On systematic mortality risk

and

quadratic hedging with mortality derivatives

University of Ulm, December 12, 2006

Thomas Møller, PFA Pension, Copenhagen
(www.math.ku.dk/~tmoller)

Outline

1. Introduction

2. Systematic mortality risk
   (With M. Dahl, IME, 2006)

3. Hedging with mortality derivatives
   (With M. Dahl and M. Melchior, work in progress)

   - Theory and modeling
   - Numerical results

Since 2003, I work in PFA Pension
– Danish life insurance company/pension fund
– Mutual company
– Balance: approximately 27 billion euro
– participating life insurance contracts (defined contributions)

Background: until 2003, Assistant professor,
Laboratory of Actuarial Math, Univ. Cph

Chief Analyst, Actuarial Innovation
– market-valuation of liabilities
– new savings products
– actuarial support for risk-management and investment depts
– actuarial research & supervision

Brief motivation – Systematic mortality risk

- Large improvements in the mortality in many countries during the last years
- Risk for life insurers with (guaranteed) annuities (mortality tables were not conservative enough!)
- Future mortality is difficult to predict (unpredictable!)
- A new market for mortality derivatives is appearing (mortality/survivor swaps, longevity bonds etc)

Necessary to model e.g. the mortality intensity as a stochastic process

Some recent literature on systematic mortality risk

Marocco/Pitacco (1998)
Olivieri/Pitacco (2002)

Milevsky/Promislow (2001)

Dahl (2004)
Dahl/Møller (2006)

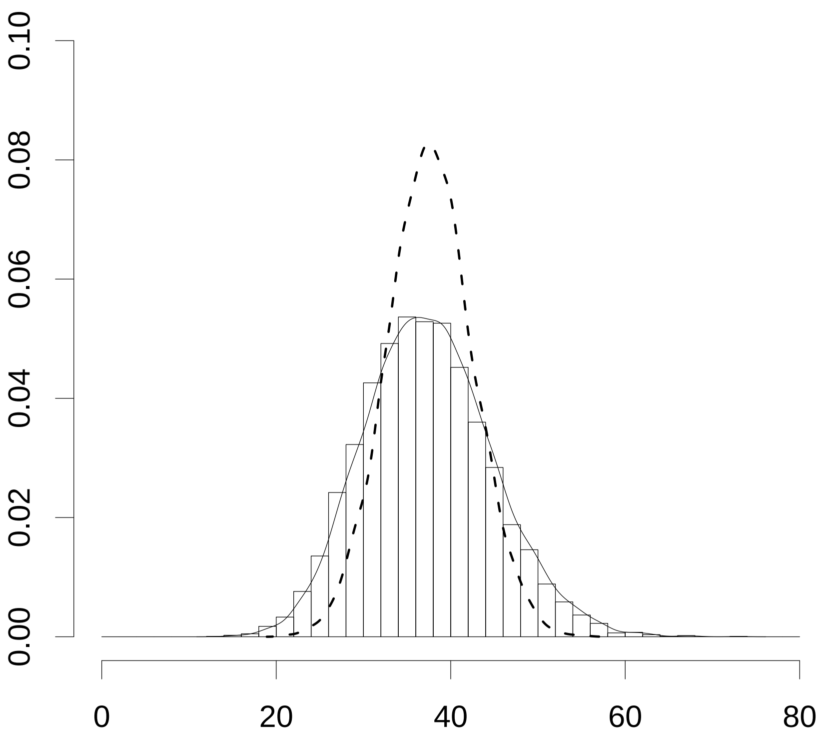
Cairns, Blake and Dowd (2004)

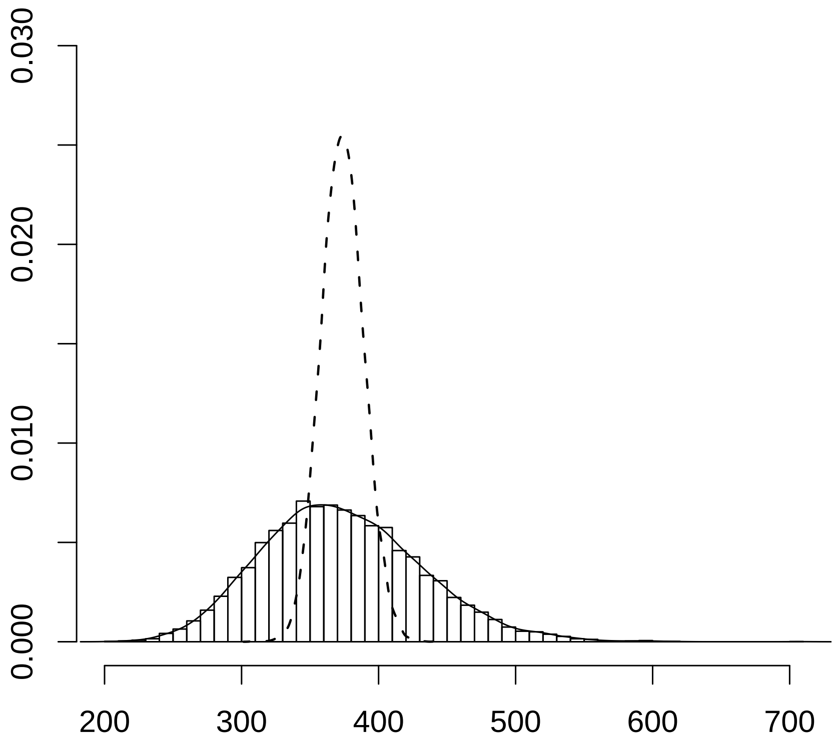
Miltersen/Persson (2006)

Biffis and Millossovich (2006)

Long term simulation of number of survivors

Example: age 30, simulate number of survivors at age 85

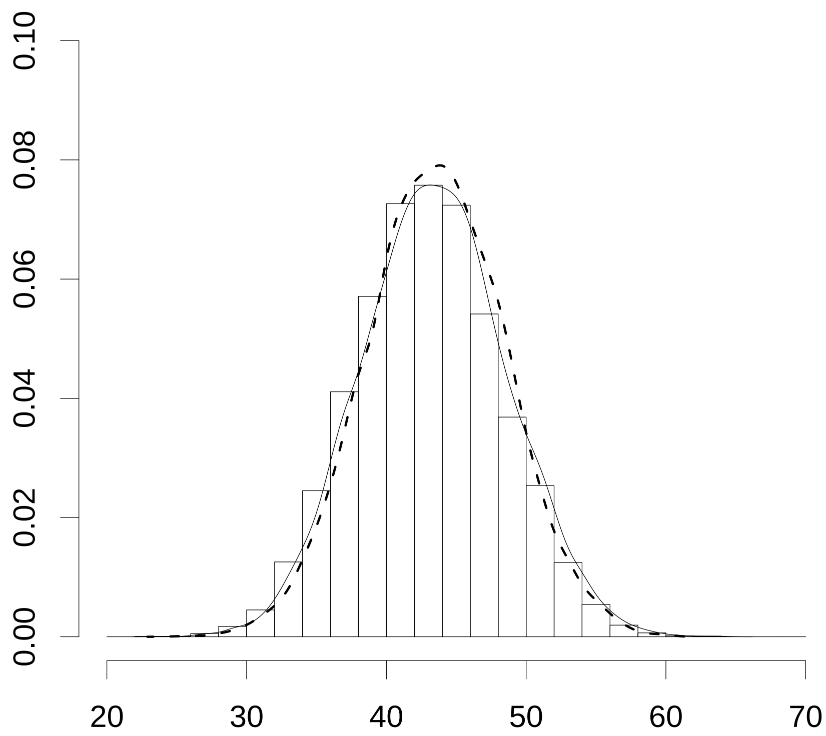


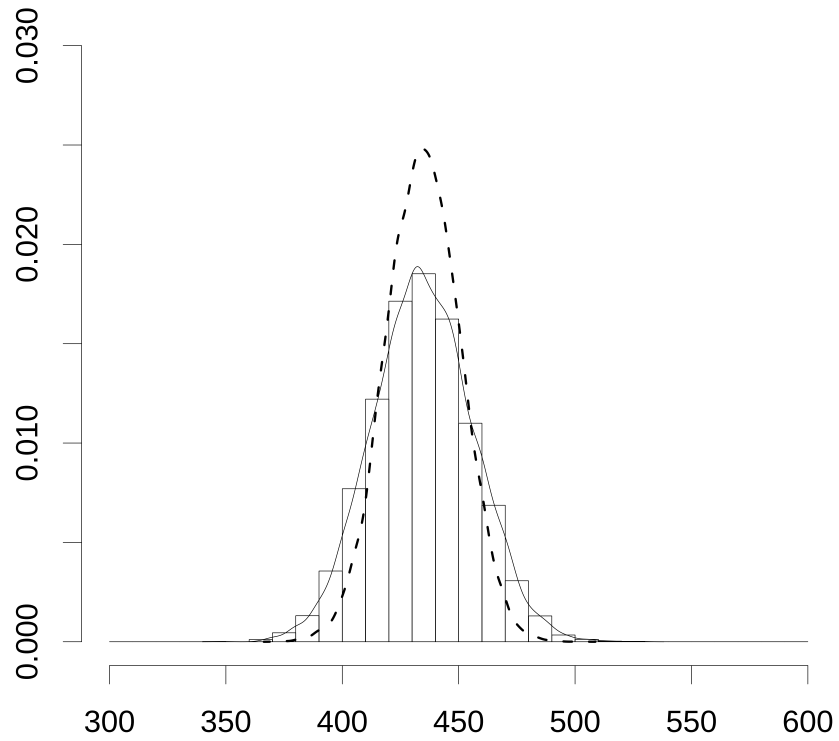


100 policy-holders	1.000 policy-holders
Expected no. of survivors: 37	Expected no. of survivors: 374
Std.dev without systematic risk: 4.9	Std.dev without systematic risk: 15.4
Std.dev with systematic risk: 7.3	Std.dev with systematic risk: 57.7

Simulation for a portfolio of retired

Example: start age 75, simulate number of survivors at age 85





100 policy-holders	1.000 policy-holders
Expected no. of survivors: 43.5	Expected no. of survivors: 435
Std.dev without systematic risk: 4.9	Std.dev without systematic risk: 15.8
Std.dev with systematic risk: 5.2	Std.dev with systematic risk: 21.5

The mortality intensity is a stochastic process
(joint work with M. Dahl)

Known at time 0:
$\mu^{\circ}(x+t)$ is mortality intensity "today" at all ages $x+t$

Unknown at time 0:
$\zeta(t,x)$ is relative change in the mortality from 0 to t, age x

Mortality intensity:
$\mu(x,t) = \mu^{\circ}(x+t)\zeta(x,t)$
(In general, a stochastic process)

True survival probability from t to T given information $\mathcal{I}(t)$:

$$\mathcal{S}(x,t,T) = E^{P}\left[e^{-\int_t^T \mu(x,\tau)d\tau} \middle| \mathcal{I}(t) \right]$$

A specific model:
Time-inhomogeneous CIR model known from finance:

$$d\zeta(x,t) = (\gamma(x,t) - \delta(x,t)\zeta(x,t))dt + \sigma(x,t)\sqrt{\zeta(x,t)}dW^{\mu}(t)$$

Proposition (Affine mortality structure, Dahl, 2004)
The survival probability $\mathcal{S}(x,t,T)$ is

$$\mathcal{S}(x,t,T) = e^{A^{\mu}(x,t,T) - B^{\mu}(x,t,T)\mu(x,t)}$$

where

$$\frac{\partial}{\partial t}B^{\mu}(x,t,T) = \delta^{\mu}(x,t)B^{\mu}(x,t,T) + \frac{1}{2}(\sigma^{\mu}(x,t))^2(B^{\mu}(x,t,T))^2 - 1$$

$$\frac{\partial}{\partial t}A^{\mu}(x,t,T) = \gamma^{\mu}(x,t)B^{\mu}(x,t,T)$$

with $B^{\mu}(x,T,T) = 0$ and $A^{\mu}(x,T,T) = 0$

Martingale: $\mathcal{S}^{M}(x,t,T) = E^{P}\left[e^{-\int_0^T \mu(x,\tau)d\tau} \middle| \mathcal{I}(t) \right]$

Forward mortality intensity

$$f^{\mu}(x,t,T) = -\frac{\partial}{\partial T}\log \mathcal{S}(x,t,T) = \mu(x,t)\frac{\partial}{\partial T}B^{\mu}(x,t,T) - \frac{\partial}{\partial T}A^{\mu}(x,t,T)$$

Survival probability

$$\mathcal{S}(x,t,T) = e^{-\int_t^T f^{\mu}(x,t,u)du} \neq e^{-\int_t^T \mu(x,u)du}$$

Change of measure for mortality and financial market

Equivalent measure Q

Financial market

Standard affine model for short rate:

$$dr(t) = \left(\gamma^{r,\alpha} - \delta^{r,\alpha} r(t)\right) dt + \sqrt{\gamma^{r,\sigma} + \delta^{r,\sigma} r(t)}\, dW^{r}(t)$$

Change of measure for mortality and financial market

Equivalent measure $\frac{dQ}{dP} = \Lambda(T)$ via

$$d\Lambda(t) = \Lambda(t-)\Big(h^r(t)dW^r(t) + h^\mu(t)dW^\mu(t) + g(t)dM(x,t)\Big)$$

Require affine under Q. Zero coupon bond prizes

$$P(t,T) = e^{A^r(t,T) - B^r(t,T)r(t)}$$

where $A^r(t,T)$ and $B^r(t,T)$ solve

$$\frac{\partial}{\partial t}B^r(t,T) = \delta^{r,\alpha,Q}B^r(t,T) + \frac{1}{2}\delta^{r,\sigma}(B^r(t,T))^2 - 1$$
$$\frac{\partial}{\partial t}A^r(t,T) = \gamma^{r,\alpha,Q}B^r(t,T) - \frac{1}{2}\gamma^{r,\sigma}(B^r(t,T))^2$$

with $B^r(T,T) = 0$ and $A^r(T,T) = 0$

Two portfolios of insured lives

$T_{j,1}, \ldots, T_{j,n}$ are i.i.d. given ζ_j with

$$P(T_{j,1} > t | \mathcal{I}(T)) = e^{-\int_0^t \mu_j(x,s)ds}, \qquad j = 1, 2$$

(1: own pf, 2: other pf)

Counting processes and martingales

$$N_j(x,t) = \sum_{i=1}^{n} 1_{(T_{j,i} \leq t)}$$

$$M_j(x,t) = N_j(x,t) - \int_0^t (n - N_j(x,u-))\mu_j(x,u)du$$

Insurance payment process (Benefits – premiums on pf 1)

$$dA(t) = (n - N_1(x,\overline{T}))\Delta A_0(\overline{T})d1_{(t \geq \overline{T})} + a_0(t)(n - N_1(x,t))dt + a_1(t)dN_1(x,t)$$

(a_i, A_0 deterministic functions)

Modeling of the mortality in two portfolios

$$d\zeta_j(x,t) = (\gamma_j(x,t) - \delta_j(x,t)\zeta_j(x,t))dt + \sigma_j(x,t)\sqrt{\zeta_j(x,t)}dW^\mu(t)$$

Here:
W^μ two-dimensional Brownian motion and $\sigma_j(x,t) \in \mathbf{R}^2$

Possibility for correlation between systematic mortality risk in the two portfolios

Simple example: $\begin{pmatrix} \sigma_1(x,t) \\ \sigma_2(x,t) \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ 0 & \sigma_{22} \end{pmatrix}$

First: focus on risk in portfolio 1. Hedge with bonds

Later: Hedge with bonds and mortality swaps

Q-martingale:

$$
\begin{aligned}
V^*(t) &= E^Q\left[\int_{[0,T]} e^{-\int_0^\tau r(u)du} dA(\tau) \middle| \mathcal{F}(t)\right] \\
&= \int_{[0,t]} e^{-\int_0^\tau r(u)du} dA(\tau) + e^{-\int_0^t r_u du}\, \widetilde{V}^Q(t)
\end{aligned}
$$

Here, the market reserve is

$$
\begin{aligned}
\widetilde{V}^Q(t) &= E^Q\left[\int_{(t,T]} e^{-\int_t^\tau r(u)du} dA(\tau) \middle| \mathcal{F}(t)\right] \\
&= (n - N_1(x,t)) V^Q(t, r(t), \mu_1(x,t))
\end{aligned}
$$

where

$$
\begin{aligned}
V^Q(t, r(t), \mu_1(x,t)) = &\int_t^T P(t,\tau) \mathcal{S}_1^Q(x,t,\tau) \Big(a_0(\tau) + a_1(\tau) f^{\mu_1, Q}(x,t,\tau) \Big) d\tau \\
&+ P(t,\overline{T}) \mathcal{S}_1^Q(x,t,\overline{T}) \Delta A_0(\overline{T}) 1_{(t<\overline{T})}
\end{aligned}
$$

Risk-minimization
(Föllmer/Sondermann, Schweizer)

Here: with payment streams (Møller, 2001)

Market: savings account and long zero coupon bond

Discounted price processes: $X(t) = P^*(t, T)$, $Y(t) = 1$

Trading strategy: Process $\varphi = (\xi, \eta)$ with ξ predictable
(+ technical conditions)

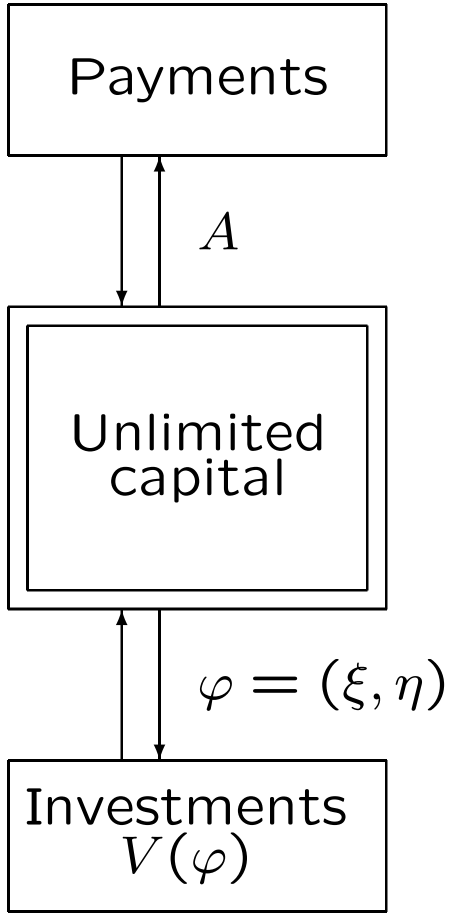
Value process: $V(t, \varphi) := \xi(t)X(t) + \eta(t)$

Payment process $A = (A(t))_{0 \le t \le T}$ square integrable
$A(t) - A(s)$ is amount paid by insurer during $(s, t]$

Cost process
$C(t, \varphi) = V(t, \varphi) - \int_0^t \xi(u)\, dX(u) + A^*(t)$

Risk process
$R(t, \varphi) = E^Q \left[(C(T, \varphi) - C(t, \varphi))^2 \middle| \mathcal{F}(t) \right]$

Criterion of risk-minimization
Minimize $R(t, \varphi)$ over φ for all t



Terminal cost
$C(T, \varphi) = V(T, \varphi) - \int_0^T \xi(u)\, dX(u) + A^*(T)$
$\rightsquigarrow$ V value after payments. Fix $V(T, \varphi) = 0$

Kunita-Watanabe decomposition:

$$V^*(t) := E^Q[A^*(T) \mid \mathcal{F}(t)] = V^*(0) + \int_0^t \xi^{A,Q}(u)\, dX(u) + L^{A,Q}(t)$$

where

- $\xi^{A,Q}$ is predictable
- $L^{A,Q}$ is a square integrable martingale
- X and $L^{A,Q}$ are orthogonal

Theorem.

$\exists!$ risk-minimizing strategy $\varphi = (\xi, \eta)$ with $V(T, \varphi) = 0$:

$$\begin{aligned} \xi(t) &= \xi^{A,Q}(t) \\ \eta(t) &= V^*(t) - A^*(t) - \xi^{A,Q}(t) X(t) \end{aligned}$$

The minimum risk process

$$R(t, \varphi) = E^Q \left[(L^{A,Q}(T) - L^{A,Q}(t))^2 \middle| \mathcal{F}(t) \right]$$

Intrinsic value process:

$$dV^{*,Q}(t) = \nu_1^{V,Q}(t)dM_1^Q(x,t) + \eta_1^{V,Q}(t)dW^{r,Q}(t) + \rho_1^{V,Q}(t)dW^{\mu,Q}(t)$$

where

$$\begin{aligned}
\nu_1^{V,Q}(t) &= B(t)^{-1}a^d(t) - \widetilde{V}_p^{*,Q}(t) \\
\eta_1^{V,Q}(t) &= \sqrt{\gamma^{r,\sigma}}\frac{\partial}{\partial r}\widetilde{V}^{*,Q}(t) \\
\rho_{1,j}^{V,Q}(t) &= \sigma_{1,j}^{\mu}(x,t)\sqrt{\mu_1(x,t)}\frac{\partial}{\partial \mu_1}\widetilde{V}^{*,Q}(t)
\end{aligned}$$

Risk-minimizing strategy determined from Galtchouk-Kunita-Watanabe decomposition:

$$dV^*(t) = \xi^{A,Q}(t)dX(t) + dL^{A,Q}(t)$$

Risk-minimizing strategy in a pure bond market

$$(\xi_B^*(t), \eta_B^*(t)) = (\xi^{A,Q}(t), \widetilde{V}^{*,Q}(t) - \xi^{A,Q}(t)P^*(t,T))$$

where

$$\xi^{A,Q}(t) = \frac{\eta_1^{V,Q}(t)}{-\sqrt{\gamma^{r\sigma}}B^r(t,T)P^*(t,T)}$$

The unhedgeable risk

$$dL^Q(t) = \nu_1^{V,Q}(\tau)dM_1^Q(x,\tau) + \rho_{1,1}^{V,Q}(\tau)dW_1^{\mu,Q}(\tau) + \rho_{1,2}^{V,Q}(\tau)dW_2^{\mu,Q}(\tau)$$

(See Dahl/Møller (2006))

Sources of risk from GKW-decomposition:

$$dV^*(t) \;=\; \xi^{A,Q}(t)dX(t) + \nu^Q(t)dM_1^Q(t) + \rho_1^{V,Q}(t)dW^{\mu,Q}(t)$$

Financial risk: $\xi^{A,Q}dX$

Unsystematic mortality risk: $\nu^Q dM_1^Q$

Systematic mortality risk: $\rho_1^{V,Q}dW^{\mu,Q}$

Properties of the optimal strategy: $\xi = \xi^{A,Q}$

✔ eliminates the financial risk

✗ is unable to deal with other risks

Extending the market with mortality swaps
(joint work with M. Dahl and M. Melchior)

Underlying payment processes:

$$dA_j^{\text{swap}}(x,t) = (n_j - N_j(x,t))dt - n_j \cdot {}_tp_x^j dt$$

(Defined for portfolios $j = 1, 2$)

Traded price process:

$$Z_j^{*,Q}(x,t) = E^Q\left[\int_0^T e^{-\int_0^\tau r(u)du} dA_j^{\text{swap}}(x,\tau) \middle| \mathcal{F}(t)\right]$$

We assume this process is traded on extended market (B^*, P^*, Z_j^*)

$j = 1$: same portfolio (same systematic and unsystematic risk)
$j = 2$: another portfolio (systematic risk correlated)

Motivation/idea

- Mortality swaps are available in the reinsurance markets
- The mortality swap contains systematic and unsystematic risk
- If we use Z_1^*, we hedge with 1 process driven by 3 sources of risk $(M_1, W^{\mu,1}, W^{\mu,2})$
- Can use this process to "balance" the systematic and unsystematic risks in the insurance portfolio
- Using a swap on another portfolio introduces a new unsystematic risk M_2, but eliminates part of the systematic risk

Dynamics for the traded process

$$dZ_1^{*,Q}(t) = \nu_1^{Z,Q}(t)dM_1^Q(x,t) + \eta_1^{Z,Q}(t)dW^{r,Q}(t) + \rho_1^{Z,Q}(t)dW^{\mu,Q}(t)$$

where

$$\begin{aligned}
\nu_1^{Z,Q}(t) &= -\int_t^T P^*(t,\tau)\mathcal{S}_1^Q(x,t,\tau)d\tau \\
\eta_1^{Z,Q}(t) &= -\sqrt{\gamma^{r\sigma}}(n_1 - N_1(x,t))\int_t^T B^r(t,\tau)P^*(t,\tau)\mathcal{S}_1^Q(x,t,\tau)d\tau \\
&\quad + \sqrt{\gamma^{r\sigma}}\int_t^T B^r(t,\tau)P^*(t,\tau)\,{}_\tau p_x^1\, n_1\, d\tau \\
\rho_{1,j}^{Z,Q}(t) &= -\sigma_{1,j}^{\mu}(x,t)\sqrt{\mu_1(x,t)}(n_1 - N_1(x,t))(1+g_1(t)) \\
&\quad \times \int_t^T B_1^{\mu,Q}(t,\tau)P^*(t,\tau)\mathcal{S}_1^Q(x,t,\tau)d\tau
\end{aligned}$$

Useful for finding the risk-minimzing strategy

GKW-decomposition of $V^{*,Q}$ in the extended market (B, P, Z_1)

$$dV^{*,Q}(t) = \xi_1^Q(t)dP^*(t,T) + \vartheta_1^Q(t)dZ_1^{*,Q}(x,t) + dL_1^Q(t)$$

where

$$\begin{aligned} dL^Q(t) &= \Big(\nu_1^{V,Q}(t) - \vartheta_1^Q(t)\nu_1^{Z,Q}(t)\Big)dM_1^Q(x,t) \\ &\quad + \Big(\rho_{1,1}^{V,Q}(t) - \vartheta_1^Q(t)\rho_{1,1}^{Z,Q}(t)\Big)dW_1^{\mu,Q}(t) \\ &\quad + \Big(\rho_{1,2}^{V,Q}(t) - \vartheta_1^Q(t)\rho_{1,2}^{Z,Q}(t)\Big)dW_2^{\mu,Q}(t) \end{aligned}$$

and

$$\begin{aligned} \xi_1^Q(t) &= \frac{\eta_1^{V,Q}(t) - \vartheta_1^Q(t)\eta_1^{Z,Q}(t)}{-\sqrt{\gamma^{r,\sigma}}B^r(t,T)P^*(t,T)} \\ \vartheta_1^Q(t) &= \frac{\nu_1^{V,Q}(t) + \rho_{1,1}^{V,Q}(t)(\kappa_{1,1}^Q(t))^{-1}\rho_{1,2}^{V,Q}(t)(\kappa_{1,2}^Q(t))^{-1}}{\nu_1^{Z,Q}(t) + \rho_{1,1}^{Z,Q}(t)(\kappa_{1,1}^Q(t))^{-1} + \rho_{1,2}^{Z,Q}(t)(\kappa_{1,2}^Q(t))^{-1}} \end{aligned}$$

Here: $\kappa_{1,j}^Q(t) = \frac{\nu_1^{Z,Q}(t)\lambda_1^Q(x,t)}{\rho_{1,j}^{Z,Q}(t)}$

Interpretation:

Optimal number of swaps: The strategy balances the three sources of risk: the unsystematic mortality risk and the two factors driving the systematic risk

$$\vartheta_1^Q(t) = \frac{\nu_1^{V,Q}(t) + \rho_{1,1}^{V,Q}(t)(\kappa_{1,1}^Q(t))^{-1} + \rho_{1,2}^{V,Q}(t)(\kappa_{1,2}^Q(t))^{-1}}{\nu_1^{Z,Q}(t) + \rho_{1,1}^{Z,Q}(t)(\kappa_{1,1}^Q(t))^{-1} + \rho_{1,2}^{Z,Q}(t)(\kappa_{1,2}^Q(t))^{-1}}$$

Optimal position in bonds: Identical to the previous position (without swaps) added a position which eliminates the new interest rate risk in the swaps

$$\xi_1^Q(t) = \frac{\vartheta_1^Q(t)\eta_1^{Z,Q}(t) - \eta_1^{V,Q}(t)}{\sqrt{\gamma^{r\sigma}}B^r(t,T)P^*(t,T)}$$

GKW-decomposition of $V^{*,Q}$ in the extended market (B, P, Z_2)

$$dV^{*,Q}(t) = \xi_2^Q(t)dP^*(t,T) + \vartheta_2^Q(t)dZ_2^{*,Q}(x,t) + L_2^Q(t)$$

where

$$\begin{aligned} dL_2^Q(t) &= \nu_1^{V,Q}(t)dM_1^Q(x,t) - \vartheta_2^Q(t)\nu_2^{Z,Q}(t)dM_2^Q(x,t) \\ &\quad + \Big(\rho_{1,1}^{V,Q}(t) - \vartheta_2^Q(t)\rho_{2,1}^{Z,Q}(t)\Big)dW_1^{\mu,Q}(t) \\ &\quad + \Big(\rho_{1,2}^{V,Q}(t) - \vartheta_2^Q(t)\rho_{2,2}^{Z,Q}(t)\Big)dW_2^{\mu,Q}(t) \end{aligned}$$

and

$$\begin{aligned} \xi_2^Q(t) &= \frac{\eta_1^{V,Q}(t) - \vartheta_2^Q(t)\eta_2^{Z,Q}(t)}{-\sqrt{\gamma^{r,\sigma}}B^r(t,T)P^*(t,T)} \\ \vartheta_2^Q(t) &= \frac{\rho_{1,1}^{V,Q}(t)(\kappa_{2,1}^Q(t))^{-1} + \rho_{1,2}^{V,Q}(t)(\kappa_{2,2}^Q(t))^{-1}}{\nu_2^{Z,Q}(t) + \rho_{2,1}^{Z,Q}(t)(\kappa_{2,1}^Q(t))^{-1} + \rho_{2,2}^{Z,Q}(t)(\kappa_{2,2}^Q(t))^{-1}} \end{aligned}$$

Here: $\kappa_{2,j}^Q(t) = \frac{\nu_2^{Z,Q}(t)\lambda_2^Q(x,t)}{\rho_{2,j}^{Z,Q}(t)}$

Interpretation:

Optimal number of swaps: (Similar interpretation). The strategy balances the three sources of risk: the unsystematic mortality risk and the two factors driving the systematic risk

Optimal position on bonds: Similar intepretation as in previous model

Note: The investment in the alternative swap introduces new unsystematic risk related to the insurance portfolio

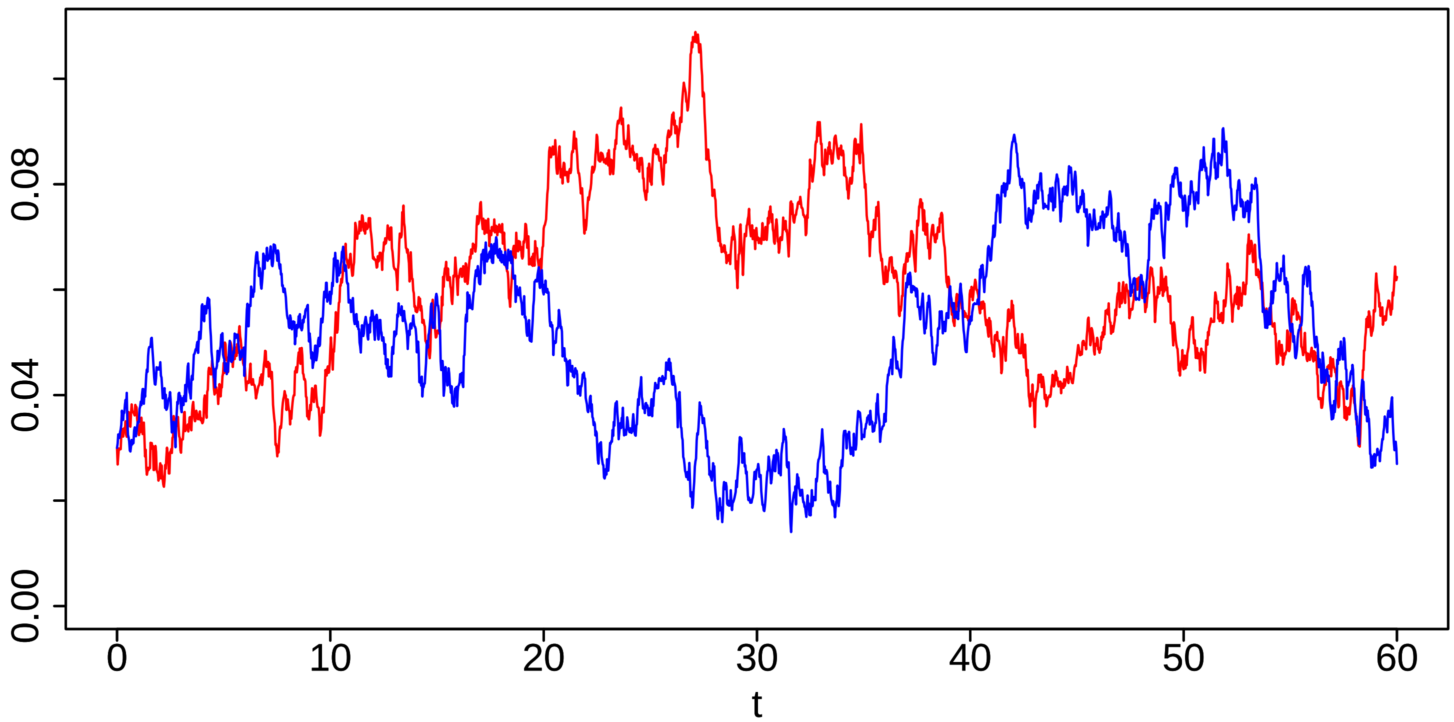
$$
\begin{aligned}
dL_2^Q(t) &= \nu_1^{V,Q}(t)dM_1^Q(x,t) - \vartheta_2^Q(t)\nu_2^{Z,Q}(t)dM_2^Q(x,t) \\
&\quad + \left(\rho_{1,1}^{V,Q}(t) - \vartheta_2^Q(t)\rho_{2,1}^{Z,Q}(t)\right)dW_1^{\mu,Q}(t) \\
&\quad + \left(\rho_{1,2}^{V,Q}(t) - \vartheta_2^Q(t)\rho_{2,2}^{Z,Q}(t)\right)dW_2^{\mu,Q}(t)
\end{aligned}
$$

Have also derived GKW-decomposition of $V^{*,Q}$ in the extended market (B, P, Z_1, Z_2)

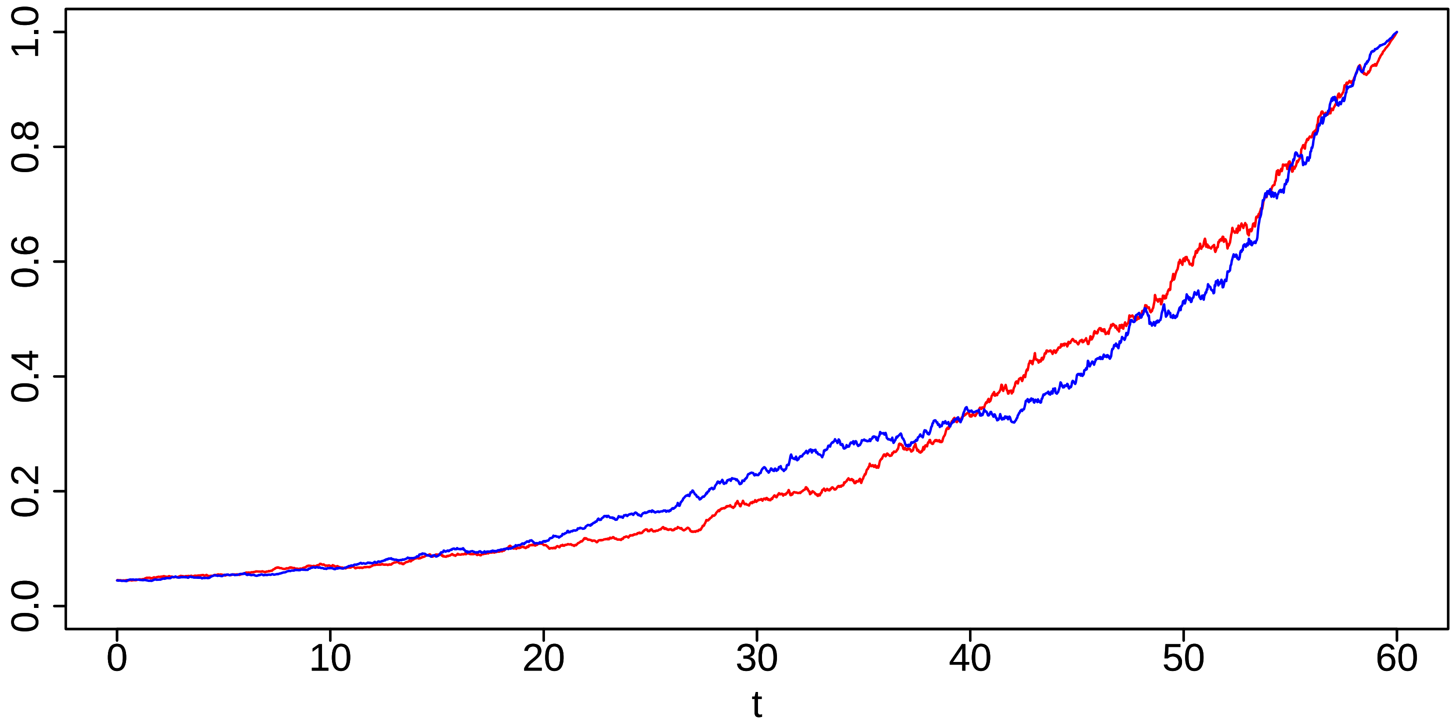
$$dV^{*,Q}(t) = \xi^Q(t)dP^*(t,T) + \vartheta^Q(t)dZ_1^{*,Q}(x,t) + \psi^Q(t)dZ_2^{*,Q}(x,t) + L^Q(t)$$

More involved expressions.

Now use both mortality swaps to hedge dynamically the risk inherent in the life insurance portfolio



Realization of the short rate over a period of 60 years in two different stochastic scenarios (red and blue line)



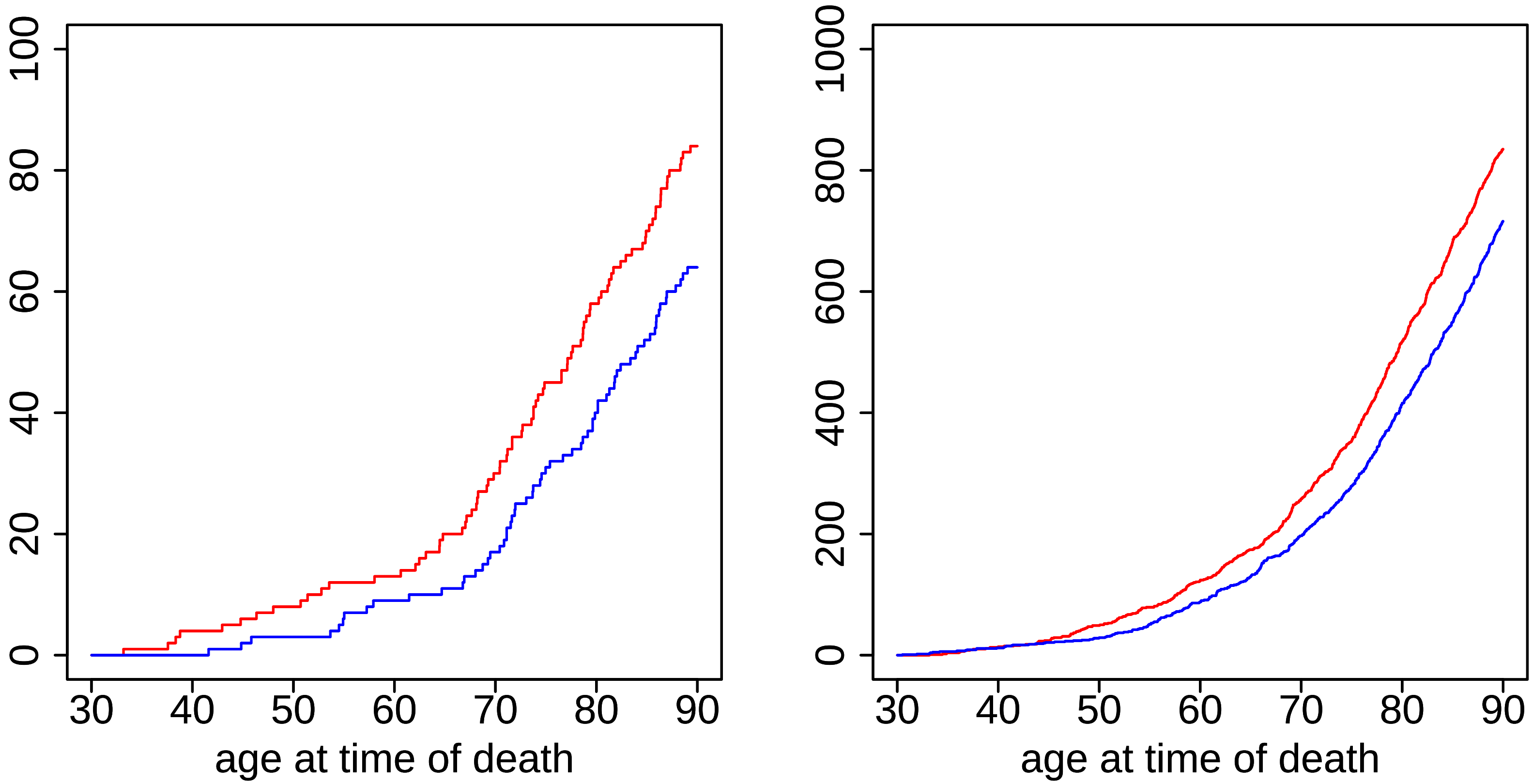
Price of a zero coupon bond with maturity $T = 60$ years in two different stochastic scenarios (red and blue line).

Portfolio (j)	$\mu_j(x,0)$	$\gamma_j(x,t)$	$\delta_j(x,t)$	$\sigma_{j,1}(x,t)$	$\sigma_{j,2}(x,t)$
1	$\mu_1^0(x)$	0.0001800	0.0080	0.006	0.018
2	$\mu_2^0(x)$	0.0001805	0.0081	0.000	0.019

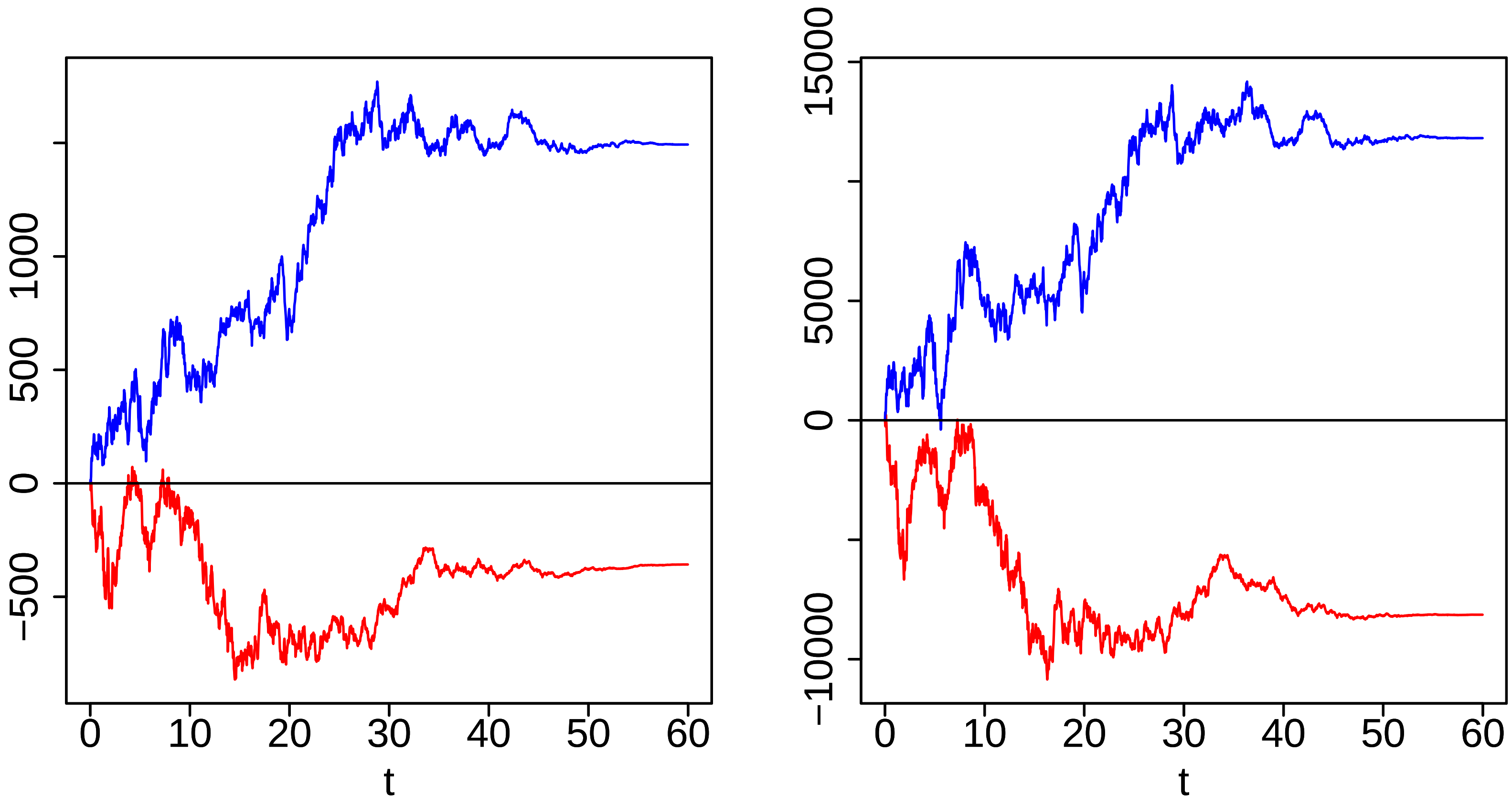
Parameters for mortality intensities.

We consider two portfolios, $n_1 = 100$, $n_2 = 1{,}000$.

Hedging with mortality derivatives: Numerical results

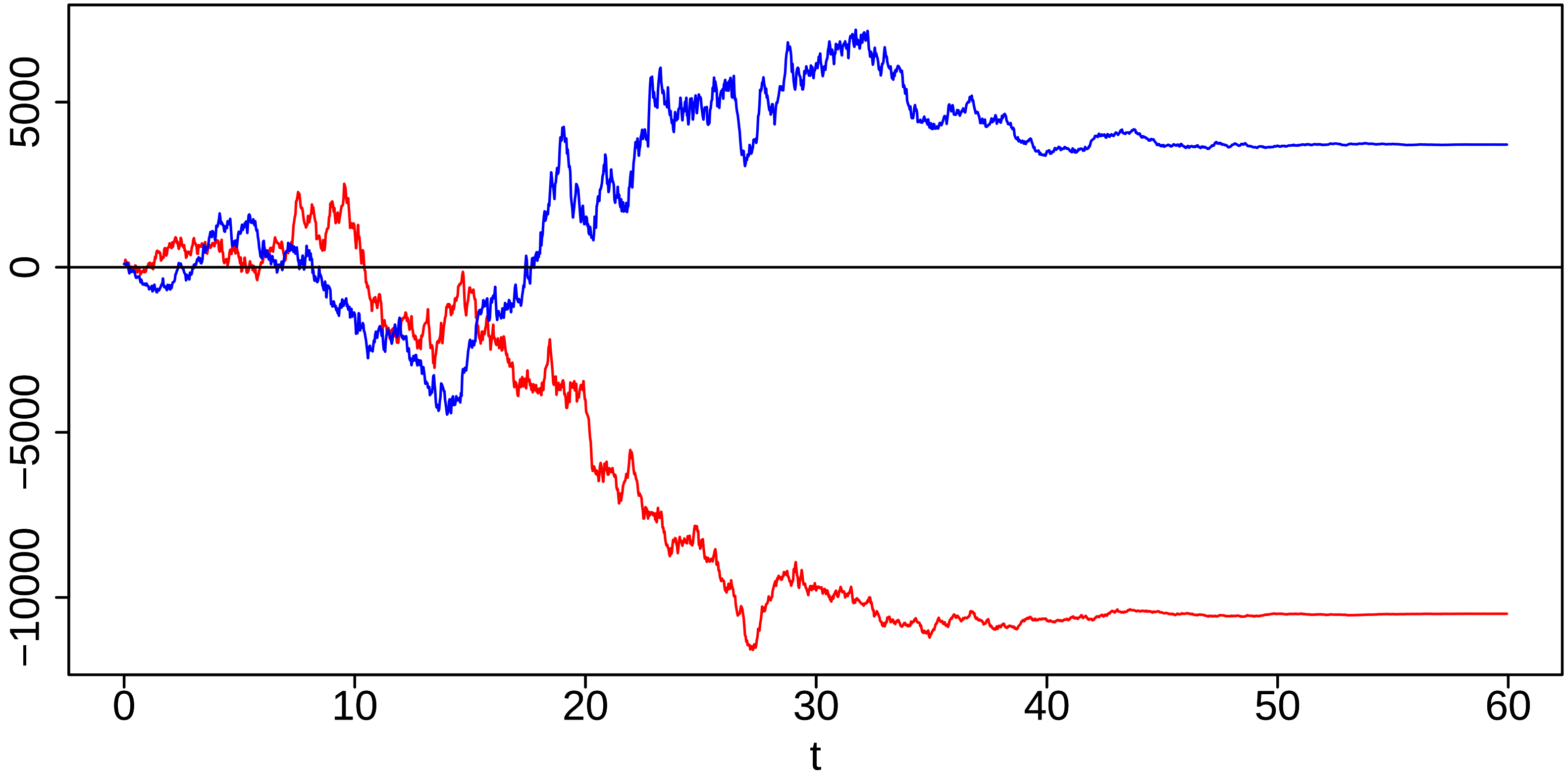


Deaths in the insurance portfolio (left plot) and deaths in the population (right plot) in two stochastic scenarios (red lines and blue lines)



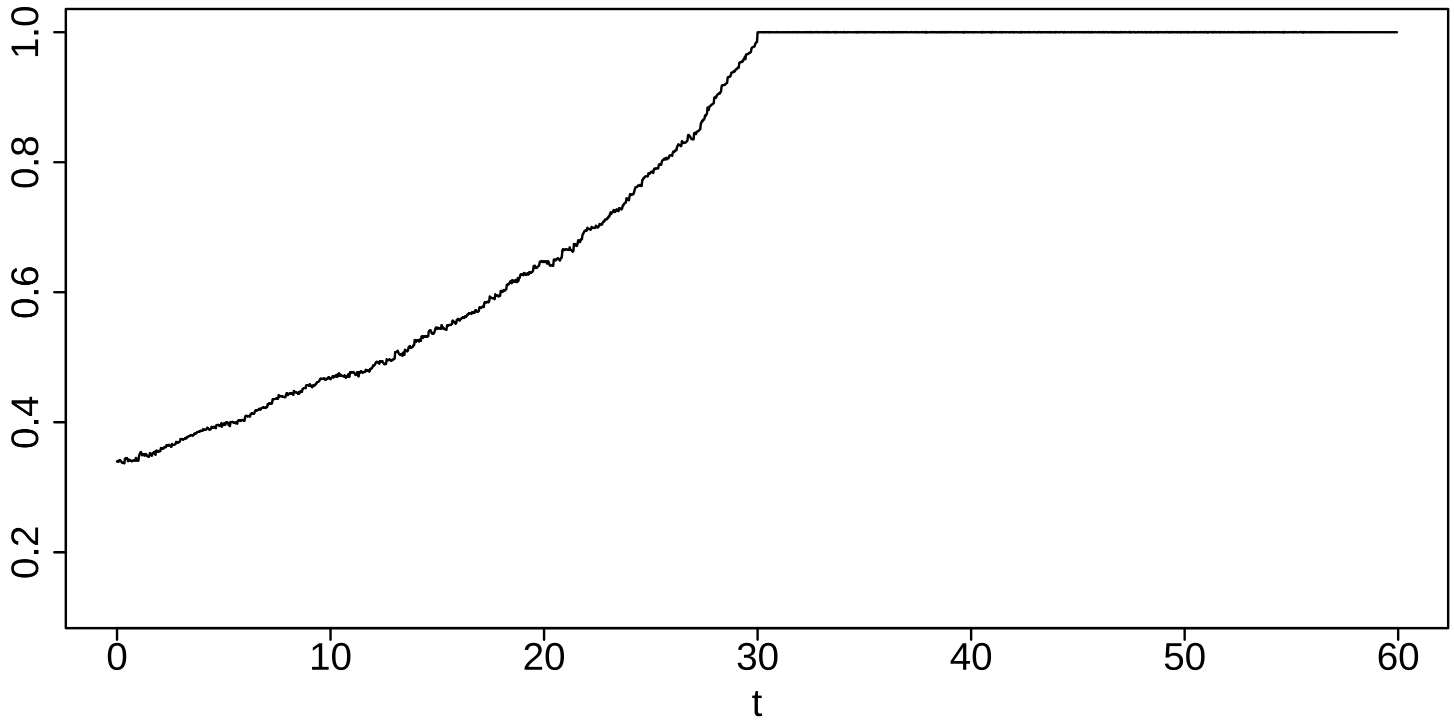
The new hedging instruments

Intrinsic value processes for survivor swap on the insurance portfolio (left plot) and survivor swap on the population (right plot) in two scenarios (red and blue lines)

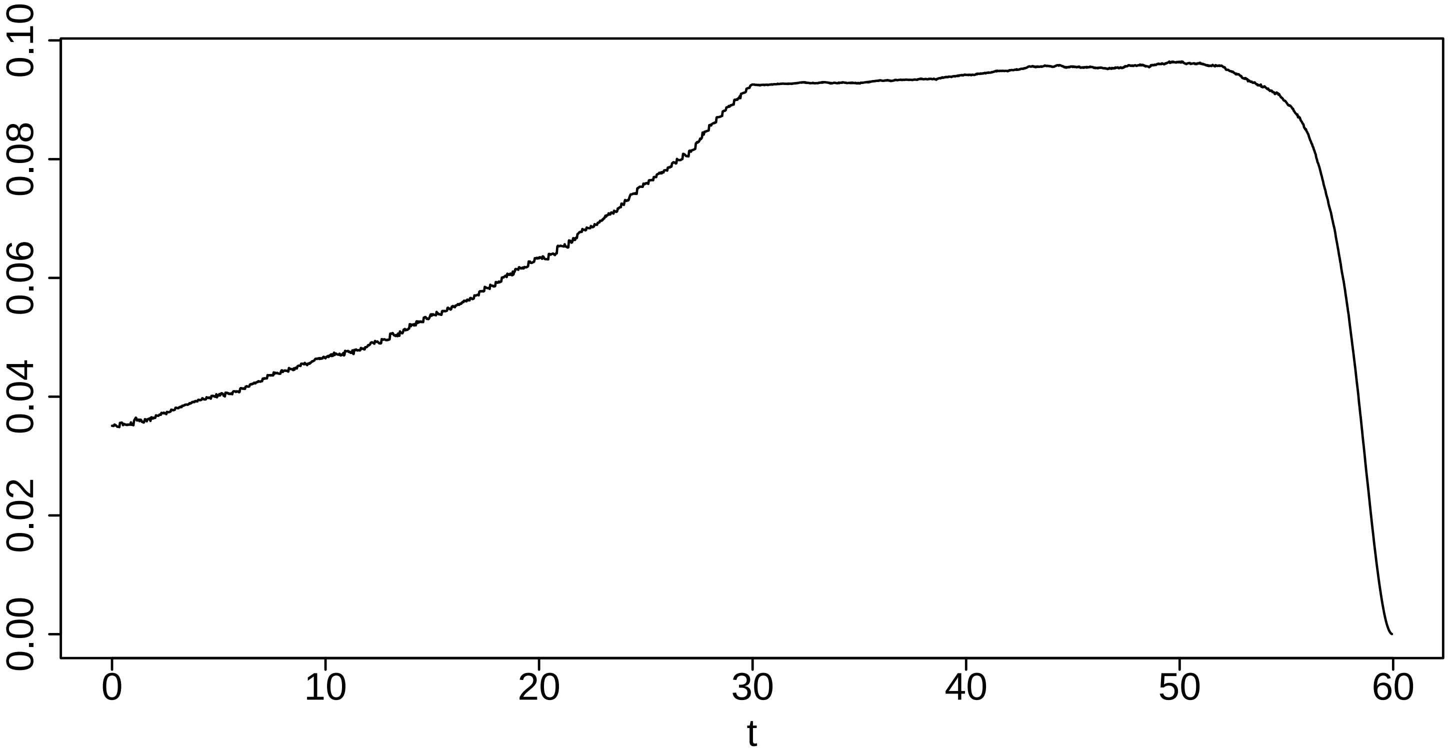


The liability - to be hedged!

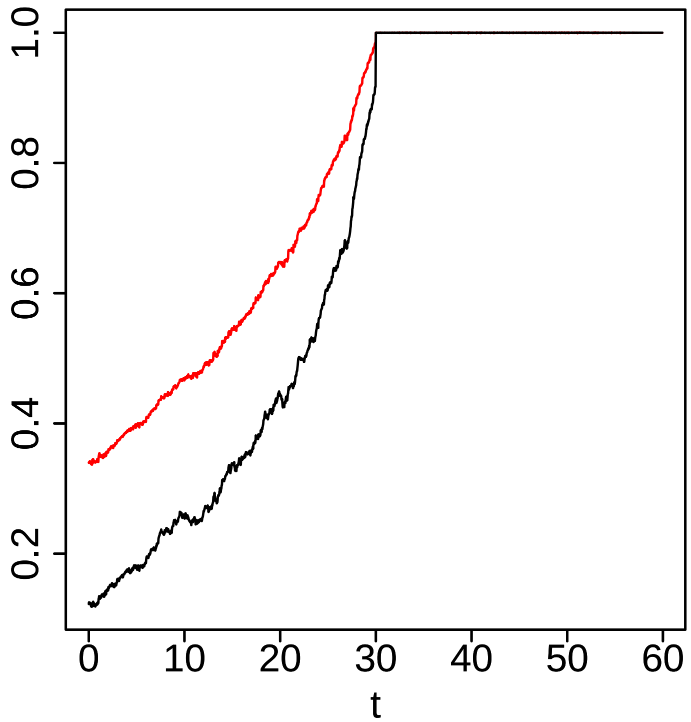
Intrinsic value processes for the insurance contract in two different stochastic scenarios (red and blue line)

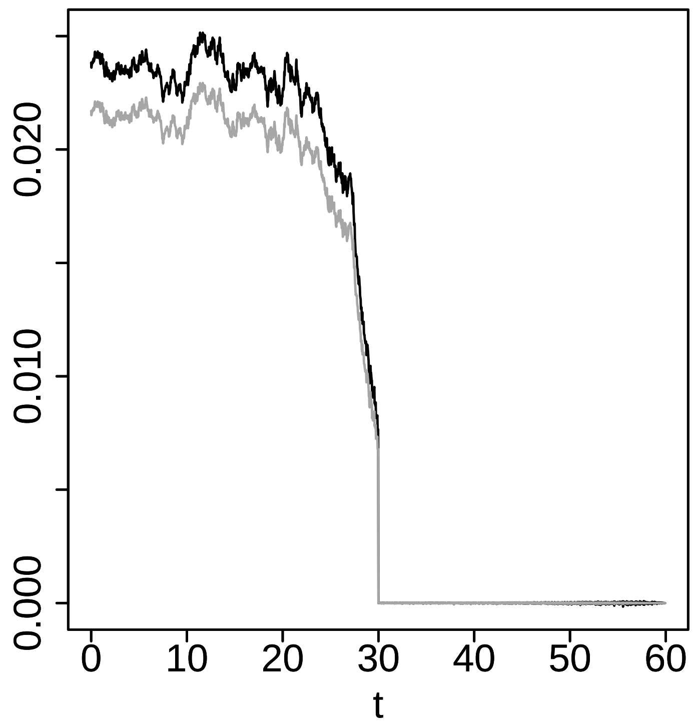


Picture: Number of survivor swaps on the insurance portfolio held at time t in the market (B, P, Z_1) (in scenario 1)



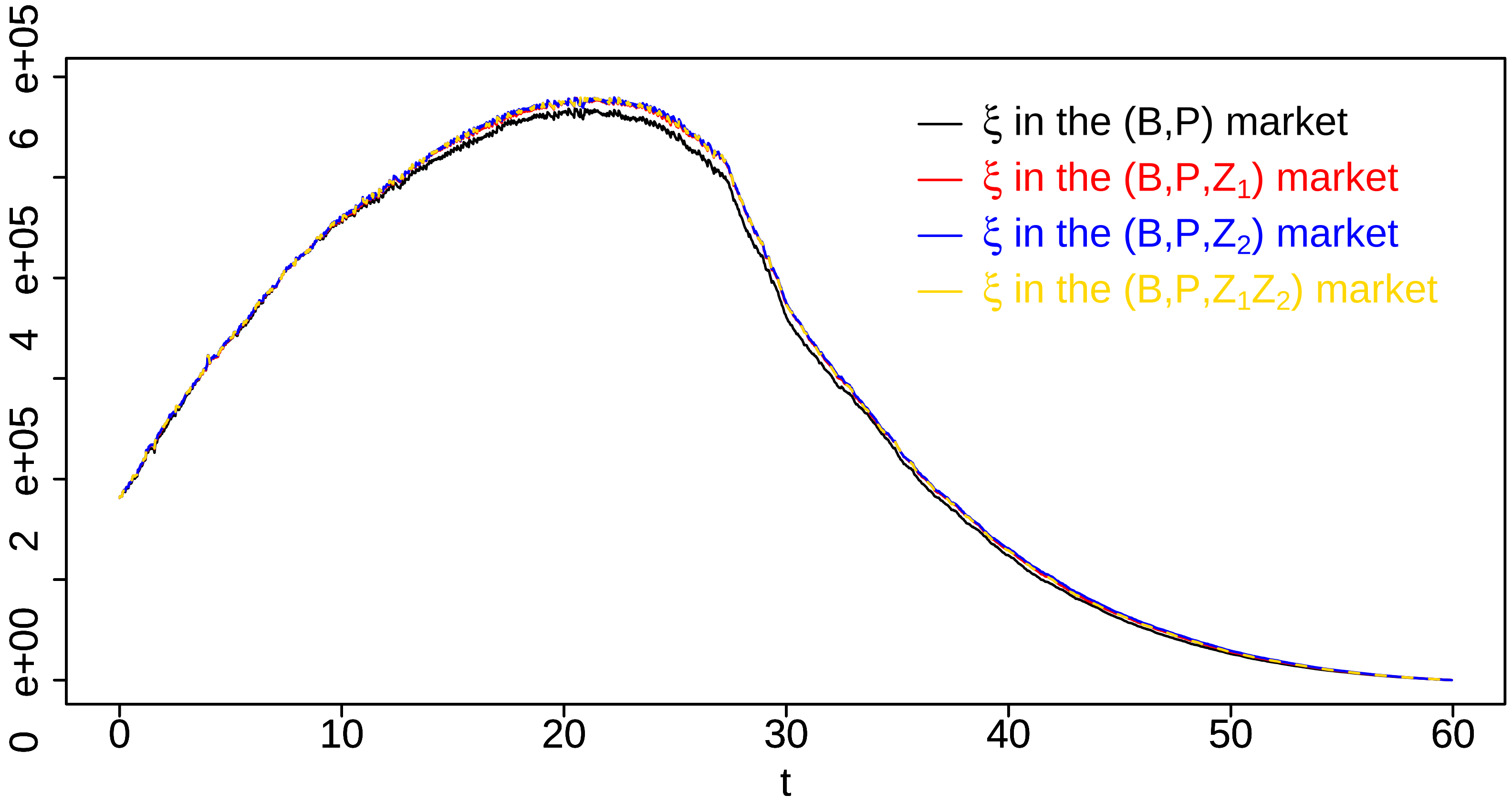
Picture: Number of survivor swaps on the the population held at time t in the market (B, P, Z_2) (in scenario 1)





Left plot: Black line is number of survivor swaps on the insurance portfolio in the (B, P, Z_1, Z_2) market. Red line is number of survivor swaps on the insurance portfolio in the (B, P, Z_1) market

The right plot: Black line is number of survivor swaps on the population in the (B, P, Z_1, Z_2) market. Grey line is the difference between the investments in the survivor swap on the insurance portfolio from the (B, P, Z_1) market and the (B, P, Z_1, Z_2) market scaled by a factor 10



Number of zero coupon bonds held. Hedge for the interest rate risk inherent in the insurance portfolio and for the interest rate risk in the mortality swaps

n_1	n_2	$\frac{\sqrt{R(0,\Psi_V^*)}}{n_1}$	$\frac{\sqrt{R(0,\Psi_B^*)}}{n_1}$
100	1,000	0.632	0.111
100	10,000	0.633	0.111
1,000	10,000	0.628	0.062
1,000	100,000	0.622	0.062
10,000	100,000	0.628	0.055

n_1	n_2	$\frac{\sqrt{R(0,\Psi_1^*)}}{n_1}$	$\frac{\sqrt{R(0,\Psi_2^*)}}{n_1}$	$\frac{\sqrt{R(0,\Psi^*)}}{n_1}$
100	1,000	0.048	0.101	0.033
100	10,000	0.048	0.096	0.020
1,000	10,000	0.032	0.033	0.018
1,000	100,000	0.032	0.030	0.015
10,000	100,000	0.013	0.011	0.010

The minimum obtainable risk in the various markets

We have also studied

❏ Strategies in discrete time for the mortality swap combined with continuous time hedging for the bond (have derived optimality result)

We are currently

❏ Finishing the paper

❏ Extending the numerical work further

❏ Comparing with alternative mortality derivatives

Hedging with mortality derivatives: References

Dahl (2004). Stochastic mortality in life insurance: Market reserves and mortality-linked insurance contracts, *Insurance: Mathematics and Economics* **35**, 113–136

☞ Dahl, Melchior and Møller (2006). On systematic mortality risk and Quadratic hedging with mortality swaps, *in progress*

☞ Dahl and Møller (2006). Valuation and hedging of life insurance liabilities with systematic mortality risk, *Insurance: Mathematics and Economics* **39**, 193–217

Møller (2001). Risk-minimizing hedging strategies for insurance payment processes, *Finance and Stochastics* **5**, 419–446

Møller (2002). On valuation and risk management at the interface of insurance and finance, *British Actuarial Journal* **8**, 787–828

Møller and Steffensen (2007). *Market-valuation methods in life and pension insurance*, forthcoming, Cambridge University Press