Modelling the Claims Development Result for Solvency Purposes, Chain Ladder Model

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Overview

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1. Introduction
2. Chain-Ladder Method
3. Claims Development Result
4. Conclusions

1. Introduction

Non-life insurance company: **Accounting year** $I + 1 = 2008$

Budget statement at 1/1/2008
Profit & Loss (P&L) statement at 31/12/2008

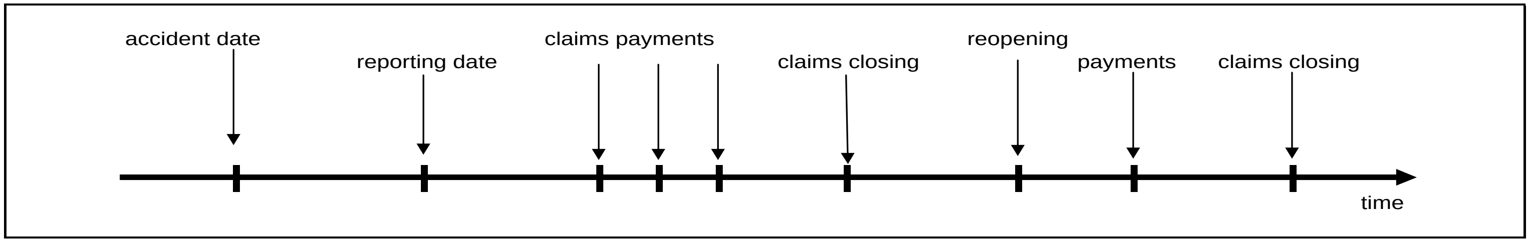
	Budget 1/1/2008	P&L 31/12/2008
premium earned	4'000'000	4'020'000
claims incurred current accident year	-3'300'000	-3'340'000
loss experience prior accident years	**0**	**-40'000**
administrative expenses	-1'000'000	-1'090'000
investment income	500'000	510'000
income before taxes	200'000	60'000

Questions

- What is the position "**loss experience prior accident years**"?
- What are (best estimate) **claims reserves**?
- Quantification of **uncertainties** in the claims reserves?
- **Long term view versus the short term view**

Pay attention in Solvency 2 developments:
These questions can not be answered with simple concepts.

Claims Settlement Process



Often it takes **several years** until a claim is finally settled. Reasons:

1. **Reporting delay**: time lag between accident date and reporting date (notification at insurance company)

2. **Settlement delay**: time interval between reporting date and final settlement (severity of claim, recovery process, court decisions, etc.)

3. **Reopenings** due to new (unexpected) claim developments

Conclusions: Claims Reserving

- Every claim generates a (random) payment cashflow.
- The claims reserves should suffice to meet this random cashflow

$\Longrightarrow$ **claims reserving is a prediction problem**.

- Determine the **prediction uncertainty**:

deterministic claims reserves $\Longleftrightarrow$ **stochastic claims payments**

These are the task of the reserving actuary / risk manager.

Prediction Uncertainty (1/2)

- X future cash flow (**random variable**) to be **predicted**.
- $\mathcal{D}_I$ information available at time I.
- Assume $\widehat{X}$ is a $\mathcal{D}_I$-measurable predictor for X.

The (conditional) **mean square error of prediction (MSEP)** is defined by

$$\mathrm{msep}_{X|\mathcal{D}_I}\left(\widehat{X}\right) = E\left[\left(X - \widehat{X}\right)^2 \middle| \mathcal{D}_I\right].$$

Prediction Uncertainty (2/2)

Due to the $\mathcal{D}_I$-measurability of $\widehat{X}$ we have

$$\text{msep}_{X|\mathcal{D}_I}\left(\widehat{X}\right) = \text{Var}\left(X|\,\mathcal{D}_I\right) + \left(E\left[X|\,\mathcal{D}_I\right] - \widehat{X}\right)^2.$$

- $\text{Var}\left(X|\,\mathcal{D}_I\right)$ is called **Process Variance**.
- $\left(E\left[X|\,\mathcal{D}_I\right] - \widehat{X}\right)^2$ is called **Parameter Estimation Error**.

Hence, $\widehat{X}$ is a **predictor** for X and an **estimator** for $E\left[X|\,\mathcal{D}_I\right]$.

Task. Determine $\widehat{X}$, process variance and parameter estimation error.

2. Chain-Ladder Method

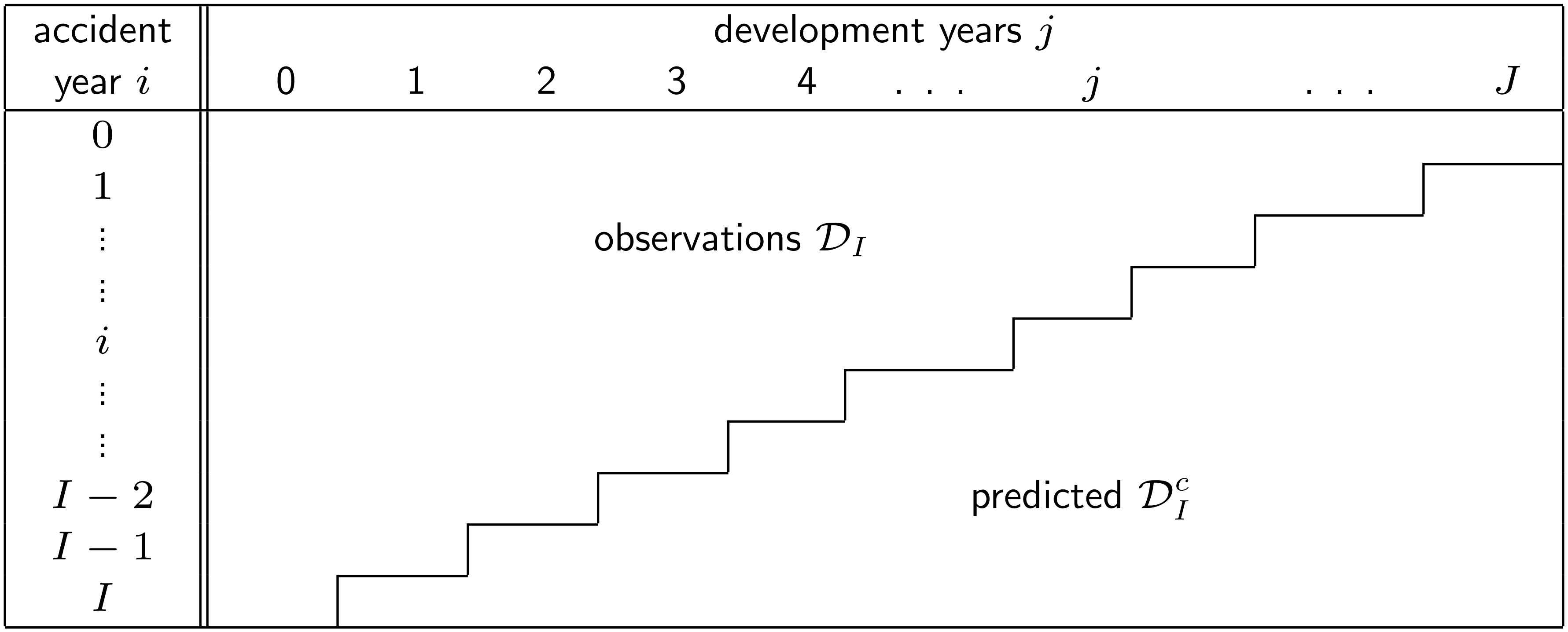
We use the following notations:

- **accident years** are denoted by $i \in \{0, \dots, I\}$
- **development years** are denoted by $j \in \{0, \dots, J\}$
- **incremental claims** are denoted by $X_{i,j}$
- **cumulative claims** are denoted by

$$C_{i,j} = \sum_{k=0}^{j} X_{i,k}.$$

Here: $C_{i,j}$ denote cumulative payments.

Loss Development Triangle at Time I



- **upper triangle** $\mathcal{D}_I = \{C_{i,j} : \; i + j \le I\}$ (**observations**)

- **lower triangle** $\mathcal{D}_I^c = \{C_{i,j} : \; i + j > I, \; i \le I\}$ (**to be predicted**)

Example 1: Cumulative Payments

	0	1	2	3	4	5	6	7	8	9
0	5'946975	9'668212	10'563929	10'771690	10'978394	11'040518	11'106331	11'121181	11'132310	11'148124
1	6'346756	9'593162	10'316383	10'468180	10'536004	10'572608	10'625360	10'636546	10'648192	
2	6'269090	9'245313	10'092366	10'355134	10'507837	10'573282	10'626827	10'635751		
3	5'863015	8'546239	9'268771	9'459424	9'592399	9'680740	9'724068			
4	5'778885	8'524114	9'178009	9'451404	9'681692	9'786916				
5	6'184793	9'013132	9'585897	9'830796	9'935753					
6	5'600184	8'493391	9'056505	9'282022						
7	5'288066	7'728169	8'256211							
8	5'290793	7'648729								
9	5'675568									

Observed historical **cumulative payments**

$$\mathcal{D}_{I=9} = \{C_{i,j} :\ i + j \le I = 9\}.$$

Distribution-Free CL Model Assumptions

- Different accident years i are independent.
- $\{C_{i,j}\}_{j\geq 0}$ is a Markov chain with

$$E\left[C_{i,j}\middle|\, C_{i,j-1}\right] = f_{j-1}\, C_{i,j-1}, \qquad \text{for all } i,j.$$

$$\mathrm{Var}\left(C_{i,j}\middle|\, C_{i,j-1}\right) = \sigma^2_{j-1}\, C_{i,j-1}, \qquad \text{for all } i,j.$$

Expected ultimate claim $C_{i,J}$, given $\mathcal{D}_I$, is

$$E\left[C_{i,J}\middle|\, \mathcal{D}_I\right] = C_{i,I-i} \prod_{j=I-i}^{J-1} f_j.$$

Estimation of CL Factors

At time I we have information

$$\mathcal{D}_I = \{C_{i,j} : \; i + j \le I\}.$$

Hence, we estimate the parameters f_j and σ_j^2 by

$$\widehat{f}_j^I = \frac{\sum_{i=0}^{I-j-1} C_{i,j+1}}{\sum_{i=0}^{I-j-1} C_{i,j}},$$

$$\widehat{\sigma}_j^2 = \frac{1}{I-j-1} \sum_{i=0}^{I-j-1} C_{i,j} \left(\frac{C_{i,j+1}}{C_{i,j}} - \widehat{f}_j^I \right)^2.$$

CL Estimator / CL Predictor

Henceforth, we predict the ultimate claim $C_{i,J}$ at time I by

$$\widehat{C}_{i,J}^{I} = C_{i,I-i} \prod_{j=I-i}^{J-1} \widehat{f}_j^{I}.$$

Note that (see Wüthrich-Merz [Wiley 2008]):

- $\widehat{C}_{i,J}^{I}$ is a conditionally **unbiased estimator** for $E\left[C_{i,J}\middle|\mathcal{D}_I\right]$.
- $\widehat{C}_{i,J}^{I}$ is $\mathcal{D}_I$-measurable.

Prediction Uncertainty

Conditional MSEP for predictor $\widehat{C}_{i,J}^{I}$

$$\begin{aligned}
\mathrm{msep}_{C_{i,J}|\mathcal{D}_I}\left(\widehat{C}_{i,J}^{I}\right) &= E\left[\left(\widehat{C}_{i,J}^{I} - C_{i,J}\right)^2 \middle| \mathcal{D}_I\right] \\
&= \mathrm{Var}\left(C_{i,J} \middle| \mathcal{D}_I\right) + \left(E\left[C_{i,J} \middle| \mathcal{D}_I\right] - \widehat{C}_{i,J}^{I}\right)^2 \\
&= \mathrm{Var}\left(C_{i,J} \middle| \mathcal{D}_I\right) + C_{i,I-i}^2 \left(\prod_{j=I-i}^{J-1} f_j - \prod_{j=I-i}^{J-1} \widehat{f}_j^I\right)^2 \\
&= \text{process variance} + \text{parameter estimation error.}
\end{aligned}$$

Not treated: model choice/error!

MSEP Estimator

There are different approaches, see Mack [ASTIN Bulletin 1993], Buchwalder et al. [ASTIN Bulletin 2006] or Gisler-Wüthrich [ASTIN Colloquium 2007].

The Mack approach gives the following formula for the MSEP:

$$\widehat{\mathrm{msep}}_{C_{i,J}|\mathcal{D}_I}\left(\widehat{C}_{i,J}^I\right)$$

$$= \left(\widehat{C}_{i,J}^I\right)^2 \left[\sum_{j=I-i}^{J-1} \frac{\widehat{\sigma}_j^2 / \left(\widehat{f}_j^I\right)^2}{\widehat{C}_{i,j}^I} + \sum_{j=I-i}^{J-1} \frac{\widehat{\sigma}_j^2 / \left(\widehat{f}_j^I\right)^2}{\sum_{k=0}^{I-j-1} C_{k,j}}\right].$$

This is the **long term view**: uncertainty over the whole runoff period (this is not the one-year solvency view).

MSEP, Aggregated Accident Years

Aggregation over accident years i is more involved.

$$\widehat{\text{msep}}_{\sum_i C_{i,J}|\mathcal{D}_I}\left(\sum_{i=1}^{I}\widehat{C}_{i,J}^{I}\right) = \sum_{i=1}^{I}\widehat{\text{msep}}_{C_{i,J}|\mathcal{D}_I}\left(\widehat{C}_{i,J}^{I}\right)$$

$$+\ 2\sum_{1\le i<l\le I}\widehat{C}_{i,J}^{I}\ \widehat{C}_{l,J}^{I}\ \sum_{j=I-i}^{J-1}\frac{\widehat{\sigma}_j^2/\left(\widehat{f}_j^I\right)^2}{\sum_{k=0}^{I-j-1}C_{k,j}}.$$

Question: How is this related to the one-year solvency view?

Example 1, revisited

	0	1	2	3	4	5	6	7	8
$\widehat{f}_j^I$	1.4925	1.0778	1.0229	1.0148	1.0070	1.0051	1.0011	1.0010	1.0014
$\widehat{\sigma}_j$	135.253	33.803	15.760	19.847	9.336	2.001	0.823	0.219	0.059

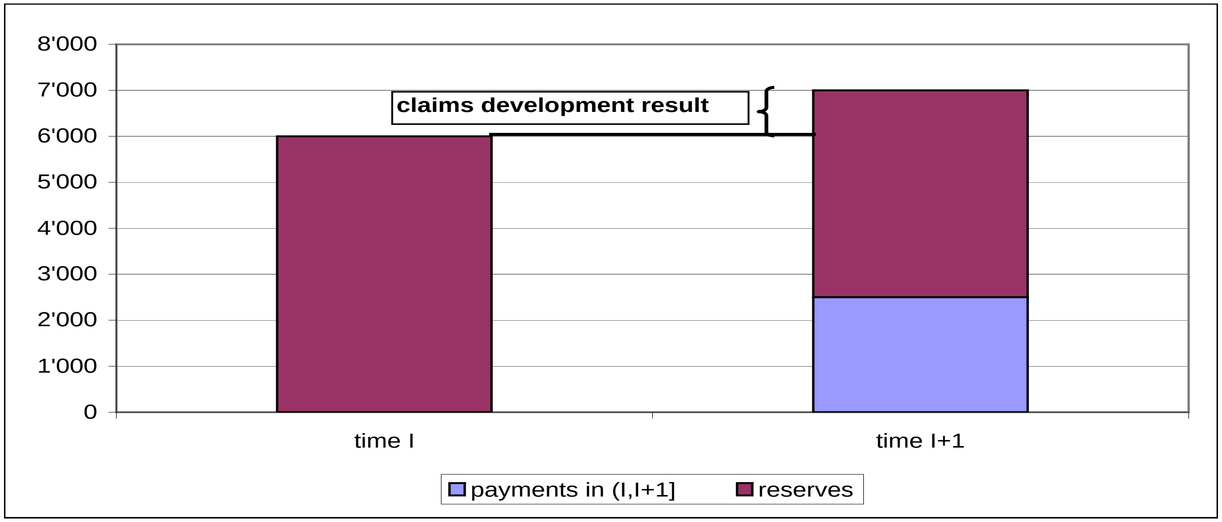
i	$\widehat{R}_i^{\mathcal{D}_I}$	process std.dev.		estimation std.dev.		$\widehat{\text{msep}}_{C_{i,J}\mid\mathcal{D}_I}(\widehat{C}_{i,J}^I)^{1/2}$	
0	0						
1	15'126	191	1.3%	187	1.2%	267	1.8%
2	26'257	742	2.8%	535	2.0%	914	3.5%
3	34'538	2'669	7.7%	1'493	4.3%	3'058	8.9%
4	85'302	6'832	8.0%	3'392	4.0%	7'628	8.9%
5	156'494	30'478	19.5%	13'517	8.6%	33'341	21.3%
6	286'121	68'212	23.8%	27'286	9.5%	73'467	25.7%
7	449'167	80'077	17.8%	29'675	6.6%	85'398	19.0%
8	1'043'242	126'960	12.2%	43'903	4.2%	134'337	12.9%
9	3'950'815	389'783	9.9%	129'769	3.3%	410'817	10.4%
cov. term				116'810		116'810	
Total	6'047'061	424'379	7.0%	185'024	3.1%	462'960	7.7%

CL best estimate reserves at time I: $\widehat{R}_i^{\mathcal{D}_I} = \widehat{C}_{i,J}^I - C_{i,I-i}$.

3. Claims Development Result

So far: **long term view**, uncertainty over the whole runoff period.

Short term view: changes over the next accounting year.
Loss experience prior accident years in P&L.



Therefore: study the **claims development result (CDR)**.

Budget and P&L Statement

Non-life insurance company: **Accounting year** $I + 1 = 2008$

	Budget 1/1/2008	P&L 31/12/2008
premium earned	4'000'000	4'020'000
claims incurred current accident year	-3'300'000	-3'340'000
loss experience prior accident years	**0**	**-40'000**
administrative expenses	-1'000'000	-1'090'000
investment income	500'000	510'000
income before taxes	200'000	60'000

- How do we quantify the uncertainty in the position loss experience prior accident years?

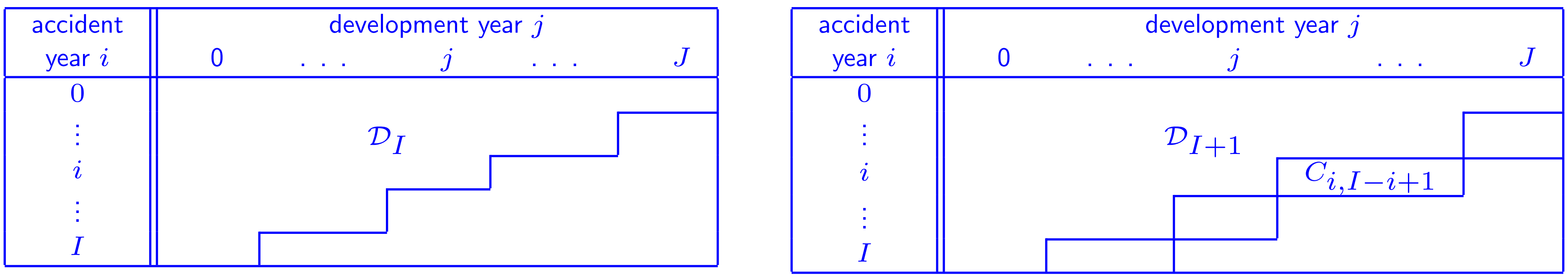
- What are the risk drivers?

Updating Information

The CDR is mainly concerned with updating the information.

$$\mathcal{D}_I = \{C_{i,j} : \; i+j \le I \text{ and } i \le I\},$$

$$\mathcal{D}_{I+1} = \{C_{i,j} : \; i+j \le I+1 \text{ and } i \le I\} \supset \mathcal{D}_I.$$



Loss development triangle at time I and at time $I+1$.

This provides predictors $\widehat{C}_{i,J}^{I}$ and $\widehat{C}_{i,J}^{I+1}$.

CL Factor Estimation at Times I and $I+1$

CL factor estimators at times I and $I+1$:

$$\widehat{f}_j^I = \frac{\sum_{i=0}^{I-j-1} C_{i,j+1}}{\sum_{i=0}^{I-j-1} C_{i,j}} \qquad \text{and} \qquad \widehat{f}_j^{I+1} = \frac{\sum_{i=0}^{I-j} C_{i,j+1}}{\sum_{i=0}^{I-j} C_{i,j}}.$$

Ultimate claims predictors at times I and $I+1$:

$$\widehat{C}_{i,J}^I \;=\; C_{i,I-i} \prod_{j=I-i}^{J-1} \widehat{f}_j^I,$$

$$\widehat{C}_{i,J}^{I+1} \;=\; C_{i,I-i+1} \prod_{j=I-i+1}^{J-1} \widehat{f}_j^{I+1}.$$

Best Estimate Reserves at Times I and $I+1$

Assume $C_{i,j}$ denotes cumulative payments.

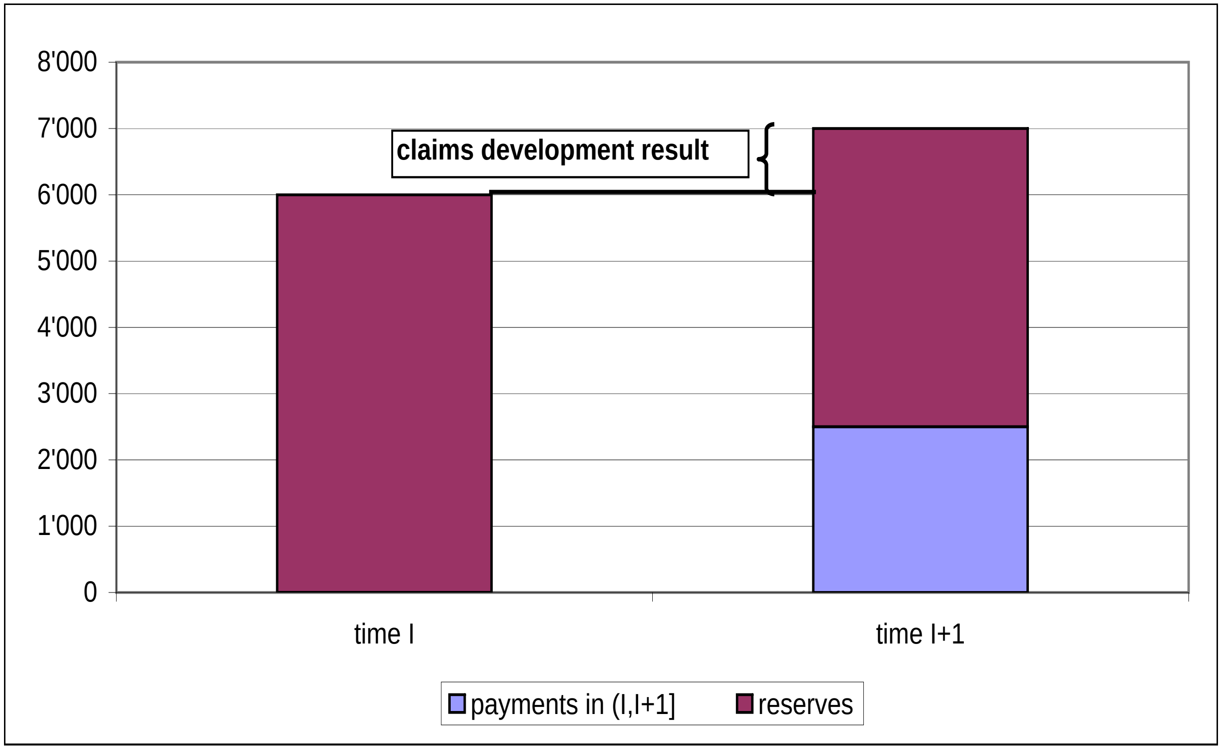
$$\widehat{R}_i^{\mathcal{D}_I} \;=\; \widehat{C}_{i,J}^{I} - C_{i,I-i} \;=\; C_{i,I-i}\left(\prod_{j=I-i}^{J-1} \widehat{f}_j^{I} - 1\right),$$

$$\widehat{R}_i^{\mathcal{D}_{I+1}} \;=\; \widehat{C}_{i,J}^{I+1} - C_{i,I-i+1} \;=\; C_{i,I-i+1}\left(\prod_{j=I-i+1}^{J-1} \widehat{f}_j^{I+1} - 1\right).$$

Payments within accounting year $I+1$:

$$X_{i,I-i+1} = C_{i,I-i+1} - C_{i,I-i}.$$

One-Year Claims Development Result (CDR)



$$\widehat{R}_i^{\mathcal{D}_I} \quad \overset{\text{accounting year } I+1}{\longrightarrow} \quad X_{i,I-i+1} + \widehat{R}_i^{\mathcal{D}_{I+1}}$$

Claims Development Result and MSEP

The **observable CDR** is given by

$$\widehat{\mathrm{CDR}}_i(I+1) = \widehat{R}_i^{\mathcal{D}_I} - \left(X_{i,I-i+1} + \widehat{R}_i^{\mathcal{D}_{I+1}} \right).$$

- Fluctuation of the observable CDR around 0:

budget statement vs. P&L statement

$$\begin{aligned} \mathrm{msep}_{\widehat{\mathrm{CDR}}_i(I+1)|\mathcal{D}_I}(0) &= E\left[\left(\widehat{\mathrm{CDR}}_i(I+1) - 0 \right)^2 \middle| \mathcal{D}_I \right] \\ &= E\left[\left(\widehat{C}_{i,J}^{I} - \widehat{C}_{i,J}^{I+1} \right)^2 \middle| \mathcal{D}_I \right]. \end{aligned}$$

MSEP, Observable CDR (1/2)

Conditional MSEP for the **one-year runoff uncertainty** is (see Merz-Wüthrich [CAS E-Forum 2008]):

$$\widehat{\text{msep}}_{\widehat{\mathrm{CDR}}_i(I+1)|\mathcal{D}_I}(0)$$

$$= \left(\widehat{C}_{i,J}^I\right)^2 \left[\frac{\widehat{\sigma}_{I-i}^2 / \left(\widehat{f}_{I-i}^I\right)^2}{C_{i,I-i}} + \frac{\widehat{\sigma}_{I-i}^2 / \left(\widehat{f}_{I-i}^I\right)^2}{\sum_{k=0}^{i-1} C_{k,I-i}} + \sum_{j=I-i+1}^{J-1} \frac{C_{I-j,j}}{\sum_{k=0}^{I-j} C_{k,j}} \, \frac{\widehat{\sigma}_j^2 / \left(\widehat{f}_j^I\right)^2}{\sum_{k=0}^{I-j-1} C_{k,j}}\right].$$

Compare to the CL Mack [ASTIN Bulletin 1993] formula (total runoff uncertainty, long term view)!

MSEP, Observable CDR (2/2)

The conditional MSEP for aggregated accident years is estimated by

$$\widehat{\mathsf{msep}}_{\sum_i \widehat{\mathrm{CDR}}_i(I+1)|\mathcal{D}_I}(0) = \sum_i \widehat{\mathsf{msep}}_{\widehat{\mathrm{CDR}}_i(I+1)|\mathcal{D}_I}(0)$$

$$+2\sum_{i<l} \widehat{C}_{i,J}^I \widehat{C}_{l,J}^I \left[\frac{\widehat{\sigma}_{I-i}^2 / \left(\widehat{f}_{I-i}^I\right)^2}{\sum_{k=0}^{i-1} C_{k,I-i}} + \sum_{j=I-i+1}^{J-1} \frac{C_{I-j,j}}{\sum_{k=0}^{I-j} C_{k,j}} \frac{\widehat{\sigma}_j^2 / \left(\widehat{f}_j^I\right)^2}{\sum_{k=0}^{I-j-1} C_{k,j}} \right].$$

This is the one-year solvency view: Budget vs. P&L statement.

Example 1, revisited

i	CL reserves $\widehat{R}_i^{\mathcal{D}_I}$	$\widehat{\text{msep}}^{1/2}_{\widehat{\text{CDR}}_i(I+1)\|\mathcal{D}_I}(0)$		$\widehat{\text{msep}}_{C_{i,J}\|\mathcal{D}_I}(\widehat{C}_{i,J}^I)^{1/2}$	
1	15'126	267	1.8%	267	1.8%
2	26'257	884	3.4%	914	3.5%
3	34'538	2'948	8.5%	3'058	8.9%
4	85'302	7'018	8.2%	7'628	8.9%
5	156'494	32'470	20.7%	33'341	21.3%
6	286'121	66'178	23.1%	73'467	25.7%
7	449'167	50'296	11.2%	85'398	19.0%
8	1'043'242	104'311	10.0%	134'337	12.9%
9	3'950'815	385'773	9.8%	410'817	10.4%
cov.		94'134		116'810	
Total	6'047'061	420'220	6.9%	462'960	7.7%

- $\widehat{\text{msep}}_{\widehat{\text{CDR}}_i(I+1)|\mathcal{D}_I}(0)^{1/2}$ is the one-year CDR view (short term).
- $\widehat{\text{msep}}_{C_{i,J}|\mathcal{D}_I}(\widehat{C}_{i,J}^I)^{1/2}$ is the whole runoff uncertainty (long term).

We see that the ratio is around 90%.

Example 2 (1/2)

We consider now an example that is more volatile and has a more long-tailed claims development (commercial liability insurance).

	0	1	2	3	4	5	6	7	8	9
0	122'058	183'153	201'673	214'337	227'477	237'968	261'275	276'592	286'337	298'238
1	132'099	193'304	213'733	230'413	243'926	258'877	269'139	284'618	295'745	
2	132'130	186'839	207'919	222'818	237'617	253'623	267'766	284'800		
3	127'767	187'494	207'759	222'644	237'671	256'521	271'515			
4	127'648	179'633	196'260	213'636	229'660	245'968				
5	125'739	181'082	203'281	219'793	237'129					
6	117'470	172'967	190'535	204'086						
7	117'926	172'606	191'108							
8	118'274	171'248								
9	119'932									
$\widehat{f}_j^I$	1.4524	1.1065	1.0750	1.0679	1.0651	1.0623	1.0599	1.0372	1.0416	
$\widehat{\sigma}_j^2$	108.20	14.21	13.90	13.37	35.69	149.96	2.83	2.09	1.55	

Example 2 (2/2)

i	CL reserves $\widehat{R}_i^{\mathcal{D}_I}$	$\widehat{\text{msep}}_{C_{i,J}\vert\mathcal{D}_I}(\widehat{C}_{i,J}^I)^{1/2}$		$\widehat{\text{msep}}^{1/2}_{\widehat{\text{CDR}}_i(I+1)\vert\mathcal{D}_I}(0)$	
1	12'292	964	7.8%	964	7.8%
2	22'870	1'379	6.0%	1'101	4.8%
3	39'379	1'769	4.5%	1'248	3.2%
4	53'212	7'946	14.9%	7'783	14.6%
5	70'083	8'958	12.8%	4'233	6.0%
6	78'263	8'822	11.3%	2'840	3.6%
7	93'112	9'177	9.9%	2'946	3.2%
8	110'562	9'454	8.6%	2'993	2.7%
9	166'722	11'406	6.8%	6'482	3.9%
total	646'496	31'344	4.8%	19'300	3.0%

We see that the ratio is around 60%.

Example 3, Merz-W. [CAS E-Forum 2008]

i	$\widehat{R}_i^{\mathcal{D}_I}$	$\widehat{\text{msep}}_{\widehat{\text{CDR}}_i(I+1)\|\mathcal{D}_I}(0)^{1/2}$	$\widehat{\text{msep}}_{C_{i,J}^I\|\mathcal{D}_I}(\widehat{C}_{i,J})^{1/2}$
0	0		
1	4'378	567	567
2	9'348	1'488	1'566
3	28'392	3'923	4'157
4	51'444	9'723	10'536
5	111'811	28'443	30'319
6	187'084	20'954	35'967
7	411'864	28'119	45'090
8	1'433'505	53'320	69'552
$\text{cov}^{1/2}$		39'746	50'361
Total	2'237'826	81'080	108'401

We see that the ratio is around 75%.

4. Conclusions

- In all examples considered: the ratio between one-year CDR risk and full runoff risk was within the intervall $[50\%, 95\%]$ (range between liability insurance and property insurance).

  This is also supported by the AISAM-ACME field study 2007.

- We have measured risk with the help of the conditional MSEP. For Value-at-Risk or Expected Shortfall considerations fit distribution with appropriate moments.

- A full distributional approach can only be solved numerically, e.g. **Markov chain Monte Carlo** (MCMC) methods.

Conclusions

- Dependence is not appropriately modelled. Especially, accounting year dependence and claims inflation needs special care (MCMC methods).
- The one-year CDR view needs a **Cost-of-Capital charge (risk margin) for the risk that is beyond the one-year time horizon**.
- Similar formulas were developed for other models:
  - ⋆ Credibility Chain Ladder (Bühlmann et al., 2008)
  - ⋆ Additive Model (Merz-Wüthrich, 2008)
  - ⋆ Complementary Loss Ratio Method (Dahms et al., 2008)

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