman and Savage (1948)

“Accident” happened after Louis left and the “Proper Place” paper followed trivially.

“Putting Risk in its Proper Place”
[*American Economic Review* 2006]

“On the Utility Premium of Friedman and Savage”

“A Good Sign for Multivariate Risk Taking”
[*Management Science* 2007? … with Beatrice Rey]

Gave us much insight into the utility premium

"The ability to define what may happen in the future and to choose among alternatives lies at the heart of contemporary societies."

(Peter Bernstein, 1998, *Against the Gods*)

Caveat: di Finetti probably did it first, but ... two classic papers might be considered the "start" of modern analysis of decision making under risk:

**Milton Friedman & Leonard J. Savage (1948)**
"The Utility Analysis of Choice Involving Risk" *Journal of Political Economy*

**John W. Pratt (1964)**
"Risk Aversion in the Small and in the Large" *Econometrica*

Also, Arrow (1965-71?) – Lecture notes on probability premium

**Friedman-Savage (1948)**

Utility Premium

Income Equivalent

Exp. Wealth – I.E.

**Pratt (1964)**

Certainty Equivalent

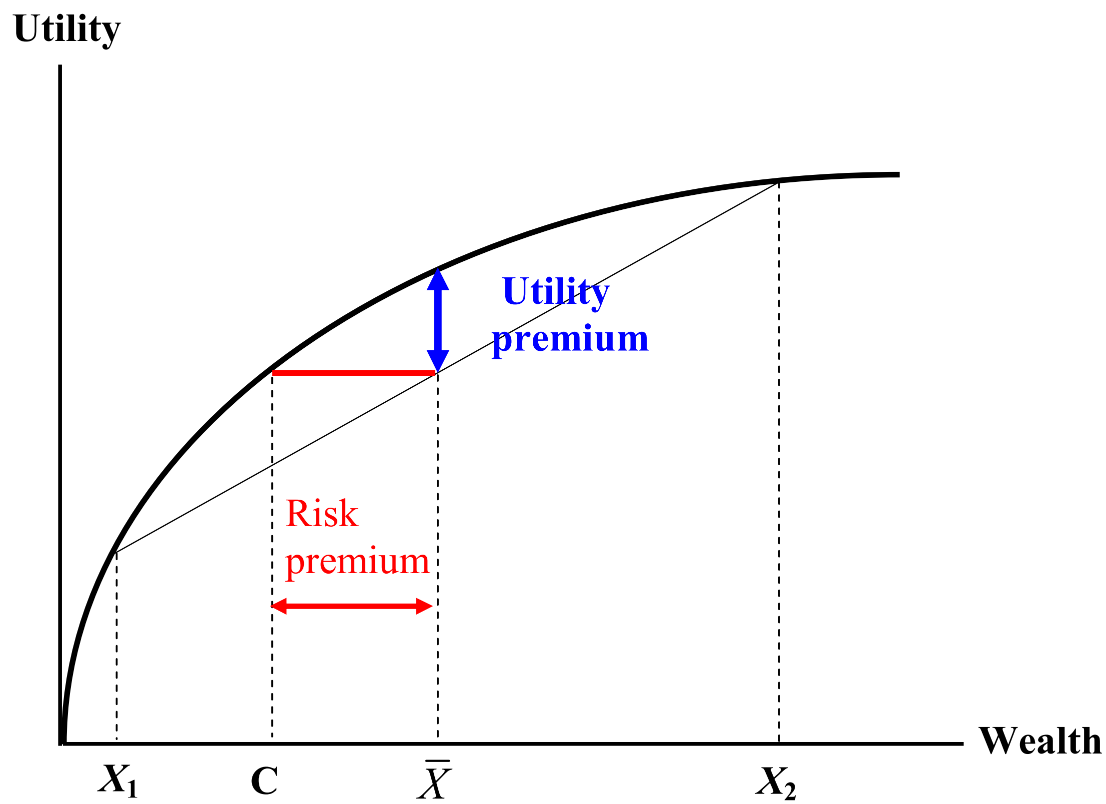
Risk Premium

***Local*** *+ Global measures*

Pratt's paper has become too famous
No need to cite Pratt when using measures of risk aversion.

Friedman-Savage has been all but forgotten.
NOTE: Even Pratt does not cite Friedman & Savage!

Unlike the RP, the UP is not interpersonally comparable. Also, no local measure.




The Utility Premium and the Risk Premium

# Outline of Presentation

Decompose Pratt's risk premium into two components

Measure of pain & WTP to remove each unit of pain

Sensitivity analysis of the UP to changes in the level of wealth

Analysis of UP and WTP "in the small"

Application to the demand for precautionary saving

Explaining "prudence" and "temperance" using the UP

Extending "prudence" and "temperance" to multivariate preferences

Let $\tilde{\varepsilon}$ denote a zero-mean random variable.

Utility premium: $v(w) \equiv u(w) - Eu(w+\tilde{\varepsilon}) = u(w) - u(w-\pi(w))$

= loss of utility **("pain")** associated with risk $\tilde{\varepsilon}$.

Very little research about the UP in the last 58 years!

**Hanson & Menezes (*WEJ*, 1971)**
"On a Neglected Aspect of the Theory of Risk Aversion"

Essentially ask: when is the UP decreasing in wealth?

$$v'(w) = u'(w) - Eu'(w+\tilde{\varepsilon}) < 0 \quad iff \quad u''' > 0.$$

Follows trivially from Jensen's inequality. $u'''> 0$ is Kimball's (1990) **"prudence"**

## Jia & Dyer (*Mgt. Science*, 1996)

Consider two risks with initial wealth $w_0$, $w_0 + \tilde{\varepsilon} \succ w_0 + \tilde{\delta}$ and ask:

When can we say that $w + \tilde{\varepsilon} \succ w + \tilde{\delta} \ \ \forall w$ ?

… or equivalently, that the UP for $\tilde{\delta}$ is always greater than the UP for $\tilde{\varepsilon}$.

Conclusions all trivial (by our use of the UP) & weak

1. Quadratic utility $u(w) = w - kw^2 \ \Rightarrow \ Eu(w + \tilde{\varepsilon}) = u(w) - k\,\mathrm{var}(\tilde{\varepsilon})$
   So that $v(w) = u(w) - [u(w) - k\,\mathrm{var}(\tilde{\varepsilon})] = k\,\mathrm{var}(\tilde{\varepsilon})$

2. CARA $v(w) = u(w) - u(w - \pi_i)$, $\pi_\delta > \pi_\varepsilon$ constant

3. Bell's one-switch $u(w) \equiv aw - be^{-cw} \Rightarrow v(w) = be^{-cw}[Ee^{-c\tilde{\varepsilon}} - 1]$

**<u>Decomposition:</u>** A tautology

$$\pi(w) = v(w) \times \frac{\pi(w)}{v(w)}$$

*Pain* x *WTP*

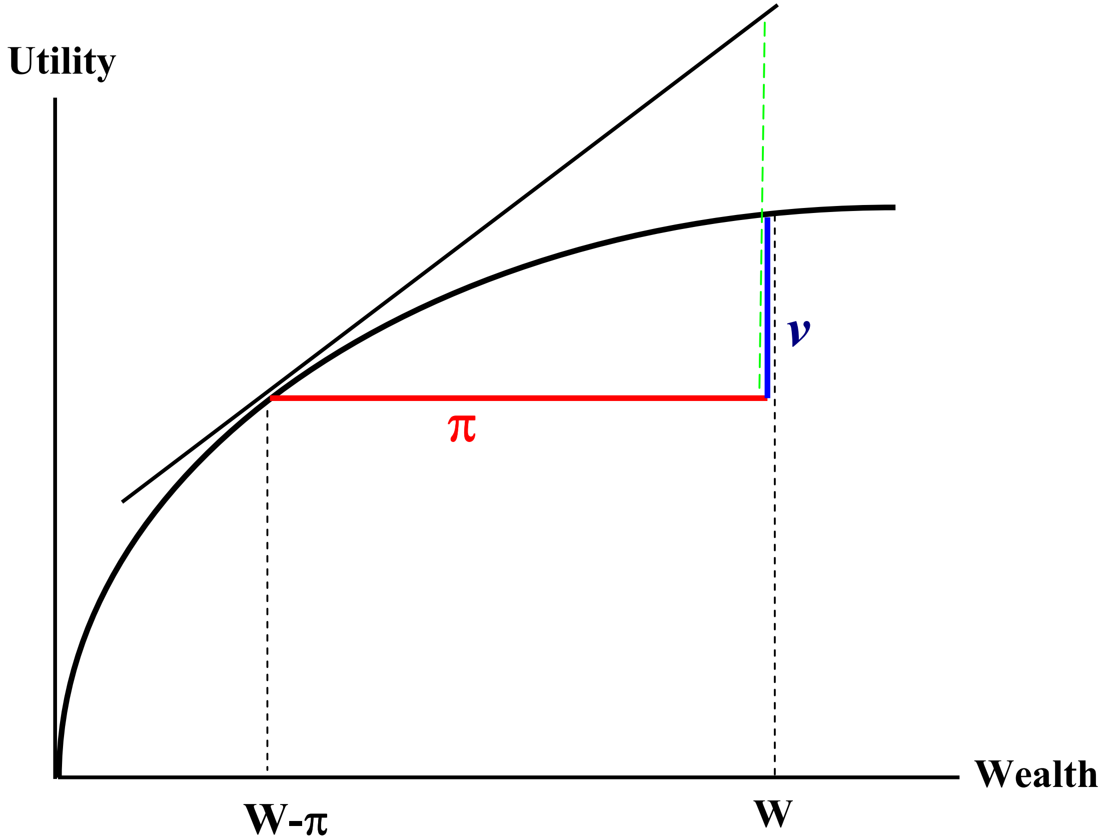
<u>Examples (as wealth increases)</u>

1. Quadratic $u(w) = w - kw^2$ $u''' = 0$ → ΔPain=0 → ΔWTP>0

2. CARA, $u''' > 0$ → ΔPain<0 → ΔWTP>0 (perfectly offsetting)

3.DARA $u''' > 0$ → ΔPain<0 → ?? ΔWTP>0 (less obvious)

$$WTP'(w) = \tfrac{1}{v^2}\{v\pi' - \pi[(u'(w) - (1-\pi')u'(w-\pi)]\}$$
$$= \tfrac{1}{v^2}\{[v - u'(w-\pi)\pi]\pi' + [u'(w-\pi) - u'(w)]\pi\}$$
$$\tfrac{1}{v^2}\ [\textbf{(negative)}(\text{negative}) + (\text{positive})] > 0.$$




$$u'(w)\pi < v < u'(w-\pi)]\pi$$

## In the small

Consider $t\tilde{\varepsilon}$ as $t \to 0^+$.

Pain (like r.a.) is a second order effect
[Segal & Spivak (1990)]

$$\frac{\partial v(w,t)}{\partial t}\Big|_{t=0} = E[u'(w+t\tilde{\varepsilon})\tilde{\varepsilon}] = 0 \quad \text{and} \quad \frac{\partial^2 v(w,t)}{\partial t^2}\Big|_{t=0} = -E[u''(w+t\tilde{\varepsilon})\tilde{\varepsilon}^2] > 0.$$

Plus, since $u'(w)\pi < v < u'(w-\pi)\pi$, we have

$$WTP = \frac{\pi}{v} \to \frac{1}{u'(w)} \quad \text{as} \quad t \to 0^+. \qquad (\text{€ / utility})$$

WTP is a first-order effect. (Thus r.a. is second-order due to *pain*.)

**Why bother? Who cares?**

Precautionary savings example with $\rho = r$.

[Kimball (1990), Leland (1968), Sandmo (1970)]

$$\underset{s}{Max}\, H(s) \equiv u(y-s) + \tfrac{1}{1+\rho} u(y+s(1+r))$$

FOC: $H'(s) = -u'(y-s) + \tfrac{1+r}{1+\rho} u'(y+s(1+r)) = 0 \quad \Rightarrow \quad s^* = 0$

Second period uncertain labor income

$$\underset{s}{Max}\, \hat{H}(s) \equiv u(y-s) + \tfrac{1}{1+\rho} Eu(y+\tilde{\varepsilon}+s(1+r))$$

$$\hat{H}'(s)\,|_{s=0} = -u'(y-s) + \tfrac{1+r}{1+\rho} Eu'(y+\tilde{\varepsilon}+s(1+r)) > 0 \;\; iff \;\; u''' > 0 \;\; [\Rightarrow \quad s^* > 0]$$

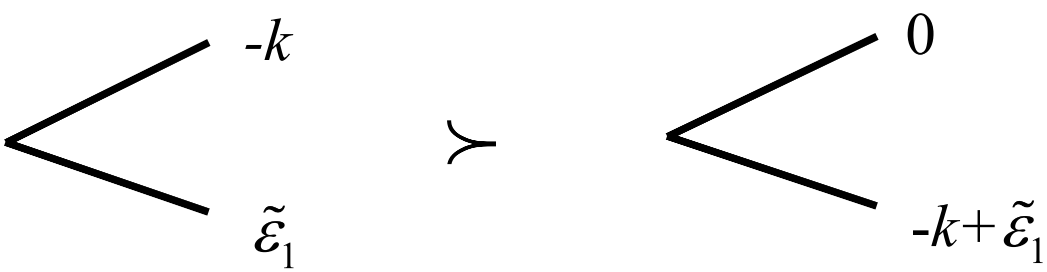
Thus, precautionary savings iff "**pain**" is decreasing in wealth. (Not DARA)

[We shift some wealth to 2nd period with the ε-risk to alleviate some of the pain.]

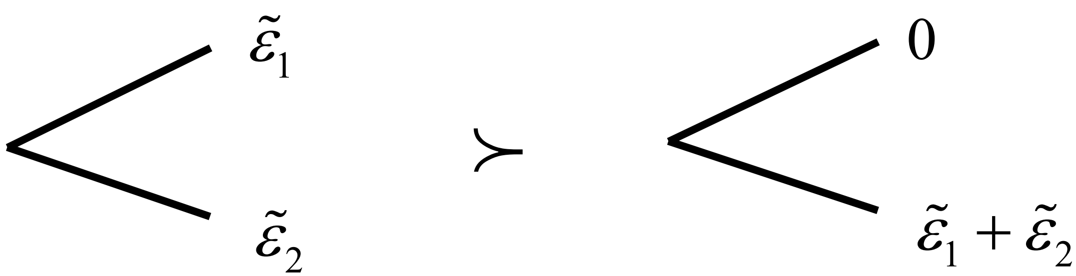
**(See UP paper for multiplicative risks)**

**<u>Lottery Preference and Risk Attitudes</u>** **("Disaggregating the harms")**

Define preferences as **<u>prudent</u>** if, for every $\boldsymbol{x}$ and zero-mean risk and $k > 0$




… **<u>temperate</u>** if, for every $\boldsymbol{x}$ and two independent, zero-mean risks
[Similar to Kimball's 1993 equivalence of Pratt & Zeckhauser's 1987 **Properness**]



Now define (only to coincide with the *AER* paper)

$$w_1(x) = -v(w) = Eu(x+\tilde{\varepsilon}_1) - u(x)$$

Remark: This is how Friedman and Savage (1948) originally defined the UP.

==================================================

$$w_1(x) = Eu(x+\tilde{\varepsilon}_1) - u(x) \le 0 \quad \text{iff} \quad u''(x) \le 0$$

$$w'_1(x) = Eu'(x+\tilde{\varepsilon}_1) - u'(x) \ge 0 \quad \text{iff} \quad u'''(x) \ge 0$$

$$w''_1(x) = Eu''(x+\tilde{\varepsilon}_1) - u''(x) \le 0 \quad \text{iff} \quad u^{iv}(x) \le 0$$

All follow trivially from Jensen's inequality.

## Prudence and utility:

Note that $w'_1(x) = Eu'(x+\tilde{\varepsilon}_1) - u'(x) \geq 0$ for all $x$ is equivalent to:

$$[Eu(x+\tilde{\varepsilon}_1) - u(x)] - [Eu(x-k+\tilde{\varepsilon}_1) - u(x-k)] \geq 0 \quad \forall k > 0$$

Or equivalently

$$\tfrac{1}{2}[Eu(x+\tilde{\varepsilon}_1) + u(x-k)] \geq \tfrac{1}{2}[Eu(x-k+\tilde{\varepsilon}_1) + u(x)]$$

**This is precisely our lottery-based definition of prudence, expressed within an EU framework!**

**With differentiable utility: Prudence** $\Leftrightarrow \; u''' > 0$

## **Temperance and utility:**

Define $w_2(x)$ as the "utility premium" for $w_1(x)$

$$w_2(x) \equiv Ew_1(x+\tilde{\varepsilon}_2) - w_1(x)$$

Thus $w_2(x) \le 0 \;\Leftrightarrow\; w_1(x)$ is concave $\;\Leftrightarrow\; u^{iv} \le 0$.

Expanding above, $w_2(x) \le 0$ is equivalent to

$$[Eu(x+\tilde{\varepsilon}_1+\tilde{\varepsilon}_2) - Eu(x+\tilde{\varepsilon}_2)] - [Eu(x+\tilde{\varepsilon}_1) - u(x)] \le 0$$

or equivalently

$$\tfrac{1}{2}[Eu(x+\tilde{\varepsilon}_1) + Eu(x+\tilde{\varepsilon}_2)] \ge \tfrac{1}{2}[Eu(x+\tilde{\varepsilon}_1+\tilde{\varepsilon}_2)] + u(x)]$$

**With differentiable utility: Temperance** $\Leftrightarrow$ $u^{iv} < 0$

## **Multiattribute Preferences**

Eisner & Strotz (*JPE*, 1961)

Modeled flight insurance (against death) in a state-dependent utility framework. They show how the sensitivity of the MU of wealth to a nonpecuniary variable matters in insurance choice.

Many examples since then.

For concreteness, we focus on wealth vs. health.

Let $x$ = *wealth* and $y$ = *health*, health an objective measure (e.g. longevity).

We assume that individuals are risk averse in each dimension separately.

## CORRELATION AVERSION

Richard (*Mgt. Science* 1975) (called it “multivariate risk aversion”)
Epstein & Tanny (*Canadian J. Econ*, 1980)

Let $c > 0$ and $k > 0$. All lottery branches have probability $p = ½$.

Preferences are **correlation averse** if, $\forall x, y$ and $\forall k > 0, c > 0$:

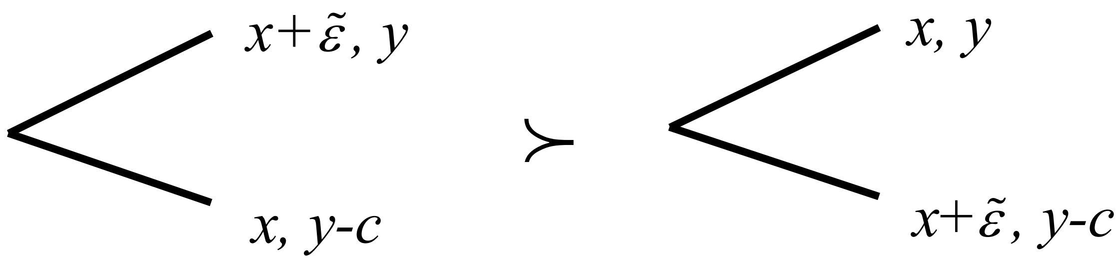
$$\left\langle \begin{array}{l} x\text{-}k,\ y \\ \\ x,\ y\text{-}c \end{array}\right. \succ \left\langle \begin{array}{l} x,\ y \\ \\ \mathrm{x}\text{-}k,\ y\text{-}c \end{array}\right.$$

Again, we prefer “disaggregating the harms.”

## CROSS PRUDENCE

Let $\tilde{\varepsilon}$ and $\tilde{\delta}$ be arbitrary, independent zero-mean random noise terms.

Cross prudence in health:




Again: prefer disaggregating the harms.

If we need to attach $\tilde{\varepsilon}$ to lottery branch, higher health helps mitigate the ill effects of risky wealth $\tilde{\varepsilon}$.

Cross prudent in wealth:

$$
\begin{tikzcd}[column sep=large]
 & x,\ y+\tilde{\delta} & & & x,\ y \\
\bullet \arrow[ur, dash] \arrow[dr, dash] & & \succ & \bullet \arrow[ur, dash] \arrow[dr, dash] & \\
 & x\text{-}k,\ y & & & x\text{-}k,\ y+\tilde{\delta}
\end{tikzcd}
$$

Again: prefer disaggregating the harms.

Higher wealth helps mitigate the ill effects of risky health $\tilde{\delta}$.

CROSS TEMPERANCE

Let $\tilde{\varepsilon}$ and $\tilde{\delta}$ be arbitrary, independent, zero-mean risks.
(Need not be i.i.d.)

$$\left\langle \begin{array}{l} x,\ y+\tilde{\delta} \\[2em] x+\tilde{\varepsilon},\ y \end{array}\right. \succ \left\langle \begin{array}{l} x,\ y \\[2em] x+\tilde{\varepsilon},\ y+\tilde{\delta} \end{array}\right.$$

Again: prefer disaggregating the harms.

If we need to attach $\tilde{\varepsilon}$ to lottery branch, prefer to attach it where there not already the risk $\tilde{\delta}$.

Term "temperance" coined by Kimball (1992)

## Relation to utility

$u(x,y) = u(wealth, health)$

Assume that preferences are monotonic and risk averse in each component: $u_1 > 0$, $u_2 > 0$, $u_{11} < 0$, and $u_{22} < 0$.

Might or might not have $u$ also concave $u_{11}u_{22} - (u_{12})^2 > 0$.

**Main Proposition**: *The following equivalences on preferences hold:*

*i) Correlation averse* $\Leftrightarrow$ $u_{12} < 0$

*ii) Cross prudent in health* $\Leftrightarrow$ $u_{112} > 0$

*iii) Cross prudent in wealth* $\Leftrightarrow$ $u_{122} > 0$

*iv) Cross temperate* $\Leftrightarrow$ $u_{1122} < 0$

Proof: We show (*ii*) here. Other cases are similar.

For a given $\tilde{\varepsilon}$, define $v(x, y) \equiv u(x, y) - Eu(x + \tilde{\varepsilon}, y)$.

Analog to the utility premium, since $y$ is fixed

Since $u_{11} < 0$, we have $v(x, y) > 0$.

Taking the derivative w.r.t. $y$, we obtain

$$v_2(x, y) \equiv u_2(x, y) - Eu_2(x + \tilde{\varepsilon}, y)$$

From Jensen's inequality

$$v_2(x, y) < 0 \;\; \textit{iff} \;\; u_2(x, y) \text{ convex in } x \;\; \textit{iff} \;\; u_{112}(x, y) > 0.$$

But $v_2 < 0$ *iff* $u(x, y) - Eu(x + \tilde{\varepsilon}, y) < u(x, y - c) - Eu(x + \tilde{\varepsilon}, y - c)$

Rearranging: $\frac{1}{2}[u(x, y - c) + Eu(x + \tilde{\varepsilon}, y)] > \frac{1}{2}[u(x, y) + Eu(x + \tilde{\varepsilon}, y - c)]$. ***QED***

**EXAMPLES** (with two-period reinterpretations of "lotteries")

Precautionary saving against risky health status (let $r = \rho = 0$)

$$\max U(s) \equiv u(x-s, y) + u(x+s, y)$$

$$\text{FOC} \quad -u_1(x-s, y) + u_1(x+s, y) = 0 \quad \Rightarrow \quad s^* = 0.$$

Now let health in period 2 be risky:
(Note: No wealth consequences of ill health are assumed.)

$$\max U(s) \equiv u(x-s, y) + Eu(x+s, y+\tilde{\delta})$$

From FOC, $s^* > 0$ if $Eu_1(x+s, y+\tilde{\delta}) > u_1(x+s, y)$.

This follows whenever $u_1$ is convex in $y$, i.e. $u_{122} > 0$.
[Thus, cross prudence in wealth yields this precautionary savings demand.]

Example: Allocating a financial risk over time.

Assume both cross temperance $u_{1122} < 0$ and that $u_{122}$ has a constant sign

[Either cross prudence in wealth or cross imprudence in wealth]

Must choose $\alpha$ before knowing realization of $\tilde{\varepsilon}$

$$\max U(\alpha) = Eu(x + \alpha\tilde{\varepsilon}, y) + Eu(x + (1-\alpha)\tilde{\varepsilon}, y) \quad \text{FOC} \Rightarrow \quad \alpha^* = \tfrac{1}{2}$$

Now let health status be risky in second period.

$$\max U(\alpha) = Eu(x + \alpha\tilde{\varepsilon}, y) + Eu(x + (1-\alpha)\tilde{\varepsilon}, y + \tilde{\delta})$$

$$\alpha^* > \tfrac{1}{2} \ \text{ if } \ U'(\tfrac{1}{2}) = Eu_1(x + \tfrac{1}{2}\tilde{\varepsilon}, y)\tilde{\varepsilon} - Eu_1(x + \tfrac{1}{2}\tilde{\varepsilon}, y + \tilde{\delta})\tilde{\varepsilon} > 0$$

Holds if $H(x,y) \equiv Eu_1(x+\frac{1}{2}\tilde{\varepsilon},y)\tilde{\varepsilon}$ is concave in $y$, $H_{22} < 0$

Since $\operatorname{sgn}[u_{122}]$ is constant and $u_{1122} < 0$, we obtain,

$$H_{22}(x,y) = \int_{-\infty}^{0} u_{122}(x+\tfrac{1}{2}\varepsilon,y)\varepsilon dF(\varepsilon) + \int_{0}^{+\infty} u_{122}(x+\tfrac{1}{2}\varepsilon,y)\varepsilon dF(\varepsilon)$$

$$< \int_{-\infty}^{0} u_{122}(x,y)\varepsilon dF(\varepsilon) + \int_{0}^{+\infty} u_{122}(x,y)\varepsilon dF(\varepsilon) = 0.$$

| | | | |
|---|---|---|---|
| $u_{122} > 0$ | less negative | + | more positive |
| $u_{122} < 0$ | more positive | + | less negative |

Thus, the cross temperate individual will accept more than half of the $\tilde{\varepsilon}$-risk in the first period.

## CONCLUDING REMARKS

UP important part of risky-decision analysis
precautionary savings, 2nd-order effect in the small

Simple 50-50 lottery preferences is amenable to experimentation.

Necessary & sufficient conditions to sign cross derivatives.

Provide new examples of applications.

*We make no normative claims about these signs***.**
(Signing the cross derivatives has strong implications.)

E.g. Simple case of correlation aversion:

Viscusi & Evans (AER, 1990) provide empirical data to support $u_{12} > 0$
Evans & Viscusi (Re. Stat., 1991) find opposite result $u_{12} < 0$.