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Summer Term 2009

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Numerical Finance Reading Course

Sheet 4 (May 14th, 2009)

Discussion: Smolyak Algorithm (Sections 3.5)

- What is the difference between the Smolyak quadrature and the standard product formula?
- What do we know about the integration error? Compare with the results for QMC integration.
- Explain Figure 3.3 (it may help to look at Exercise 1 first).
- Does the curse of dimensionality play a role in Smolyak methods?

Exercise 1: Smolyak Algorithm

The sequence of one-dimensional grids with $n_i = 2^i - 1$, $i = 1, 2, \ldots$ equidistant points $x_1, \ldots, x_{n_i}$ on $[a, b]$ forms a nested grid (Why?). We can use the (open) Newton Cotes formulas to construct a simple sparse grid. They are given by

$$\begin{aligned} n_i = 1: &\quad (b-a) f(x_1), \\ n_i = 3: &\quad \frac{b-a}{3}(2f(x_1) - f(x_2) + 2f(x_3)). \end{aligned}$$

Using these as one-dimensional quadrature formulas $Q^{(1)}$ and $Q^{(2)}$, compute the first two-dimensional Smolyak Quadrature formula $Q(1, 2)$ on $[0, 1]^2$. What does the grid look like?

Exercise 2: Comparison Monte Carlo & Quasi-Monte Carlo

Compute the integral

$$I_3[f] = \int_{[0,1]^3} x_1^2 \, x_2^2 \, x_3^2 \, \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3,$$

using

a) Monte Carlo integration,

b) Quasi-Monte Carlo integration (e.g. with the Halton sequence).

Visually compare both methods by plotting their integration errors and their theoretical convergence rates.

Gnuplot hints:

- You can define functions in Gnuplot, e.g. `f(x) = 1./x` and plot them using e.g. `plot[1:100][0:1] f(x)`. (The square brackets define the x-range and the y-range of the plot.)
- You can plot several datasets in one plot: `plot "data1", "data2"`.
- If you have several columns in your data file (e.g. N MC-Error(N) QMC-Error(N)), you can tell Gnuplot which columns to use:
  `plot "data" using 1:2 with lines, "data" using 1:3 with lines`.
- You can label the graphs:
  `plot "data" using 1:2 title "MC", "data" using 1:3 title "QMC"`.
- You can label the x- and y-axes and set a title: `set xlabel "Sample Size"`, `set ylabel "Error"`, `set title "Plot title"`.
- You can save your plot in ps, pdf or other formats (use `help terminal` for more information):

```
> set terminal postscript enhanced color
> set output "myplot.ps"
> plot  .....
> set output
> set terminal wxt
```

  This creates a file "myplot.ps" with the plot . If you don't change back to terminal `wxt`, subsequent plots will be added to this file.