A stochastic control model for individual asset-liability management

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wwwdocs.fce.unsw.edu.au/actuarial/research/papers/2005/afir_purcal.pdf

Overview

1. Motivation
2. Literature
3. Richard (1975) model
4. Extension: risky income
5. Solution method
6. Results

1 Motivation

Institutional background

- Ageing populations
- Many governments considering
  - shifting from pay-go (DB) to funded (DC)
  - winding back social security
- Companies shifting from DB to DC

$\rightarrow$ Higher demand for financial planning

Important issues in lifetime consumption, investment and insurance

- Life cycle consumption
  - life cycle theory, buffer stock theory
- Life cycle investment
  - age-phasing
- Life cycle insurance demand
  - human life value concept
- Life cycle annuity demand

2 Literature

- Yaari (1965)
- Merton (1969, 1971)
- Richard (1975)
- Bodie, Merton & Samuelson (1992)
- Koo (1998)
- Viciera (2001)
- Campbell & Viciera (2002)

3 Richard (1975) model

$$\max_{C,\pi,Z} \mathbf{E}\left[\int_{\tau}^{T} U(C(t),t)dt + B(Z(T),T)\right] \tag{1}$$

$$\frac{dQ(t)}{Q(t)} = \alpha dt + \sigma dq(t) \tag{2}$$

$$\begin{aligned} dW(t) &= -C(t)dt - P(t)dt + Y(t)dt + rW(t)dt \\ &\quad +(\alpha - r)\pi(t)W(t)dt + \sigma\pi(t)W dq(t) \end{aligned} \tag{3}$$

$$P(t) = \mu(t)\{Z(t) - W(t)\} \tag{4}$$

4 Extension: risky income

Try to improve relevance of Richard model

Income important component of wealth

Model income as stochastic process with expected exponential path:

$$\frac{dY}{Y} = \hat{\alpha}dt + \hat{\sigma}d\hat{q} \tag{8}$$

Two controlled processes are now:

$$\begin{pmatrix} dW \\ dY \end{pmatrix} = \begin{pmatrix} -C - P + Y + rW + (\alpha - r)\pi W \\ \hat{\alpha}Y \end{pmatrix} dt + \begin{pmatrix} \sigma\pi W & 0 \\ 0 & \hat{\sigma}Y \end{pmatrix} \begin{pmatrix} dq \\ d\hat{q} \end{pmatrix} \tag{9}$$

5 Solution method

Unable to find closed-form solution

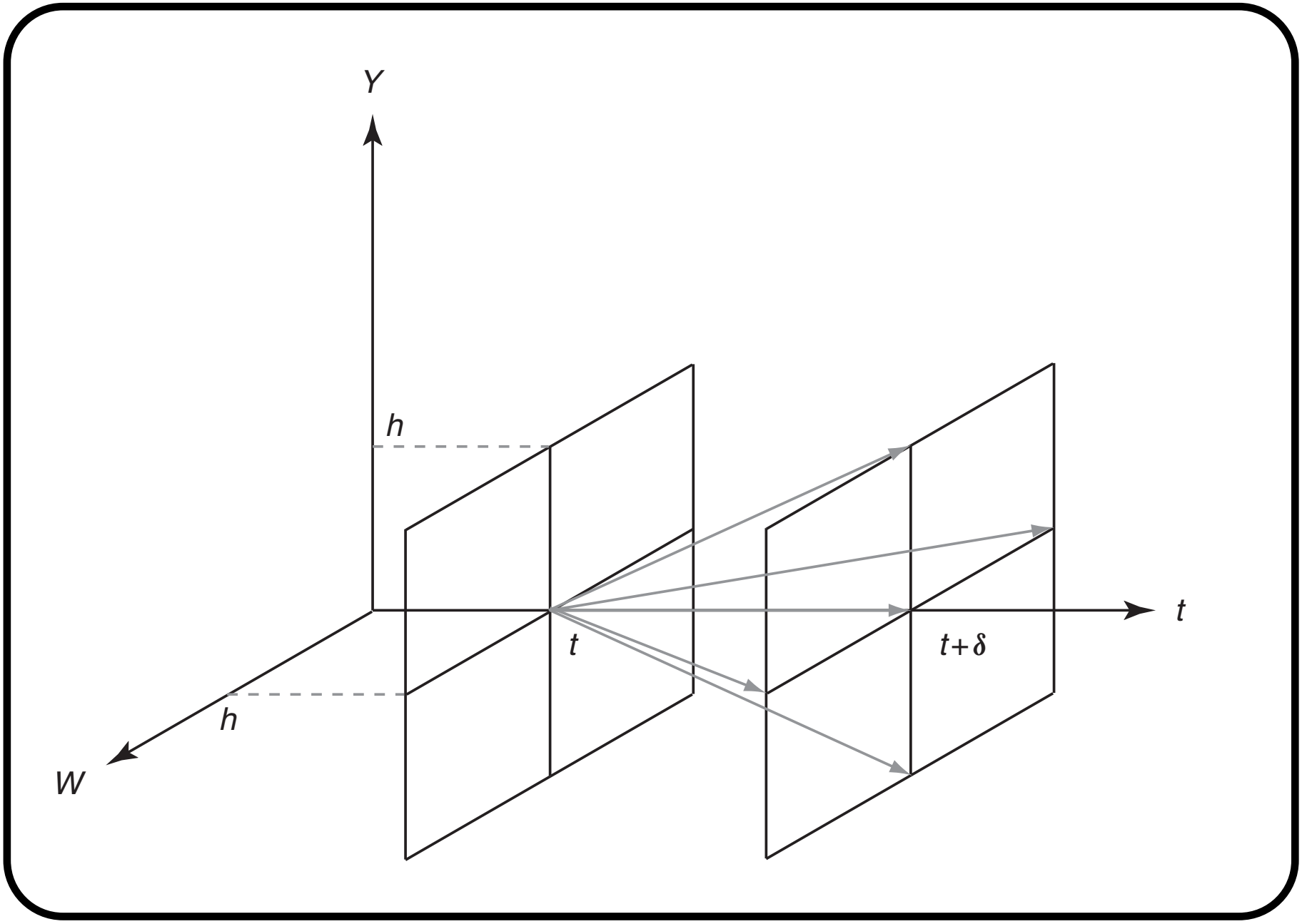
Numerical solution: *Markov Chain approximation technique* of Kushner & Dupuis (2001)

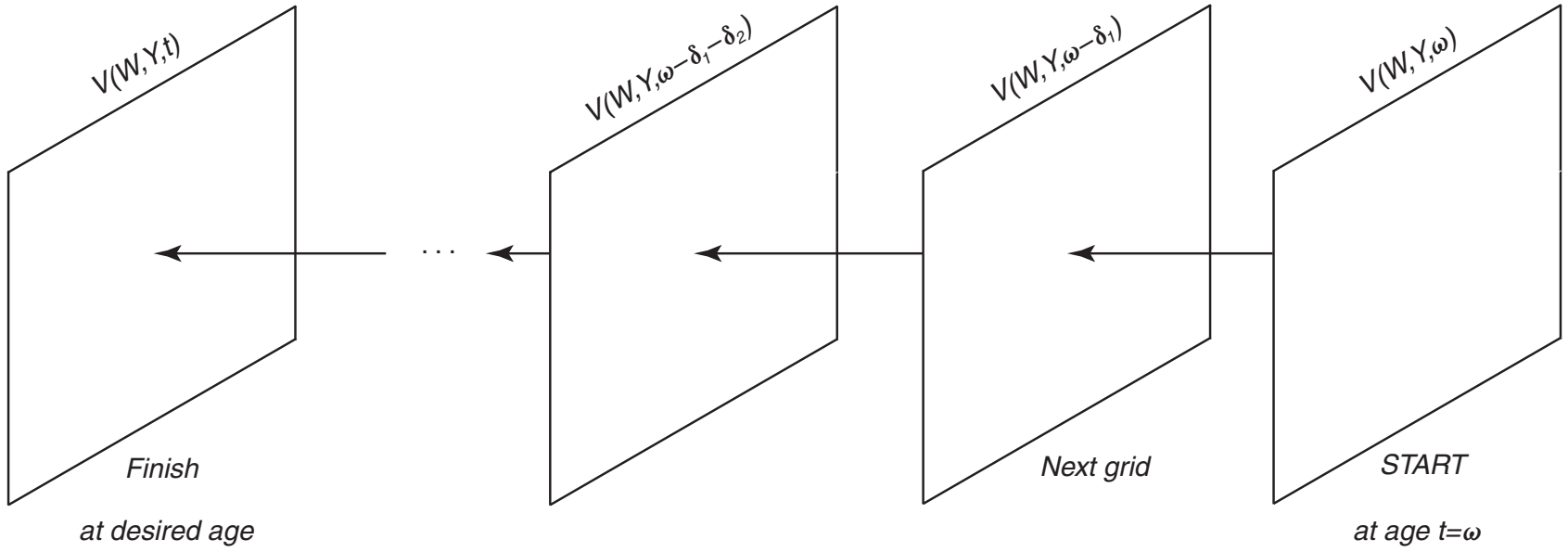
Replace partials in HJB equation with finite difference approximations (equations 9 and 10 in paper, pp. 7–9)

Rearrange:

$$
\begin{aligned}
& V(W,Y,t) \\
&= \max_{C,Z,\pi}\Bigg\{\frac{1}{1+\delta\mu+\delta\rho}\Bigg[\delta\big\{\mu(t)\phi(t)B(Z(t))+U(C(t))\big\} \\
&\qquad\qquad + \sum_{\theta=-1}^{1}\sum_{\phi=-1}^{1}\hat{p}(W+\theta h, Y+\phi h) \\
&\qquad\qquad \times V(W+\theta h, Y+\phi h, t+\delta)\Bigg]\Bigg\}
\end{aligned}
\tag{15}
$$

Solve by backward iteration, letting $h \to 0$ and $\delta \to 0$ together.

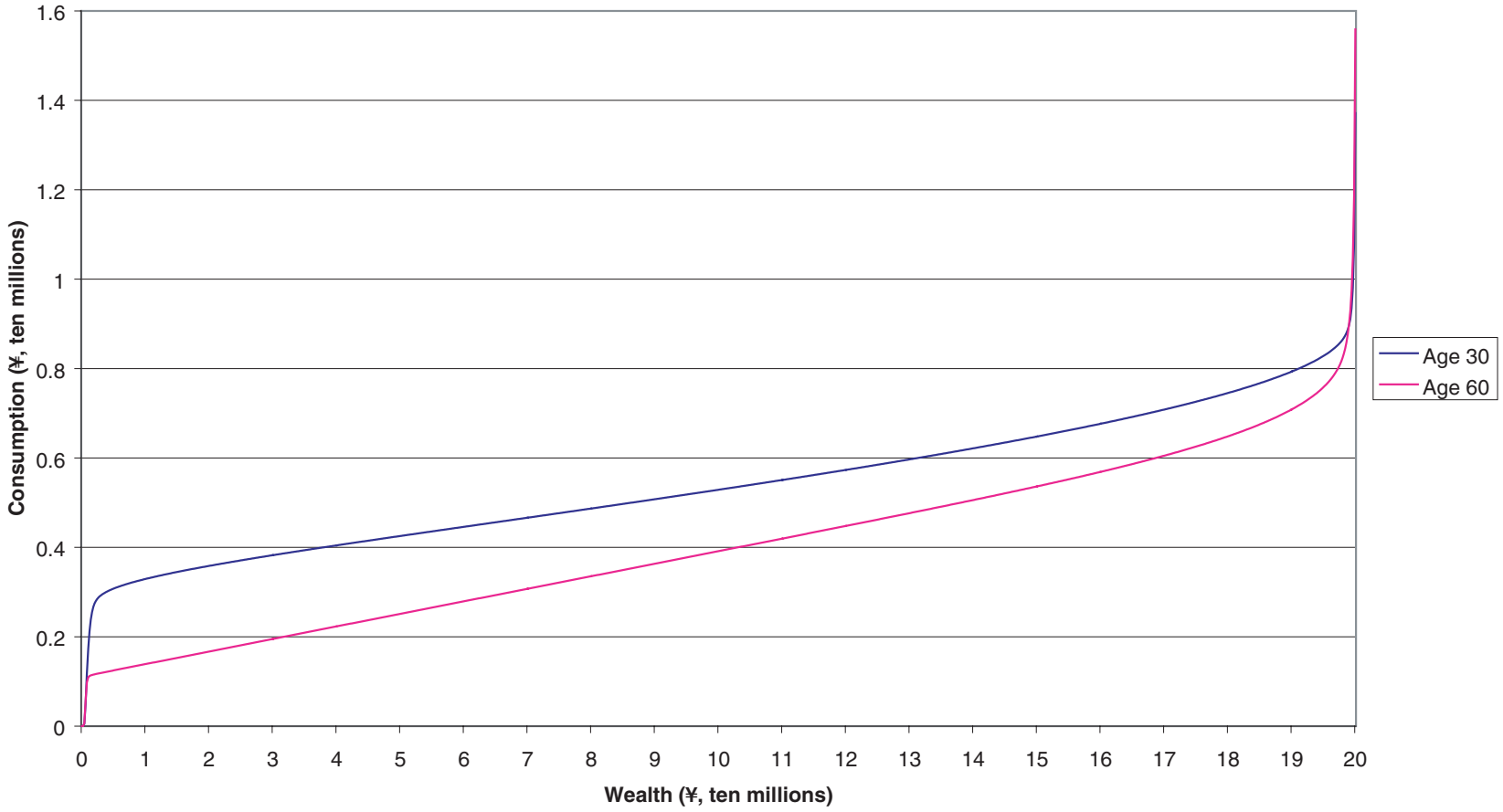

Y
h
h
W
t
t+δ
t


V(W,Y,t)
V(W,Y,ω−δ1−δ2)
V(W,Y,ω−δ1)
V(W,Y,ω)
Finish
at desired age
Next grid
START
at age t=ω

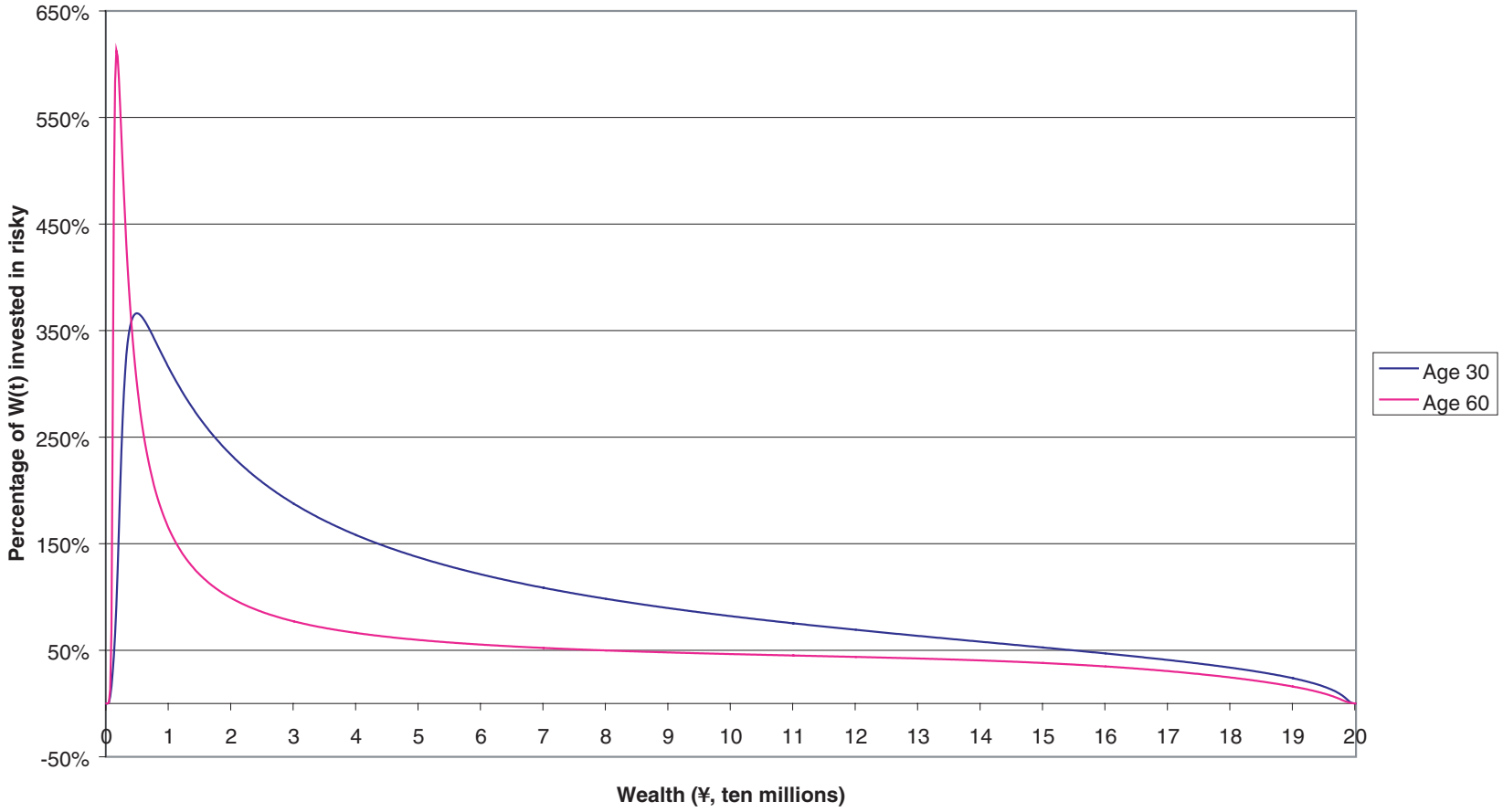
6 Results

1. Comparative statics
   (a) Salary fixed
   (b) Wealth fixed
2. Expected paths

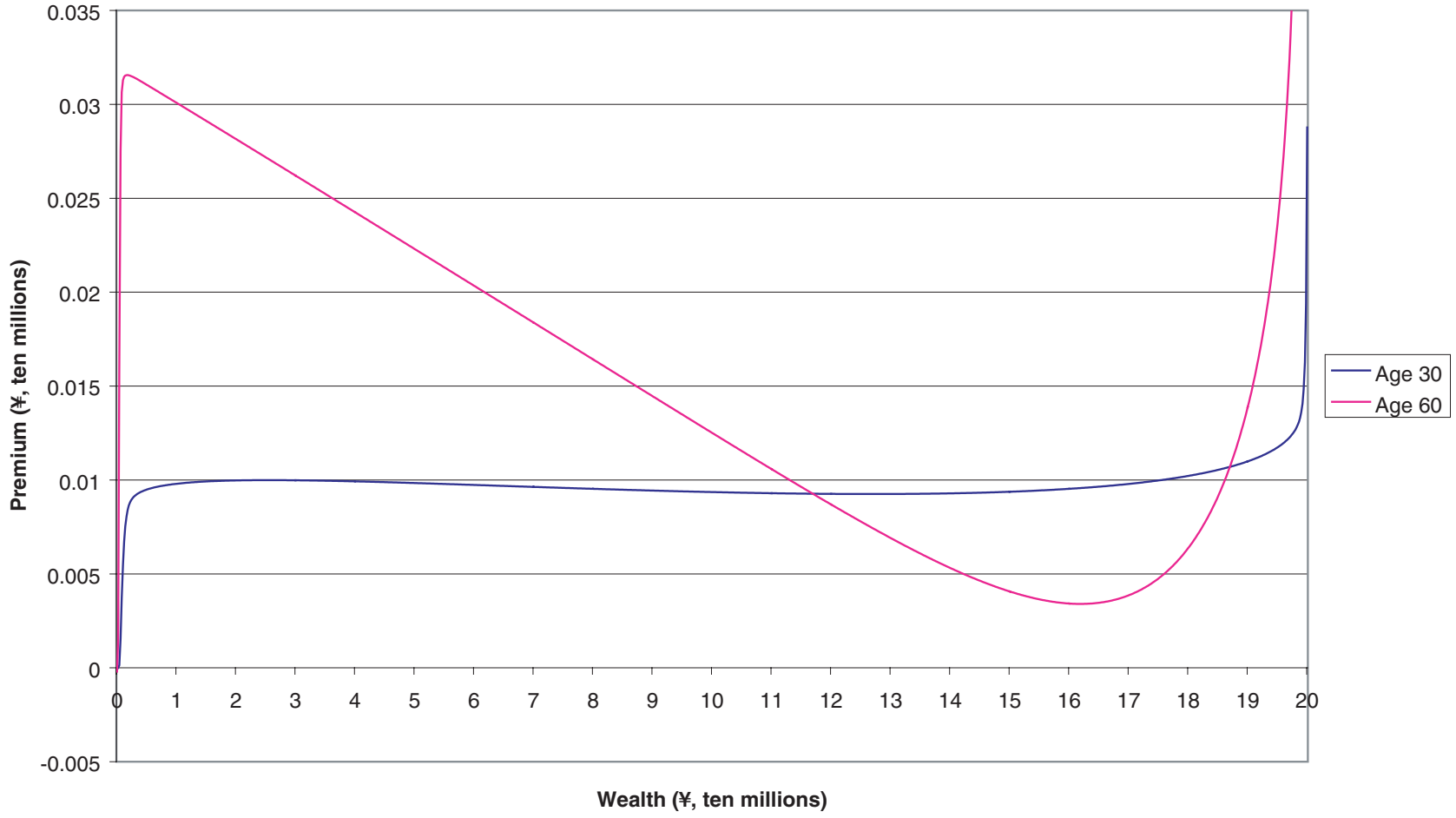
Fixed salary results



Consumption and wealth relationship, salary fixed.

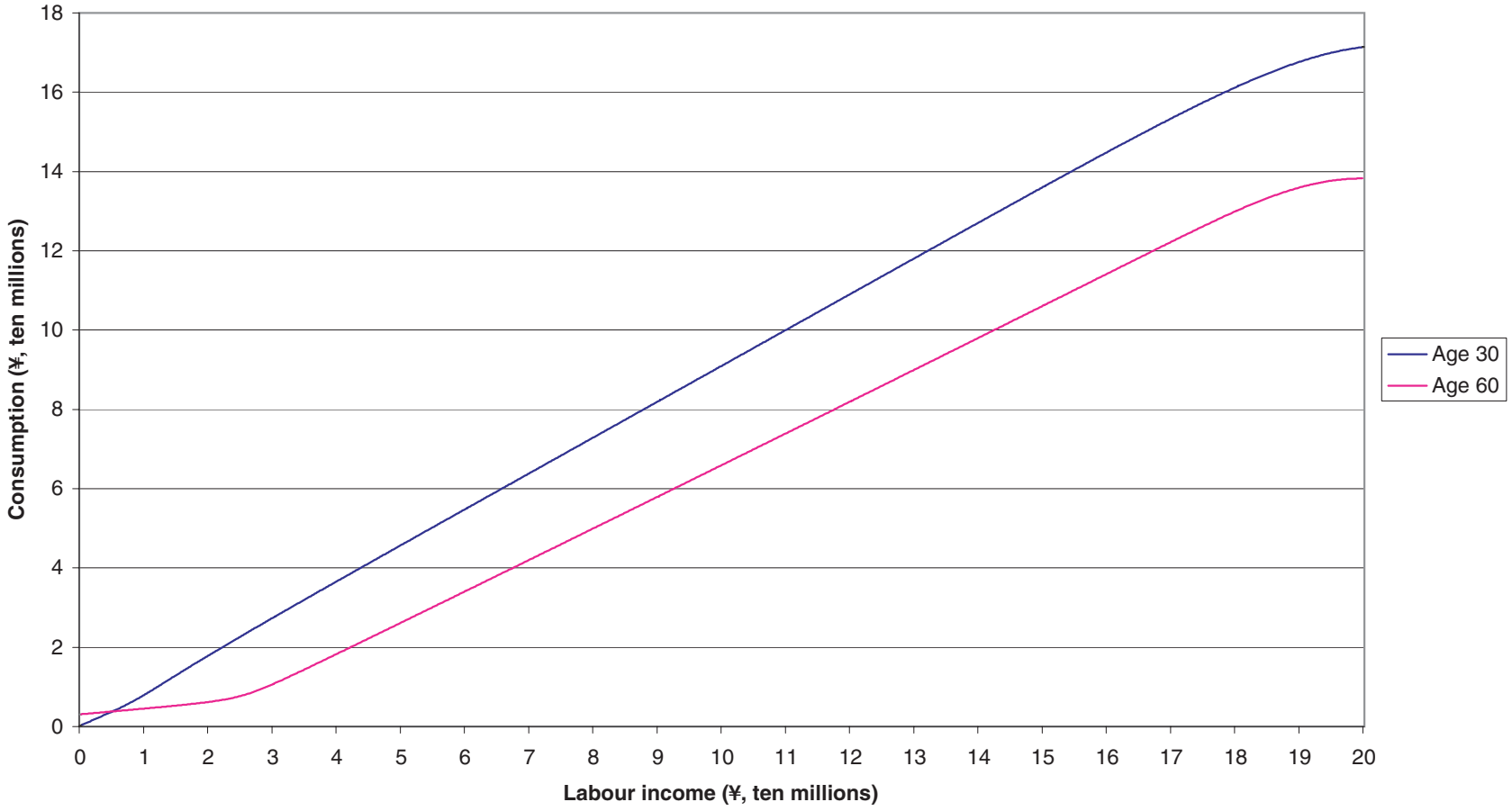


Investment and wealth relationship, salary fixed.

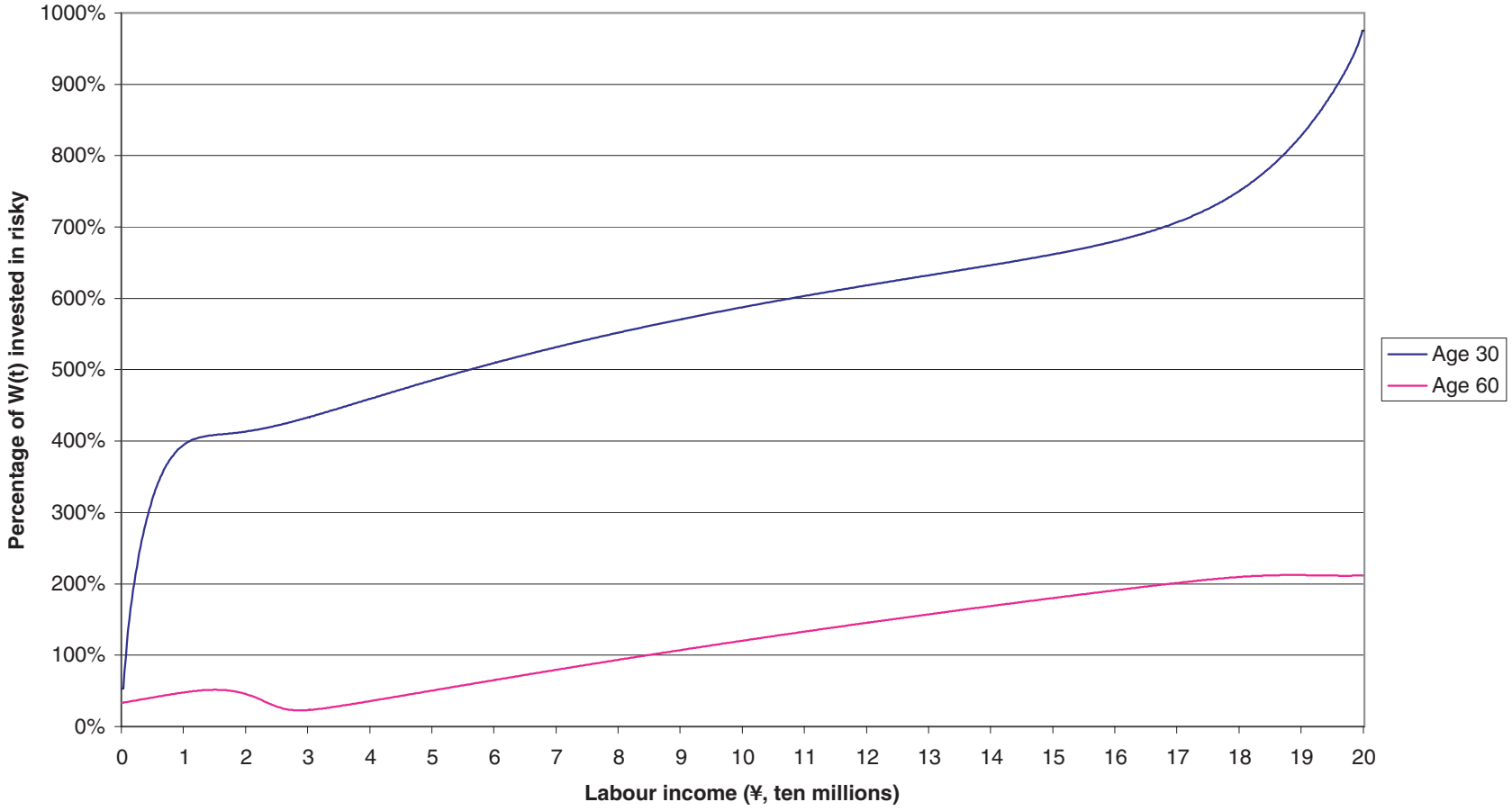


Premium and wealth relationship, salary fixed.

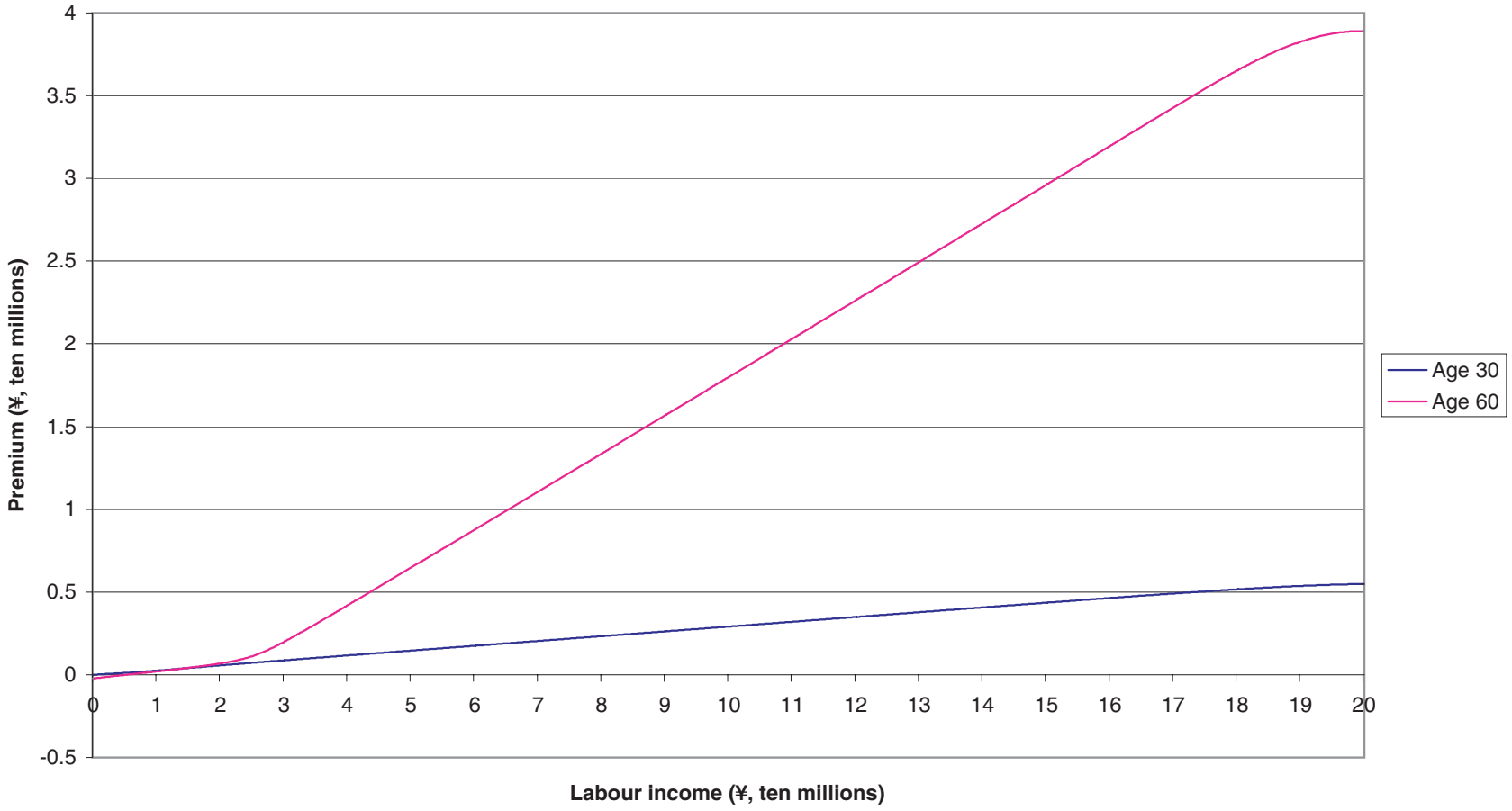
Fixed wealth results



Consumption and salary relationship, wealth fixed.

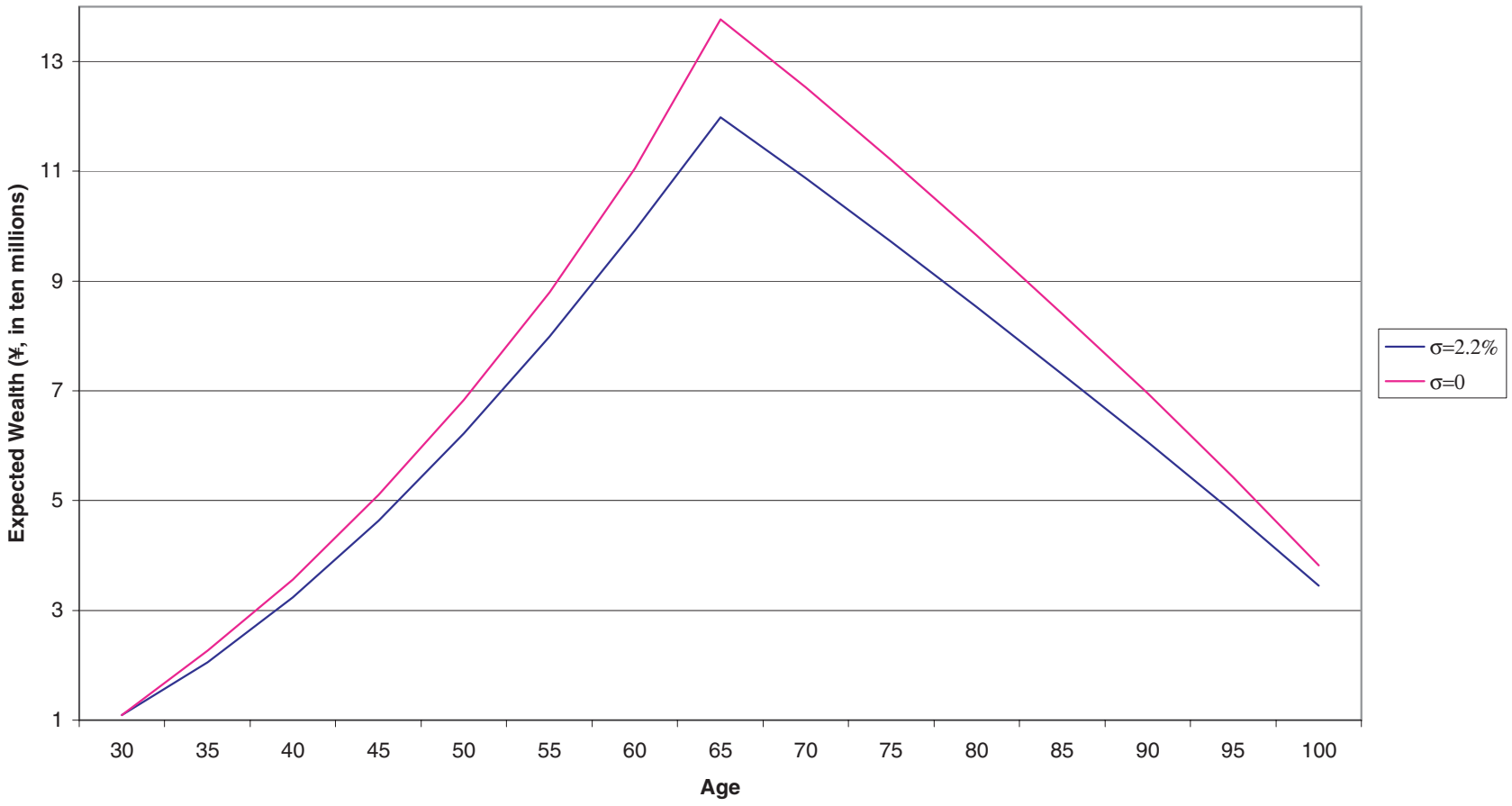


Investment and salary relationship, wealth fixed.

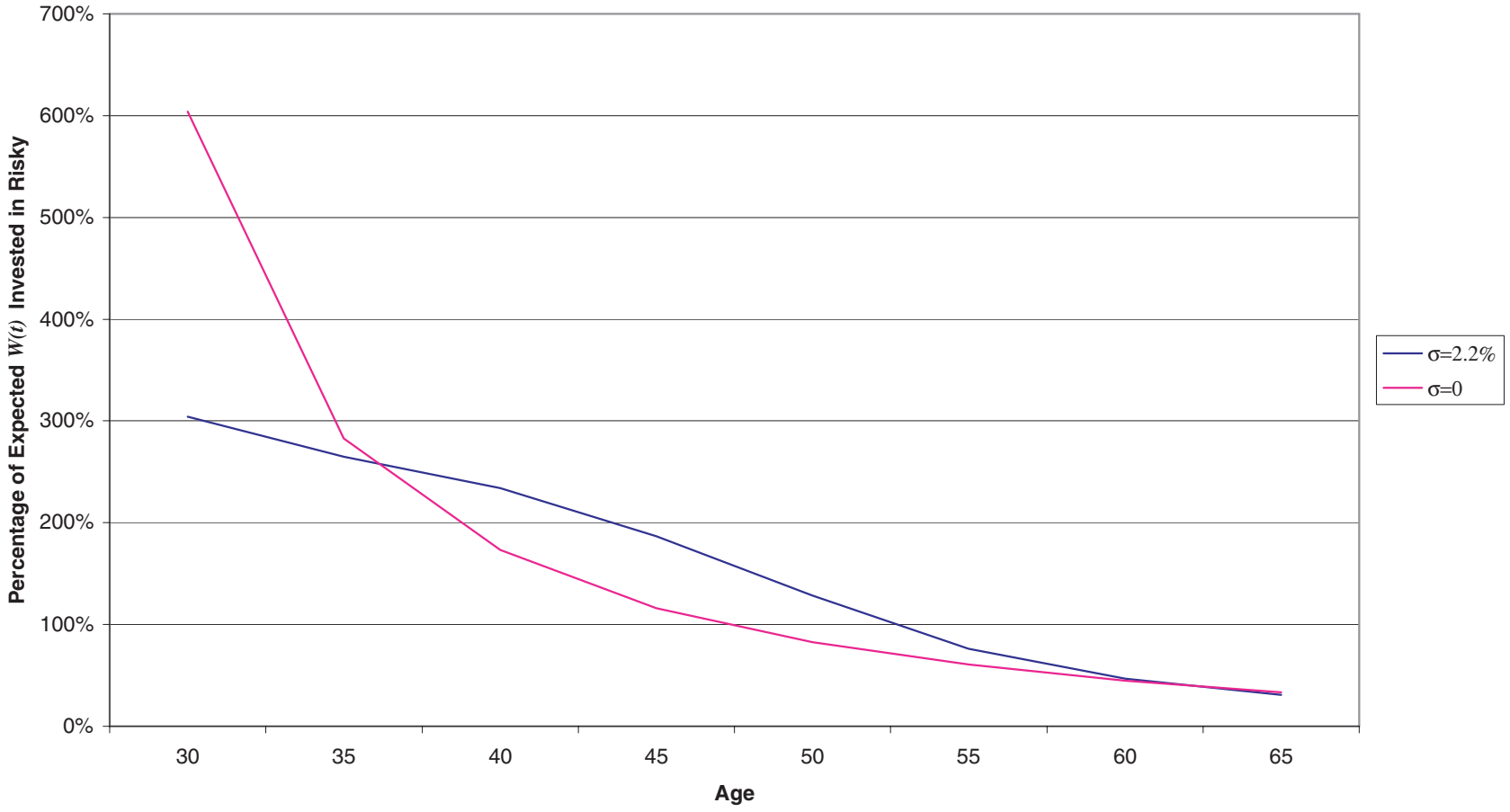


Premium and salary relationship, wealth fixed.

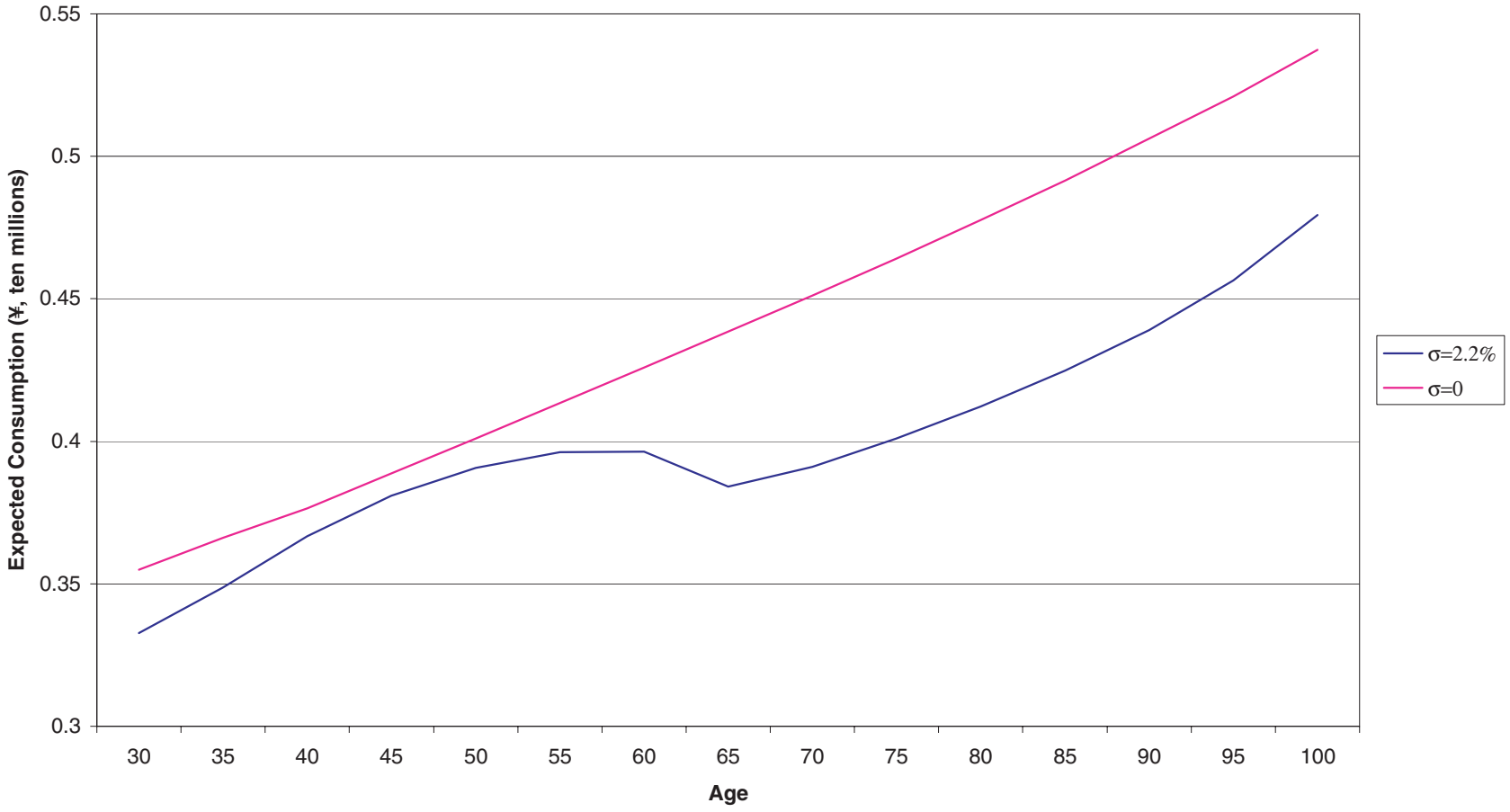
Expected paths results



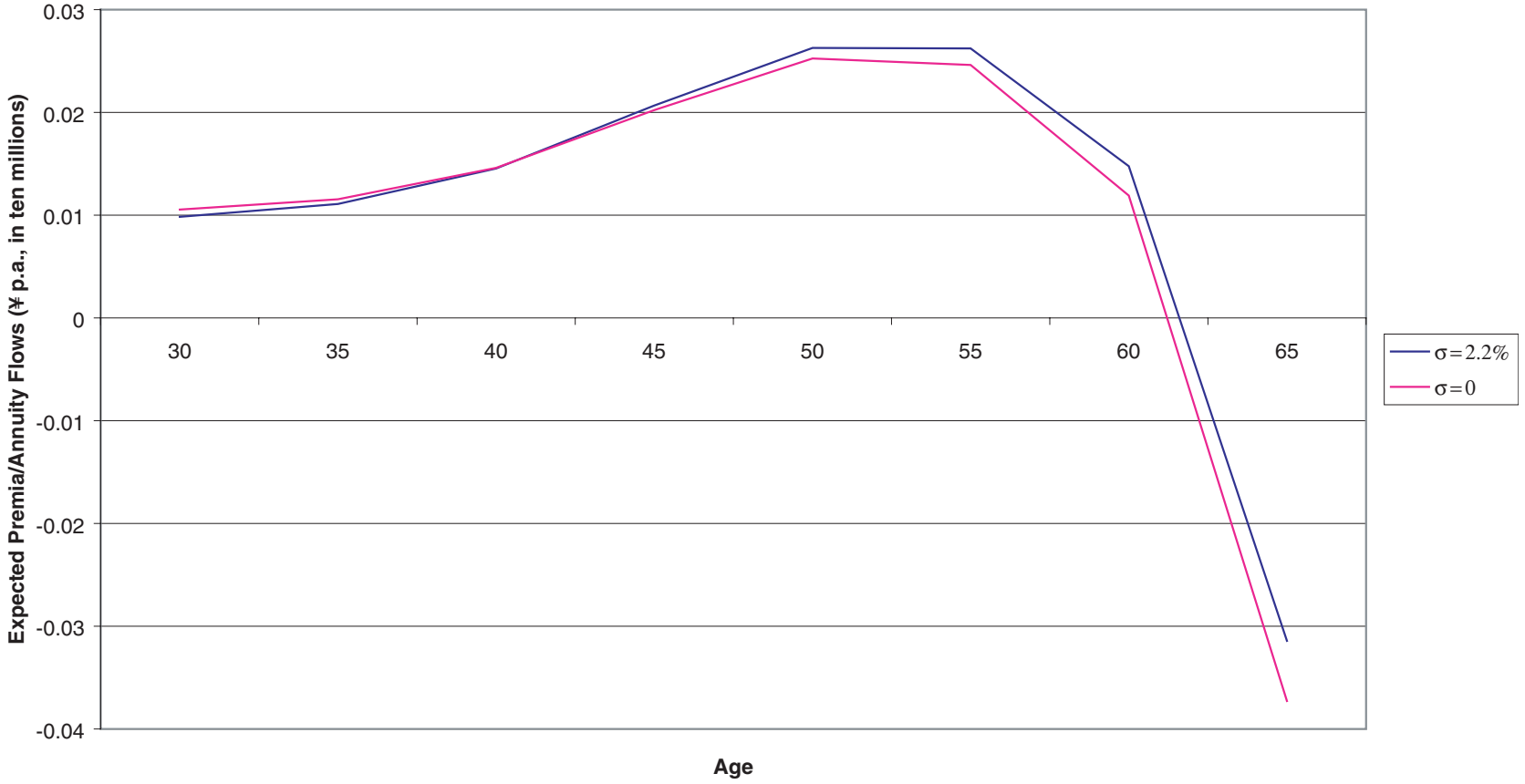
Expected wealth path.



Expected investment path.



Expected consumption path.



Expected premium path.

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