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Numerical Finance Reading Course

Sheet 3 (May 6th, 2009)

Discussion: Quasi-Monte Carlo Integration and (t,m,j)-nets
(Sections 3.3, 3.4)

- What is the idea behind Quasi-Monte Carlo integration? Why is the Koksma-Hlawka inequality so important in this context?
- QMC methods can accelerate the Monte Carlo convergence rate $O(\frac{1}{\sqrt{n}})$ to nearly $O(\frac{1}{n})$. Explain. What does the convergence rate depend on?
- What are (t,m,j)-nets? How do they relate to general low discrepancy sequences?
- What is the role of the parameters t and b in (t,m,j)-nets? Is it better for t and b to be large or small?

Exercise 1: Hardy-Krause Variation

Calculate the Hardy-Krause variation $V(f)$ for the function

$$f(x_1, x_2) = \frac{3}{1 + x_1 + 2x_2}.$$

Hint: You will first need to specify $J_k^{(2)}$, $k = 1, 2$ and to calculate the Vitali variations for all index sets $I \in J_k^{(2)}$, $k = 1, 2$.

Exercise 2: Halton Sequence

a) Implement the radical inverse function $\phi_b(i)$, using the idea that

$$i = \sum_{k=0}^{j} d_k b^k = (d_k b^{k-1} + \cdots + d_1)b + d_0.$$

You should not need to use more than one loop in your algorithm.

b) For a d-dimensional Halton sequence

$$x_i = (\phi_{p_1}(i), \ldots, \phi_{p_d}(i)), \qquad i = 1, 2, \ldots,$$

one usually takes $p_1, \ldots, p_d$ to be the first d prime numbers. Generate 1000 points of the 2-dimensional Halton sequence (i.e. $p_1 = 2$, $p_2 = 3$). Also generate 1000 Halton points using $p_1 = 109$, $p_2 = 113$ (the 29th and 30th prime number, respectively). Compare the two point sets by plotting them (Gnuplot: `plot "data"`). What does this tell you about the behaviour of the Halton sequence in higher dimensions?
(Hint: the second sequence corresponds to the projection of a 30-dimensional sequence onto the last two coordinates).