Modeling the Spot Price of Electricity in Deregulated Energy Markets

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Outline

• Empirical Analysis of Electricity Spot Prices
  – Physical vs. financial assets
  – Trajectorial vs. statistical properties of market prices
  – Endogenous factors

• A Jump-Diffusion Model
  – Desirable features
  – Modules: trend, noise, and spikes
  – The model

• Model Calibration
  – Step 1: Fitting structural elements
  – Step 2: Parameter estimation
  – Empirical results
Empirical Analysis of Electricity Spot Prices

I. Introduction

Market context  Deregulation of energy market → price fluctuation → new typology of risk → hedging needed

Issue  Determine and quantify these relations

Method  Empirical analysis → modelling (qualifying risk) → calibration (quantifying risk) → test (performance analysis)
II. A Special Underlying

**Arbitrage pricing** = derivative price is the minimal capital required to set up a self-financing hedging portfolio of tradeable assets

→ Hypothesis = the underlying is transferable in time (at a cost = interest rate) and space (at some transaction cost).

**Electricity**

- Limited space transferability (capacity constraints, line losses, physical market segmentation).

- Almost impossible time transferability (power cannot be easily stored).

⇒ singular price dynamics → new typology of nondiversifiable risk.
III. Trajectorial Properties of Market Prices
III. Trajectorial Properties of Market Prices

1) Drift = Periodical trend + Mean reversion
2) Volatility = Local perturbations
3) Spikes = Periodical occurrence + Jump reversion

*Warning*: spike = sequence of upward jumps followed by downward jumps ≠ two jumps with opposite sign.
IV. Statistical Properties of Market Prices

- Daily price return of 1 MWh.


<table>
<thead>
<tr>
<th></th>
<th>ECAR</th>
<th>PJM</th>
<th>COB</th>
<th>NP</th>
<th>APX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.000</td>
<td>-0.001</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.353</td>
<td>0.236</td>
<td>0.159</td>
<td>0.099</td>
<td>0.359</td>
</tr>
<tr>
<td>Skew.</td>
<td>-0.557</td>
<td>0.395</td>
<td>0.159</td>
<td>0.461</td>
<td>-0.17</td>
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</tbody>
</table>
V. Driving Factors

**Market:** Inelastic demand function
V. Driving Factors

**Economy:** Real economy growth, generation asset prices

**Weather:** Hot waves, cold winters

**Physical:** Grid balancing $\rightarrow$ marginal prices; transmission constraints $\rightarrow$ line congestion; production system:

- thermal (PJM) $\rightarrow$ non storable $\rightarrow$ capacity *shortfalls* due to demand excess

- hydro (WSCC) $\rightarrow$ stable $\rightarrow$ *shortfalls* due to outages
A Jump-Diffusion Model

I. Desirable Features: A Model Should be...

Representative and Flexible (fit trajectory and statistical properties across different markets + embed all risk factors)

Tractable (Markovian, reduced-form) → Valuation and hedging

Easily implementable → Scenario simulation

Statistically stable → Robustness of hedging prescriptions

Simple → Acceptance by market operators

Tested with respect to these criteria
II. Trend and Local Volatility

Variable

\[ E(t) := \log (\text{Spot Price } (t)) \]

Trend

\[ \mu(t) = \alpha + \beta t + \gamma \cos [\varepsilon + 2\pi t] + \delta \cos [\zeta + 4\pi t] \]

Mean reversion

\[ dE(t) = D\mu(t) \, dt + \theta_1 [\mu(t) - E(t^-)] \, dt + \ldots \]

Local shock

\[ \ldots + \sigma dW(t) + \ldots \]
III. Spikes

Jump component

\[ ... + h(E(t^-)) \, dJ(t) \]

- Jump sign \( \rightarrow h(t^-) = 1 \), if \( E(t^-) < \mu(t) + \Delta \); \(-1\) otherwise.

- Jump size \( \rightarrow J_i \sim p(x; \theta_3, \psi) \propto e^{\theta_3 f(x)}, 0 \leq x \leq \psi. \)

- Jump occurrence \( \rightarrow N(t) \) counting process with freq. \( \nu(t) = \theta_2 \times s(t). \)

- Jump frequency shape \( \rightarrow s(t). \)

- Cumulative jump size \( \rightarrow J(t) = \sum_{i=1}^{N(t)} J_i. \)
IV. The Model

Dynamics

\[ dE(t) = \mu'(t) \, dt + \theta_1 [\mu(t) - E(t^-)] \, dt + \sigma dW(t) + h(E(t^-)) \, dJ \]

Input parameters

<table>
<thead>
<tr>
<th>Structural elements (market specific)</th>
<th>Parameters (time series specific)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu(t) ) Mean trend</td>
<td>( \sigma ) Local volatility</td>
</tr>
<tr>
<td>( s(t) ) Jump frequency shape</td>
<td>( \theta_1 ) Mean reversion force</td>
</tr>
<tr>
<td>( \Delta ) Jump direction switch</td>
<td>( \theta_2 ) Jump frequency magnitude</td>
</tr>
<tr>
<td>( p(x; \theta_3) ) Jump size distribution</td>
<td>( \theta_3 ) Jump size parameter</td>
</tr>
</tbody>
</table>
Model Calibration

I. Fitting Structural Elements

**Trend** → \( \mu(t) = \alpha + \beta t + \gamma \cos[\varepsilon + 2\pi t] + \delta \cos[\zeta + 4\pi t] \)

**Jump frequency shape** → \( s(t) = \left( \frac{2}{1 + \left| \sin\left(\pi(t - \tau)/k\right) \right|} - 1 \right)^2 \)

- \( \tau = \) max. freq. date (summer peak → \( \tau = 0.5 \))
- \( k = \) period (annual periodicity → \( k = 1 \))

**Jump direction switch**: \( \Delta = \alpha\)-quantile of detrended prices

**Jump size distribution**: \( p(x; \theta_3) = \) truncated exponential
II. Parameter Estimation

Idea

Continuous time process $X$

$\downarrow$ Discretization

Discrete time approxim. $X^n$ \xrightarrow{\text{Transition}} \text{Exact Likelihood (approx.proc.) } \mathcal{L}_{X^n}$

Continuous time process $X$ \xrightarrow{\text{Girsanov}} Exact Likelihood (cont. observ.) $\mathcal{L}$

$\downarrow$ Piecewise constant obs.

Approx. Likelihood (discr. observ.) $\mathcal{L}_{X}^n$

Data disentangling $\Delta E(t) \rightarrow \Delta E^c(t) + \Delta E^d(t)$
II. Parameter Estimation

Local volatility $\sigma \rightarrow$ modified local covariance estimator

$$\sigma = \sqrt{\sum_{i=0}^{n-1} \left( \Delta E^c(t_i) - |\theta_1 \times (\mu(t_i) - E(t_i))| \right)^2}$$

Parameters $\theta_1, \theta_2, \theta_3 \rightarrow$ new estimator

$$L_{\theta^0,E}(\theta) = \sum_{i=0}^{n-1} \frac{(\mu(t_i)-E_i)\theta_1}{\sigma^2} \left[ \Delta E^c_i - \mu'(t_i) \Delta t \right] - \frac{\Delta t}{2} \sum_{i=0}^{n-1} \left( \frac{(\mu(t_i)-E_i)\theta_1}{\sigma} \right)^2 \left( - (\theta_2 - 1) \sum_{i=0}^{n-1} s(t_i) \Delta t + \log \theta_2 N(t) \right) + \sum_{i=0}^{n-1} \left[ - (\theta_3 - 1) \frac{\Delta E^d_i}{h(E_i)} \right] + N(t) \log \left( \frac{1-e^{-\theta_3 \psi}}{\theta_3(1-e^{-\psi})} \right)$$
III. Empirical Results: Price Paths

Trajectorial properties (ECAR, PJM, COB)
III. Empirical Results: Price Paths

Trajectorial properties (ECAR market under varying resolution)
## IV. Empirical Results: Statistics

<table>
<thead>
<tr>
<th></th>
<th>ECAR EMP</th>
<th>ECAR SIMUL</th>
<th>PJM EMP</th>
<th>PJM SIMUL</th>
<th>COB EMP</th>
<th>COB SIMUL</th>
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<tbody>
<tr>
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<td>-0.0001</td>
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<td>0.0000</td>
<td>0.0009</td>
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<tr>
<td>Std. Dev.</td>
<td>0.3531</td>
<td>0.3382</td>
<td>0.2364</td>
<td>0.2305</td>
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<tr>
<td>Skewness</td>
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<td>0.3949</td>
<td>1.6536</td>
<td>0.1587</td>
<td>0.9610</td>
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</table>
V. Comparative Analysis of Alternative Model Specifications

**Reduction:** upward jumps only

**Extension:** price dependent jump frequency (random intensity)

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<thead>
<tr>
<th></th>
<th>ECAR</th>
<th>Up-jump (det. freq.)</th>
<th>Sgn-jump (det. freq.)</th>
<th>Sgn-jump (random freq.)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average</strong></td>
<td>-0.0002</td>
<td>0.0000</td>
<td>-0.0001</td>
<td>-0.0000</td>
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<td><strong>Std. Dev.</strong></td>
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<td><strong>Kurtosis</strong></td>
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<td>8.3542</td>
<td>22.5825</td>
<td>28.0288</td>
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</tbody>
</table>
VI. Conclusion: our class of models ...

(1) Matches both trajectorial and statistical properties of price dynamics
(2) Fits all markets
(3) Embeds risk factors (noise, spikes)
(4) Reproduces forecastable trends (drift, periodical jumps)
(5) Is Markovian
(6) Can be easily estimated and simulated
References


About the Author

Andrea Roncoroni is Assistant Professor of Finance at ESSEC Business School (France). He is regular lecturer at University Paris Dauphine, Bocconi University, and the Italian Stock Exchange. He holds PhD’s in Applied Mathematics and Finance. His research interests cover quantitative modeling and risk management in energy markets. He has consulted for private companies (Gaz de France, Fideuram Capital Asset Management) and public institutions (International Energy Agency, Italian Authority for Electricity and Gas). He is author of the book ”Implementing Models in Quantitative Finance: Methods and Cases” (with G. Fusai), Springer-Verlag, forthcoming 2006.

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