

Numerical Finance Reading Course

Sheet 2 (April 29nd, 2009)

Discussion: Discrepancy, Product Formulas and Monte Carlo Integration (Sections 2.3, 3.1, 3.2)

- What is the general difference between statistical tests and the notion of discrepancy to ensure the uniform distribution of a sequence?
- What are low-discrepancy sequences? What may be their advantages in comparison with grids?
- Product Formulas: What is meant by the *curse of dimensionality*?
- Monte Carlo techniques are said to break the curse of dimensionality. Why?

Exercise 1: Discrepancy

Prove Proposition 2.3.3. In part (b), consider only $m = 1$.

Exercise 2: Monte Carlo Integration

- a) Evaluate the following integral by Monte Carlo integration and show the convergence rate:

$$\int_{-1}^1 \sqrt{1-x^2} dx.$$

- b) Antithetic variables use the fact that if $u \sim U[0,1]$ then also $\tilde{u} := 1 - u \sim U[0,1]$. Using $u_1, \tilde{u}_1, u_2, \tilde{u}_2, \dots$ in a simulation might reduce the variance σ_F if $\text{Cov}(F(u), F(\tilde{u})) < 0$, as is the case for example for monotone functions F .

Compute the integral

$$\int_0^1 (x-b)^3 dx$$

by Monte Carlo integration for $b = 0.55$ and $b = 0.5$ with and without the use of antithetic variables and compare the error and the convergence rates.

Hint: To produce log-log-plots in gnuplot use `set logscale xy`.