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Quiz 6

High Performance Computing I (WS 2016/2017)

Deadline: 12 January 2018, 2pm

For a regular $m \times n$ matrix A the LU-factorization has the form

$$PA = LU$$

where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$), and U is upper triangular (upper trapezoidal if $m < n$). An implementation can overwrite A compactly with L and U and we therefore denote such an operation by

$$A \leftarrow P^T(L\backslash U) \quad \text{(lu-variant)}.$$

Assuming an algorithm *lu_unblk* is known we can define algorithm *lu_blk* as follows:

- if $\min\{m,n\} < \text{bs}_{\min} := 2^k$ (for fixed $k \geq 0$)

$$A \quad \leftarrow \quad P\,(L\backslash U) \quad \text{lu\_unblk}$$

- else chose $\text{bs} = \max\left\{2^\ell \;:\; 2^\ell < \min\{m,n\}\right\}$ and proceed for

$$A = \left(\begin{array}{c|c} A_{0,0} & A_{0,\text{bs}} \\ \hline A_{\text{bs},0} & A_{\text{bs},\text{bs}} \end{array}\right)$$

as follows:

$$\begin{array}{rcll}
\left(\begin{array}{c} A_{0,0} \\ \hline A_{\text{bs},0} \end{array}\right) & \leftarrow & P^T \left(\begin{array}{c} L_{0,0}\backslash U_{0,0} \\ \hline L_{\text{bs},0} \end{array}\right) & \text{(lu_blk)} \\
\left(\begin{array}{c} A_{0,\text{bs}} \\ \hline A_{\text{bs},\text{bs}} \end{array}\right) & \leftarrow & P \left(\begin{array}{c} A_{0,\text{bs}} \\ \hline A_{\text{bs},\text{bs}} \end{array}\right) & \text{(swap)} \\
A_{0,\text{bs}} & \leftarrow & L_{0,0}^{-1} A_{0,\text{bs}} & \text{(triangular solver)} \\
A_{\text{bs},\text{bs}} & \leftarrow & A_{\text{bs},\text{bs}} - A_{\text{bs},0} A_{0,\text{bs}} & \text{(matrix product)} \\
A_{\text{bs},\text{bs}} & \leftarrow & \tilde{P}^T \left(L_{\text{bs},\text{bs}}\backslash U_{\text{bs},\text{bs}}\right) & \text{(lu_blk)} \\
A_{\text{bs},0} & \leftarrow & \tilde{P}\;\, A_{\text{bs},0} & \text{(swap)}
\end{array}$$

Write a function *lu_blk_var2* that implements this algorithm. You can use the test cases from session 18. Chose $\text{bs}_{\text{min}} = 4$. For the required unblocked LU-factorization you can use *lu_unblk_var1* or *lu_unblk_var2*.

Please submit your program "quiz06.cpp" as follows:

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thales$ submit hpc quiz06 quiz06.cpp
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