On the Utility Premium of Friedman and Savage

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Abstract

We re-examine the utility premium of Friedman and Savage (1948) and show how this somewhat neglected measure is actually quite useful in analyzing choice under risk. In particular, we decompose the risk premium into two subcomponents: (1) the utility premium, which measures the degree of "pain" associated with a particular risk, and (2) a measure of willingness to pay to remove a unit of "pain." We consider both additive and multiplicative risks and we show how the reaction of the utility premium to changes in wealth equates to a precautionary demand for saving.

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1 Introduction

A cornerstone for modern research on the economics of risk, at least in an expected-utility framework, has been the risk premium, which converts subjective preference towards risk into a monetary cost. To the best of our knowledge, the concept was originally introduced in a formal setting by Friedman and Savage (1948).[1] However, it was not until a seminal paper by Pratt (1964) introduced the risk premium in a more general setting that the measure became such an important tool for analyzing choice under risk. The fact that it can be linked to local properties of utility in an expected-utility framework was part of the beauty of Pratt. Using the risk premium as a starting point, many modern theories have examined how assumptions about risk attitudes can be captured via properties of the utility function.

This is in contrast to the utility premium, also introduced by Friedman and Savage (1948), which measures the degree of "pain" associated with risk, where pain is measured via a decrease in expected utility. Unlike the risk premium, the concept of the utility premium, which was exposited some sixteen years before Pratt's paper, has all but vanished from the literature on decision making under risk.

In this paper, we take another look at the utility premium and we show its rele-

[1] Friedman and Savage (1948) introduced the notion of the "income equivalent" for a given risk, which in today's terminology is the "certainty equivalent." They also discuss the difference between the mean of a random wealth distribution and this income equivalent, which of course is the measure more formally defined by Pratt (1964) as the "risk premium."

vance for decision making under risk. In particular, we decompose the risk premium into two separate subcomponents. The first is the measure of "pain" from exposure to a particular risk, i.e. the utility premium. The second is a willingness to pay to remove each unit of "pain." We then examine the effects of increasing wealth on each component for two particular cases: one where the risk is of a fixed size and one where the risk is proportional to the size of nonrandom wealth. Whereas monotonicity of the risk premium in wealth, especially "decreasing absolute risk aversion," has an abundant literature exploring its implications for decisions in portfolio choice and insurance, we show how monotonicity of the utility premium in wealth has implications for various precautionary-savings models.

2 Decomposing the (additive) risk premium

Let an individual's final wealth be represented by $w + \widetilde{\varepsilon}$, where $w > 0$ denotes the expected wealth of an individual and $\widetilde{\varepsilon}$ is a zero-mean random variable. The risk premium $\pi(w)$ for the risk $\widetilde{\varepsilon}$ at expected wealth w can be defined implicitly via

$$Eu(w + \widetilde{\varepsilon}) = u(w - \pi(w)), \tag{1}$$

where u denotes the individual's von Neumann-Morgenstern utility function and E denotes the expectation operator. We assume throughout this article that u is thrice differentiable with $u' > 0$ and $u'' < 0$. We also assume that the support of $\widetilde{\varepsilon}$ is defined such that $w + \varepsilon$ is in the domain of u.

Using "pain," as measured by a loss in utility from adding risk $\widetilde{\varepsilon}$, the utility premium can be defined as

$$v(w; \widetilde{\varepsilon}) \equiv u(w) - Eu(w + \widetilde{\varepsilon}). \tag{2}$$

It is then a tautology that we can write

$$\pi(w) = [u(w) - Eu(w + \widetilde{\varepsilon})] \times \left[\frac{\pi(w)}{u(w) - Eu(w + \widetilde{\varepsilon})}\right]. \tag{3}$$

The first term in (3) is simply the level of "pain" [utility premium] associated with risk $\widetilde{\varepsilon}$, whereas the second term is the average willingness to pay per unit of "pain" to eliminate the risk $\widetilde{\varepsilon}$.

We know a great deal about the risk premium $\pi(w)$, since it has been the focus of much research over the past forty years. We know quite a bit less about the utility premium, which has been largely ignored in the literature. One exception is Hanson and Menezes (1970), who use the utility premium to justify an assumption of $u''' \geq 0$. In particular, they showed that the utility premium is decreasing as one gets wealthier, i.e. $v'(w;\widetilde{\varepsilon}) \equiv \partial v(w;\widetilde{\varepsilon})/\partial w \leq 0$, for all w and all $\widetilde{\varepsilon}$ if and only if $u''' \geq 0$. This is easily seen to follow directly from (2) and Jensen's inequality. Another exception is Eeckhoudt and Schlesinger (2006), who show how the utility premium can be used to characterize the signs of all the derivatives of u.

Another paper of interest is by Jia and Dyer (1996). Although they did not present their analysis in terms of the utility premium *per se*, it is easy to recast their model in this direction. In particular, Jia and Dyer consider two zero-mean risks, say $\widetilde{\varepsilon}$ and $\widetilde{\delta}$, and ask the question: If $w + \widetilde{\varepsilon}$ is preferred to $w + \widetilde{\delta}$, when can we say that $w' + \widetilde{\varepsilon}$ is preferred to $w' + \widetilde{\delta}$ for all wealth levels w'? Their results are fairly weak, showing that this claim holds only for a very limited set of utility functions.[2]

The last term in (3), the willingness to pay (hereafter "WTP"), is of course defined via the risk premium and utility premium. By analyzing well-known effects of wealth changes on $\pi(w)$ and the easily calculable changes in $v(w;\widetilde{\varepsilon})$, we also determine the effects of wealth upon the WTP to remove a unit of "pain" from the risk $\widetilde{\varepsilon}$. Of course, the risk premium itself denotes a type of "willingness to pay" to remove the

[2] Jia and Dyer (1996) can be recast as follows: if $v(w;\widetilde{\varepsilon}) - v(w;\widetilde{\delta}) < 0$, when can we say that $v(w';\widetilde{\varepsilon}) - v(w';\widetilde{\delta}) < 0$? The only utility functions that can guarantee this behavior are (1) quadratic utility, $v(w;\widetilde{\varepsilon}) - v(w;\widetilde{\delta}) = k[var(\widetilde{\varepsilon}) - var(\widetilde{\delta})]$ for some positive k; (2) constant absolute risk aversion, $v(w;\widetilde{\varepsilon}) - v(w;\widetilde{\delta}) = u(w - \pi_\varepsilon) - u(w - \pi_\delta)$ where π_ε and π_δ are both constants; and (3) "one-switch" utility $u(w) = aw - b\exp(-cw)$, so that $v(w;\widetilde{\varepsilon}) - v(w;\widetilde{\delta}) = b\exp(-cw)[E(\exp(-c\widetilde{\varepsilon}) - E(\exp(-c\widetilde{\delta})]$. A somewhat similar analysis is also embedded in a paper by Bell (1995).

entire risk $\widetilde{\varepsilon}$.[3] In order to avoid potential confusion, we will use the notation WTP in this paper solely with reference to a "willingness to pay to remove a unit of pain."

3 Changes in wealth

Ever since Arrow (1965), we have thought of decreasing absolute risk aversion (DARA) as a very canonical type of behavior: As one gets richer, one would not pay as much to remove the zero-mean risk $\widetilde{\varepsilon}$ from one's wealth. Part of Arrow's original justification for this hypothesis is based upon the behavioral consequences of this assumption: "If absolute risk aversion increased with wealth, it would follow that as an individual became wealthier, he would actually decrease the amount of risky assets held." [Arrow (1971), p. 96]

Consider an increase in wealth w. We wish to examine conditions for which $\pi'(w) \equiv \partial\pi(w)/\partial w \gtreqless 0$.[4] Let $g_v(w) \equiv v'(w;\widetilde{\varepsilon})/v(w;\widetilde{\varepsilon})$ denote the growth rate of v with respect to an increase in wealth w. It follows from (2) that if u''' is monotone in sign, then $sgn(g_v) = -sgn(u''')$. Meanwhile, using (3), it is easily shown that the growth rate of the willingness to pay is $g_{wtp}(w) \equiv [\pi'(w)/\pi(w)] - g_v(w)$. Comparing

[3] In words, (3) simply states that the willingness to pay to remove the risk $\widetilde{\varepsilon}$ is equal to the average willingness to pay to remove a unit of pain times the level of pain caused by $\widetilde{\varepsilon}$.

[4] We only show our results for non-strict inequalities. Strict versions also follow, but would necessitate a fair amount of additional mathematical detail, without much in the way of additional insights.

$g_v(w)$ and $g_{wtp}(w)$ shows whether $\pi'(w) \gtreqqless 0$.

Our point in performing the above decomposition is not to determine the sign of $\pi'(w)$, which has been examined often. But, by knowing the sign of $\pi'(w)$, we are able to examine the signs of the two terms in its decomposition. Consider a few simple examples:

Example 1 *Let utility be quadratic, $u(w) = w - kw^2$, $k > 0$, where we limit the domain of wealth to levels for which u is increasing. In this case, it follows that $g_v(w) = 0$ for all w and that $g_{wtp}(w) \equiv [\pi'(w)/\pi(w)] > 0$. As wealth increases, the level of pain from risk $\widetilde{\varepsilon}$ does not change. However, the willingness to pay to remove each unit of pain is increasing in wealth. Hence, as is well known, the risk premium is increasing in wealth.*

Example 2 *Let utility exhibit constant absolute risk aversion (CARA), so that $\pi'(w) = 0$. Then $0 > g_v(w) = -g_{wtp}(w)$, so that pain is decreasing in wealth, whereas the willingness to pay to remove each unit of pain is increasing in wealth. Of course, under CARA, these two effects exactly offset one another. The amount one would pay to remove the entire risk $\widetilde{\varepsilon}$ remains constant as wealth changes.*

Example 3 *Under DARA, the level of pain is always decreasing as wealth increases, whereas the willingness to pay to remove each unit of pain is always increasing in wealth, but at a slower rate.*

Indeed, it follows from our definition that WTP must always be increasing in wealth under risk aversion. To see this, consider first that $d\pi/dw$ must always be strictly less than one. This follows trivially from (1), since $[1 - d\pi/dw] = Eu'(w + \widetilde{\varepsilon})/u'(w - \pi) > 0$. Thus,

$$\begin{aligned} \frac{dWTP}{dw} \times v^2(w; \widetilde{\varepsilon}) &= \left\{ v\frac{d\pi}{dw} - \pi[u'(w) - (1 - \frac{d\pi}{dw})u'(w - \pi)] \right\} \\ &> [1 - \frac{d\pi}{dw}]\pi[u'(w) - u'(w - \pi)] > 0. \end{aligned} \tag{4}$$

The first inequality in (4) follows under risk aversion, since we have $\pi u'(w) < v$. Hence, WTP is always increasing in wealth.

The above analyses and examples are all taken for a fixed risk $\widetilde{\varepsilon}$, but oftentimes we are interested in behavior towards a "small risk." To this end, consider the risk $t\widetilde{\varepsilon}$ for $t > 0$ and let $t \to 0^+$. From (1) and (2), we now have

$$Eu(w + t\widetilde{\varepsilon}) = u(w - \pi(w, t)) \tag{5}$$

and

$$v(w; t\widetilde{\varepsilon}) \equiv u(w) - Eu(w + t\widetilde{\varepsilon}). \tag{6}$$

Assuming that u is at least twice differentiable, we know from Segal and Spivak (1990) that risk aversion is a second-order effect. In particular, it follows from (5) that $\partial\pi/\partial t|_{t=0} = 0$ and $\partial^2\pi/\partial t^2|_{t=0} > 0$. That is, a risk-averse individual is indistinguishable from a risk-neutral one in behavior towards an infinitesimal risk. It is trivial to see from (6) that "pain" is also a second-order phenomenon under risk aversion, with $\partial v/\partial t|_{t=0} = -E[u'(w + t\widetilde{\varepsilon})\widetilde{\varepsilon}]|_{t=0} = -cov[u'(w + t\widetilde{\varepsilon}), \widetilde{\varepsilon}]|_{t=0} = 0$ and $\partial^2 v/\partial t^2|_{t=0} = -E[u''(w + t\widetilde{\varepsilon})\widetilde{\varepsilon}^2]|_{t=0} > 0$.

Now consider the willingness to pay to remove a unit of pain, which we can write as WTP$\equiv \pi(w,t)/v(w;t\widetilde{\varepsilon})$. We can easily determine how WTP behaves for $t \to 0^+$ by applying L'Hôpital's rule twice to obtain

$$\text{WTP} \to [\partial^2\pi/\partial t^2|_{t=0}]/[\partial^2 v/\partial t^2|_{t=0}] > 0.$$

In other words, willingness to pay to remove a unit of pain is a first-order effect. The individual is willing to pay to alleviate any pain stemming from a risk, even an infinitely small amount of pain; but an infinitesimal risk generates no pain whatsoever. Note that under risk aversion, for any value of $t > 0$, we have $\pi(w,t)u'(w) < v(w;t\widetilde{\varepsilon}) < \pi(w,t)u'(w - \pi(w,t))$. It thus follows that we can bound WTP via $[u'(w - \pi(w,t))]^{-1} <WTP< [u'(w)]^{-1}$. Hence as $t \to 0^+$ we have WTP$\to [u'(w)]^{-1}$. For a small risk, the willingness to pay to remove a unit of pain is inversely proportional to marginal utility.[5]

4 Marginal Changes in Risk

The risk premium indicates how much wealth an individual would be willing to forego to completely eliminate the risk $\widetilde{\varepsilon}$. There is also a large literature on how to value the avoidance of a marginal increase in the level of risk; or equivalently, how to value a marginal decrease in the level of risk.[6] To this end, we can adapt the above analysis and consider the level of incremental pain caused by increasing the risk. The incremental pain can be defined as

$$IP(t) \equiv [Eu(w+\widetilde{\varepsilon}) - Eu(w+t\widetilde{\varepsilon})]/(t-1), \tag{7}$$

where $t \geq 1$. This measure is analogous to the utility premium, but for the incremental level of risk $(t-1)\widetilde{\varepsilon}$. Taking the limit as $t \to 1^+$ and applying L'Hôpital's

[5]Risk aversion, which is a second-order effect under differentiable utility, can actually be decomposed into two multiplicative effects: (1) "pain", which is a second-order effect, and (2) WTP, which is a first-order effect. We thus see that risk aversion is a second-order effect only because infinitessimal risk causes no pain.

[6]A classical example is the valuation of a statistical life, as modeled by Jones-Lee (1976). A good overview of the extensive literature is by Viscusi (1993).

rule we obtain the marginal pain caused by increasing the level of $\widetilde{\varepsilon}$-risk:

$$MP \equiv -E[u'(w+\widetilde{\varepsilon})\widetilde{\varepsilon}]. \tag{8}$$

The amount that an individual is willing to forego to eliminate the marginal risk is easily found by setting $Eu(w + t\widetilde{\varepsilon})$ equal to a constant and totally differentiating. Evaluated at $t = 1$, this leads to

$$[dw/dt]|_{t=1} = [-Eu'(w+\widetilde{\varepsilon})\widetilde{\varepsilon}]/Eu'(w+\widetilde{\varepsilon}). \tag{9}$$

The numerator, and hence dw/dt, is positive due to risk aversion. Using (8) and (9), we once again obtain a tautological decomposition as

$$[dw/dt] = MP \times \{[dw/dt]/MP\} = MP \times [Eu'(w+\widetilde{\varepsilon})]^{-1}. \tag{10}$$

The last term in (10) is a measure of the willingness to pay to remove one unit of pain, where the pain now is caused by the marginal increase in the $\widetilde{\varepsilon}$-risk. Notice that we have a measure of WTP "in the small," due to the fact that we are considering an infinitesimal increase in the level of risk. This is quite similar to the case where the risk itself is infinitesimal, as discussed in the previous section. Assuming risk aversion, we see once again that this willingness to pay to avoid pain always is increasing in wealth, where now the pain stems from a marginal increase in the $\widetilde{\varepsilon}$-risk. Meanwhile, differentiating MP with respect to wealth, shows that "marginal pain" will decrease [respectively increase] in wealth for any arbitrary $\widetilde{\varepsilon}$ if and only if $u''' > 0$ [respectively $u''' < 0$]. In other words, "marginal pain" is decreasing in wealth if and only if the total pain of $\widetilde{\varepsilon}$ is decreasing in wealth.

5 Multiplicative Risks

The preceding analysis is based upon a fixed additive risk $\widetilde{\varepsilon}$. Much analysis in economics and finance deals with multiplicative risks. To this end, we now let wealth be defined as $w\widetilde{y}$ where $\widetilde{y} = 1 + \widetilde{\varepsilon}$. We maintain the assumption that $E\widetilde{\varepsilon} = 0$ and we assume that the support of $w\widetilde{y}$ is limited to levels of wealth over which preferences are well defined. The multiplicative risk premium $\widehat{\pi}(w)$ for a fixed random variable $\widetilde{y}$ is defined implicitly via

$$Eu(w\widetilde{y}) = u(w(1 - \widehat{\pi}(w))). \tag{11}$$

Note that $\widehat{\pi}(w)$ itself measures a proportion of wealth w, so that the monetary amount one is willing to forgo to eliminate the risk is equal to $w\widehat{\pi}(w)$.

We define the utility premium for the multiplicative risk $\widetilde{y}$ as

$$\widehat{v}(w;\widetilde{y}) = u(w) - Eu(w\widetilde{y}). \tag{12}$$

Hence, we can once again decompose the risk premium in to a measure of "pain" and a willingness to pay per unit of pain removed:

$$w\widehat{\pi}(w) = \widehat{v}(w;\widetilde{y}) \times \frac{w\widehat{\pi}(w)}{\widehat{v}(w;\widetilde{y})}. \tag{13}$$

Unlike in the additive case, where prudence alone was equivalent to the pain of an additive risk being decreasing in wealth, prudence no longer implies this result. We wish to evaluate the sign of $\widehat{v}'(w;\widetilde{y}) \equiv \partial\widehat{v}(w;\widetilde{y})/\partial w = u'(w) - E[\widetilde{y}u'(w\widetilde{y})]$. Once again applying Jensen's inequality, we see that $\widehat{v}'(w;\widetilde{y}) < [>]\ 0$ for all w and $\widetilde{y}$, if and only if the function $f(y) \equiv yu'(wy)$ is convex [concave]. Straightforward calculations show that f is convex [concave] if and only if relative prudence, $-wu'''(w)/u''(w)$, exceeds [is less than] 2.

Similar to our previous analysis, we can define and compare the growth rates of the level of pain and the WTP with respect to changes in wealth. These growth rates are $G_{\widehat{v}}(w) = \widehat{v}'(w;\widetilde{y})/\widehat{v}(w;\widetilde{y})$ and $G_{wtp}(w) = [\widehat{\pi}'(w)/\widehat{\pi}(w)] - G_{\widehat{v}}(w)$ respectively. As an example, consider the very common assumption of constant relative risk aversion

(CRRA).

Example 4 *Let utility exhibit CRRA with level of risk aversion $\gamma > 0$, so that either $u(w) \equiv \frac{1}{1-\gamma} w^{1-\gamma}$ for $\gamma \neq 1$, or $u(w) \equiv \ln w$. It follows that relative prudence is also constant, with $-wu'''(w)/u''(w) = 1 + \gamma$. Thus if $\gamma > 1$, pain is decreasing in wealth ($G'_{\widehat{v}}(w) < 0$) and WTP is increasing in wealth ($G_{wtp}(w) < 0$). If $\gamma < 1$, then the directions of both of these effects are reversed. In either case, since $\widehat{\pi}'(w) = 0$, these two effects offset one another under CRRA.*

We can consider the case for a "small risk" by letting $\widetilde{y} \equiv 1 + t\widetilde{\varepsilon}$ and examining what happens as $t \to 0^+$. Once again, both risk aversion and the level of pain are second-order effects, with $\partial\widehat{\pi}(w,t)/\partial t|_{t=0} = 0$ and $\partial\widehat{v}/\partial t|_{t=0} = -cov[u'(w(1 + t\widetilde{\varepsilon})), w\widetilde{\varepsilon}]|_{t=0} = 0$. We can use L'Hôpital's rule, in a manner similar to the case of additive risk to show that WTP, as implicitly defined by (13), is a first-order phenomenon. Indeed, for any $t > 0$ it follows that $[u'(w(1-\widehat{\pi}))]^{-1} < \text{WTP} < [u'(w)]^{-1}$, so that WTP to remove a unit of pain is inversely proportional to marginal utility when the risk is infinitesimal.

6 Relation to Precautionary Saving

It is interesting to relate how changes in the utility premium with respect to wealth determine whether there is a precautionary savings demand in a two-period consumption-savings model. To this end, we consider two different models of precautionary saving.

Assume a two period model where both the risk-free interest rate and the rate of discounting for delayed consumption are both zero. The first model examines a consumer who has an income of w in the first period and an expected income of w in the second period. He or she decides on how much to save and how much to consume in the first period:

$$\max_s H(s; w) \equiv u(w - s) + Eu(w + \widetilde{\varepsilon} + s). \tag{14}$$

We do not necessarily restrict $s \geq 0$. Under risk aversion, it is trivial to show that the objective function H is globally concave in s. If there is no uncertainty in the second period ($\widetilde{\varepsilon} \equiv 0$), it is easy to see that the optimal savings is $s^* = 0$. For the general case, since H is concave in s, it follows that there is a precautionary demand for saving, $s^* > 0$, if and only if $[u'(w + s) - Eu'(w + \widetilde{\varepsilon} + s)]|_{s=0} < 0$, i.e. if and only if the utility premium is decreasing in wealth, $v'(w; \widetilde{\varepsilon}) < 0$. From our analysis in the previous section, this holds for every w and $\widetilde{\varepsilon}$ if and only if the decision maker is prudent.

In the second model, we assume that the individual has an initial wealth of $2w$, with no other income. The individual must decide how much to save and how much to consume in the first period; but now we assume that the interest rate for savings is stochastic. The optimization program is

$$\max_s J(s;w) \equiv u(2w-s) + Eu(s(1+\widetilde{\varepsilon})). \tag{15}$$

The objective function J is globally concave in s. For $\widetilde{\varepsilon} \equiv 0$, the optimal savings strategy is to save one-half of initial wealth, $s^* = w$. For the general case, there will be a precautionary demand for saving, $s^* > w$, if and only if $[Eu'(s) - Eu'(s(1+\widetilde{\varepsilon}))]|_{s=w} < 0$, i.e. if and only in the utility premium for multiplicative risk is decreasing in wealth, $\widehat{v}'(w;\widetilde{\varepsilon}) < 0$. From our analysis in the previous section, this holds for every w and $\widetilde{\varepsilon}$ if and only if relative prudence exceeds 2.[7]

We see in both models that a demand for precautionary saving follows if and only if the level of "pain" from the risk $\widetilde{\varepsilon}$ is decreasing in wealth. For the risky second-period income model (14), we consider the additive risk $\widetilde{\varepsilon}$. For the case of a risk-savings-rate model (15), we consider a multiplicative risk $\widetilde{\varepsilon}$. In each model, by

[7] The first proof of this result that we are aware of is by Rothschild and Stiglitz (1971), who at the time of their proof could not express their result in terms of "prudence."

shifting some wealth to the second period, we reduce the "pain" of the $\widetilde{\varepsilon}$ risk.

7 Concluding Remarks

We decomposed the risk premium into the product of two effects: (1) a measure of pain as expressed by the utility premium and (2) a willingness to pay to remove each unit of pain. Although both effects are needed to determine whether risk aversion is decreasing in wealth, having the first term (the utility premium) decreasing in wealth turns out to be equivalent to a precautionary demand for saving in a two-period model.

The decomposition in (3) also is useful in rethinking analyses about changes in absolute risk aversion with respect to changes in wealth. Consider Example 1, for instance. Quadratic utility displays the often criticized property of increasing absolute risk aversion (IARA). But as we now see, this follows solely from the WTP effect. Indeed, since WTP always is increasing in wealth, a utility displaying IARA must have one of the following three traits:

(i) "pain" also is increasing in wealth; preferences are imprudent with $u''' < 0$,

(ii) "pain" is invariant to wealth, as in the case of quadratic utility, $u''' = 0$,

(iii) "pain" is decreasing, but too slowly to offset WTP, with $0 < u''' < [u'']^2/u'$.[8]

[8] This follows directly from Pratt (1964).

Of course, in an abundance of situations, we need to know whether risk aversion is increasing or decreasing. But it is interesting to see how relevant the result of Friedman and Savage (1948) is in analyzing behavior under risk. It also is a bit surprising that a reexamination of their results, such as the one presented here, was mostly overlooked by the literature for the past 60 years.

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