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Numerical Finance Reading Course

Sheet 5 (May 21, 2009)

Discussion: Binomial Method (Sections 4.1/4.2)

- The Black-Scholes formula is an easy way to calculate option prices - what do we need numerical methods for?
- Discuss the assumptions of the binomial method. Where exactly do they come into play?
- Explain the Binomial Tree method.

Exercise 1: Risk-neutral measures

Consider the binomial model. Let S_t, B_t and V_t be the price processes of an asset, a bond and a derivative, respectively, and let $T = \Delta t = 1$. Prove that $p = \frac{e^r - d}{u - d}$ defines a risk-neutral measure P by showing that the initial value $V_\psi(0)$ of a replicating portfolio $\psi = (a, b)$ is given by

$$V_\psi(0) = V_0 = e^{-r} E_P[V_1] = e^{-r}\big(p \cdot V_1(\{u\}) + (1-p)V_1(\{d\})\big).$$

Hint: For a replicating portfolio (a, b) it holds

$$V_T = a \cdot B_T + b \cdot S_T.$$

Exercise 2: Binomial Tree Algorithm

Implement Algorithm 4.2.2 to find the fair price of a European put with $S_0 = 10$, $K = 12$, $r = 0.04$, $\sigma = 0.4$ and maturity $T = 2$.
Compare with the exact solution $V_0 = 2.922933$ that can be obtained by the Black-Scholes formula

$$C_t^E = S_t \Phi(d_1(t)) - K e^{-r(T-t)} \Phi(d_2(t)),$$

where

$$d_1(t) = \frac{\ln(\frac{S_t}{K}) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}},$$
$$d_2(t) = d_1(t) - \sigma\sqrt{T-t},$$

and the Put-Call Parity

$$C_t^E + K e^{-r(T-t)} = P_t^E + S_t.$$