Prof. Dr. Karsten Urban
Kristina Steih
Summer Term 2009

Ulm University
Institute for Numerical Mathematics

Numerical Finance Reading Course

Sheet 7 - Addendum (June 10th)

Explicit Euler Method and Stiff Problems

Consider the one-dimensional problem

$$u'(t) = -\lambda u(t), \quad u(0) = 1.$$

This is an ordinary differential equation (ODE) of the form

$$u'(t) = f(t, u(t)), \quad u(0) = 1,$$

and can be solved by the explicit Euler method on an interval $[0, T]$ in the following way. Let N be the number of time steps, so that $h = \frac{T}{N}$ and $t_i = ih$. Then the approximation u_h (with the notation $u_i = u_h(x_i)$) is calculated as

$$\begin{aligned} u_0 &= u(0) = 1, \\ u_i &= u_{i-1} + hf(t_{i-1}, u_{i-1}) = u_{i-1} - h\lambda u_{i-1}, \qquad \forall\, i = 1, \ldots, N. \end{aligned}$$

Implement the explicit Euler method to solve the above ODE for $T = 1$, $\lambda = 10, 50, 150, 200$, $N = 1, 10, \ldots, 100$. The correct solution is given by

$$u(t) = e^{-\lambda t} \qquad t \in [0, T].$$

Compute for each N, λ the error

$$\|u_h - u\|_\infty := \max_{i=0,\ldots,N} |u_i - u(t_i)|.$$

What do you see?