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Numerical Finance Reading Course

Sheet 11 (July 9th, 2009)

Discussion: American Option Pricing (Sections 7.1- 7.3.1)

- Why are American option pricing problems instances of free-boundary problems? What does this mean for finding a solution via the Black-Scholes PDE?
- Explain how the Binomial Tree method has to be modified to solve American option pricing problems.
- Write down the American option pricing problem in the form of an obstacle problem (7.2.1). How are the explicit constraints avoided?
- How does one approach obstacle problems with Finite Differences?
- What does the variational formulation of the model obstacle problem look like (and why is this so)?
- Summarize the classical iterative methods for the solution of $Ax = b$. What is meant by a SOR-method?

Exercise 1: Binomial Tree for American Options

Implement the binomial method to find the price of an American option with inital value $S_0 = 20$, strike $K = 20$, maturity $T = 2$, volatility $\sigma = 0.2$ and riskless interest rate $r = 0.05$.

a) Plot the option value for different time steps, e.g. $M = 5, 10, ..., 1000$. What do you see? Can you explain that behaviour?

b) For a fixed M (say $M = 50$), plot the tree, i.e. the approximation of $V(S_t, t)$. Compare with the payoff function. Can you see the exercise region? Modify your implementation to calculate the exercise region and add it to your plot.

Exercise 2: Iterative Methods

Prove that for a $(n \times n)$- matrix G with $\|G\| < 1$, a vector $c \in \mathbb{R}^n$ and an arbitrary starting point $x^0 \in \mathbb{R}^n$ the following error bounds hold for the iterative method

$$x^{k+1} = Gx^k + c.$$

a) $\|x^k - x\| \leq \|G\|^k \|x^0 - x\|$,

b) $\|x^k - x\| \leq \frac{\|G\|^k}{1-\|G\|} \|x^1 - x^0\|$,

where x denotes the fixpoint of the iteration, i.e., $x = Gx + c$.