Modelling Electricity Spot Prices
A Regime-Switching Approach

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Agenda

› Model Overview
› Daily Price Process
› Hourly Profile Process
› Backtesting
› Applications
› Outlook
Electricity Spot Prices

Features

› seasonality (yearly, weekly, daily)
› spikes

Explanation

› power not efficiently storable => no cash-and-carry arbitrage
› inelastic demand curve
› seasonal weather-dependent demand pattern
› events can cause market shocks
   (plant outages, low water levels, extreme temperature)
Marginal Costs of Generation

costs include CO2 emission certificates

brown coal

costs include CO2 emission certificates

nuclear

equilibrium price

gas

oil

Demand

costs include CO2 emission certificates

Price (EUR)

Adjusted load (MWh)
Historical Hourly Spot Prices EEX
Fundamental and Stochastic Approaches

**Fundamental**
- model system generation and load
- price = marginal generation costs
- needs fuel prices and data about generation capacity
- many sources of uncertainty (generation, import/export, …)

**Stochastic**
- view power prices as time series
- choose appropriate stochastic process
- calibrate to price data
- needs only prices as input data

**Hybrid**
- use both approaches (e.g. SMaPS\(^1\))

Here we concentrate on the stochastic approach!

\(^1\)M.Burger, B.Klar, A.Müller, G.Schindlmayr
Model Overview

Notation:

- $\mathbf{S}_t = (S^1_t, \ldots, S^{24}_t)$: Vector of hourly spot prices on day $t$
- $\mathbf{s}_t = \log \mathbf{S}_t$: vector of logarithmic hourly spot prices
- $s_t = \frac{1}{24} \sum_{i=1}^{24} s_i$: mean logarithmic price

$$s_t = s_t + h_t$$

- Daily mean log price $s(t)$
- Daily log profile $h(t)$
- PCA-decomposition + ARMA-process

Regime-switching AR(1)-process

Different processes for business days and non-business days
Daily Price Process: Seasonality

Seasonal component:

- dummy variables for weekdays, holidays, vacation periods ($1,...,N_d$)
- sin/cos regressors for yearly seasonality
- linear trend

\[ s_t = \sum_{d=1}^{N_d} 1_{J_d}(t) \beta^A_d + 1_{J^PT}(t) \cos(2\pi t / 365) \beta^B_d + 1_{J^PT}(t) \sin(2\pi t / 365) \beta^C_d + 1_{J^PT}(t) t \beta^D_d + 1_{J^PT}(t) 1_{J^PV}(t) \beta^E_d + y_t, \]

<table>
<thead>
<tr>
<th>coefficient</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^A_d$</td>
<td>mean level</td>
</tr>
<tr>
<td>$\beta^B_d$, $\beta^C_d$</td>
<td>Amplitudes for yearly seasonality</td>
</tr>
<tr>
<td>$\beta^D_d$</td>
<td>deterministic drift</td>
</tr>
<tr>
<td>$\beta^E_d$</td>
<td>price effect of vacation period</td>
</tr>
</tbody>
</table>
Daily Price Process: Seasonal Component EEX

- log price daily
- seasonal component

Date

2001  2002  2003  2004  2005

log price
Daily Price Process: Regime-Switching AR(1)

Model:

\[ y_k - \mu_{r_k} = \phi_{r_k} (y_{k-1} - \mu_{r_{k-1}}) + \sigma_{r_k} \epsilon_k \]

\( r_k = \) regime at time \( k \)

transition matrix (for two regimes):

\[ P = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix} \]

\( p_{11} = 0.94, p_{21} = 0.28, p_{12} = 0.06, p_{22} = 0.72 \)

\( \mu, \phi, \sigma \) calibration: Hamilton filter (max. likelihood optimization)

\( \text{example: EEX (Jan 01 – May 05)} \)

\[ \begin{array}{ccc}
\text{regime 1} & \mu & \phi & \sigma \\
-0.004 & 0.74 & 0.11 \\
\text{regime 2} & -0.02 & 0.68 & 0.30 \\
\end{array} \]
Daily Price Process: Regime Identification

residuals

probability
spike regime
Daily Price Process: Autocorrelation of Residuals

Sample Autocorrelation Function (ACF)

Sample Autocorrelation

Lag

-0.2
0
0.2
0.4
0.6
0.8

0
2
4
6
8
10
12
14
16
18
20

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Daily Price Process: Q-Q Plot of Residuals

QQ Plot of Sample Data versus Standard Normal

- no regime-switching
- regime-switching

Quantiles of Input Sample

Standard Normal Quantiles

-4 -3 -2 -1 0 1 2 3 4

-6 -4 -2 0 2 4 6

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Hourly Profiles: PCA Decomposition

Regression:
seasonal component

\[ h_t = \hat{h}_t + \Delta h_t \]

PCA decomposition for 24h-residuals

\[ \Delta h_t = ZP^T \]
\[ Z = (\Delta h) P \]

stochastic model for factor loads

ARMA\((p,q)\) for \(z^i_k\)

For spike regime: take random historical profile according to season and weekday
Hourly Profiles: Seasonal Component
Hourly Profiles: Principal Component Vectors
Hourly Profiles: PCA Explained Variance
Hourly Profiles: Factor Load Autocorrelation of Residuals

ARMA(1,1): Sample Autocorrelation Function (ACF)
The Long-Term Dynamics

Model:  \( s_t = f(t) + y_t + h_t + l_t \)
- \( f(t) \): seasonal (deterministic) component
- \( y_t \): regime-switching process
- \( h_t \): hourly profile process
- \( l_t \): long term process

Future price: for \( T \gg t \)
- short term dynamics:  \( E_t[y_T] \approx E[y_T] \quad E_t[h_T] \approx E[h_T] \)
- long term dynamics
  \[
  F_{i,T} = E_t[S_T] \approx C(T)E_t[\exp(l_t)]
  \]
- long-term approximation: Black’s future price model

Calibration
- historical volatility or implied volatility (depending on application)
Simulation Results: Sample Paths

![Graph showing hourly price in EUR/MWh over time with multiple paths for different dates from January 06 to January 07.]
Simulation Results: Histogram Prices 100-250 €/MWh
Backtesting: Calibration Stability

moving two-year calibration period

Parameter base regime:
- $\phi_0$
- $\lambda_0$
- $\sigma_0$

Parameter spike regime:
- $\phi_1$
- $\lambda_1$
- $\sigma_1$

Transition probabilities:
- $p_{00}$
- $p_{10}$
## Backtesting: 1-Day-Forecasting Quality

<table>
<thead>
<tr>
<th>Regime-Switching-Modell</th>
<th>1 2R</th>
<th>2 3R</th>
<th>3 $2^B1_{nB}$</th>
<th>4 $2^B2_{nB}$</th>
<th>5 $3^B1_{nB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Business Days</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAE</td>
<td>4,520</td>
<td>4,492</td>
<td>4,157</td>
<td>4,132</td>
<td>4,130</td>
</tr>
<tr>
<td>RMSE</td>
<td>6,877</td>
<td>6,839</td>
<td>6,429</td>
<td>6,455</td>
<td>6,385</td>
</tr>
<tr>
<td>MAPE</td>
<td>11,488</td>
<td>11,436</td>
<td>10,476</td>
<td>10,360</td>
<td>10,430</td>
</tr>
</tbody>
</table>

| **Non-Business Days (excl. Holidays)** |         |         |               |               |               |
| MAE                     | 3,106   | 3,130   | 3,615         | 3,678         | 3,716         |
| RMSE                    | 3,905   | 4,032   | 4,635         | 4,719         | 4,746         |
| MAPE                    | 11,562  | 11,562  | 13,537        | 13,708        | 13,892        |

| **Holidays**            |         |         |               |               |               |
| MAE                     | 4,250   | 5,396   | 5,535         | 5,446         | 4,764         |
| RMSE                    | 5,597   | 6,249   | 6,308         | 6,295         | 5,626         |
| MAPE                    | 18,017  | 23,315  | 23,803        | 23,769        | 20,580        |
Backtesting: Quantile-Statistics

How do the probability distributions compare?

› histogram to analyze, how often the real spot price falls into which quantile of the model distribution

› period: 01.07.2004 – 30.06.2005

› calibration off-sample (uses data from 01.01.2001- 30.06.2005)
Applications: Option Pricing

- **Electricity Option**
  - **Option on Forwards**
    - Underlying: Forward contract for delivery month/quarter/year
  - **Virtual Power Plants**
    - Hourly exercise
    - Multi-commodity
    - Technical constraints
  - **Daily/Hourly Option**
    - Strip of options for daily/hourly exercise
    - Underlying: daily product (base/peak) or single hour
  - **Swing Option**
    - Daily/hourly exercise
    - Energy constraints
Applications: Option Pricing and Hedging

Hourly call option

› period: 01/01/2006 – 01/01/2007
› strike: 60 €/MWh
› capacity: 10 MW

Pricing results

› price: 380,000 €
› inner value: 190,000 €
› profit-at-risk (95%): 160,000 €
› mean exercise 16 GWh (1600 h)
Applications: Mean Exercise Schedule
Applications: Hedging Strategies

energetic hedging
  › calculate mean exercise schedule
  › sell energetic equivalent base and peak contracts

delta hedging
  › calculate delta sensitivities with respect to base/peak forward prices
  › construct delta neutral portfolio

variance-minimizing hedge
  › calculate hedge ratios by minimizing portfolio variance
Applications: Analyzing Hedging Strategies

![Diagram showing frequency of P&L outcomes for different hedging strategies: no hedge, delta hedge, and energy hedge.](chart.png)
Outlook

› better coupling of business days and non-business days
› improve dynamics of hourly profiles, especially during spike regime
› integration of spot and future price models
› multi-commodity model: integrate fuel and CO2 prices