Statistical Analysis of Spatial Point Patterns on Deep Seismic Reflection Data: A Preliminary Test

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SUMMARY

Spatial point patterns generated from bitmaps of images of processed reflection seismic profiles are analyzed to quantify the reflectivity patterns. The point process characteristics for two different regions of a deep seismic reflection profile in northwestern Canada demonstrate that in both cases the points are not randomly distributed and that the point pattern distribution is different between the regions. The cluster effects for small point pair distances are stronger for the region of data where there is strong sedimentary layering than for the region where the layering is less distinct. As a result, it appears that future developments in point pattern analysis may provide a new tool for analysing spatial variations in reflection data.

Key Words: Deep Proterozoic Basin, Spatial Point Patterns, Point Process Characteristics, Estimation

1 INTRODUCTION

The purpose of this paper is to present spatial point pattern analyses of seismic reflection data in an effort to enhance automated analyses of such images and to seek patterns that may not be obvious by visual inspection or by conventional pattern recognition techniques. While it should be possible to apply these methods to any digital data set (seismic in this case), we have chosen to use part of a regional deep seismic reflection profile recorded by the Canadian Lithoprobe program for the following reasons. First, the data have patterns that are reasonably well understood in terms of their geological context. Second, the data are of good quality, with numerous coherent reflections and pattern variations. Third, the profile represents a portion of a much larger data set (both locally and throughout Canada) that are available for further study if the results are encouraging. The major objectives are thus to determine whether or not seismic data processing schemes have significant difference in the behaviour of point process characteristics, and to establish whether or not discernible point process characteristics can be noted between different regions on seismic reflection data across a deep sedimentary basin.

One approach to quantifying patterns observed on binary images is spatial point pattern analysis. In the context of bitmaps, spatial point pattern analysis is concerned with analysis of locations of events. Events here refer to centres of gravity of coherent structures observed in bitmaps. The collection of events is called a point pattern. The analysis follows two main steps: One is to determine if the point pattern is a complete spatially random (CSR) process and two is to establish if there are point process models that fit the data. Spatial point pattern analysis is applied to data in different scientific disciplines (Mecke *et al*. 1994; Stanford & Referty, 2000; Mecke & Stoyan, 2000, 2005; Hirsch *et al.* 2000; Foxall & Baddeley, 2002; Scanlon *et al.* 2003; Lucio & Castelucio de Brito, 2004; Beil *et al*. 2005a, 2005b, 2006; Mecke & Arns, 2005; Fleischer *et al*. 2006a, 2006b; Mattfeldt *et al*. 2006). In geophysics, spatial point pattern analysis of earthquake data has been recently investigated (Veen & Schoenberg, 2005). In particular, Beil et al. (2205a, 2005b, 2006) and Fleischer et al. (2006a, 2006b) have shown that it is possible to derive the spatial point patterns from various types of microscopic images of data and to analyse them using point process characteristics. For example, for the quantitative analysis of keratin filament networks of pancreatic cancer cells, Beil et al. (2005b) used scanning electron microscopy (SEM) images of cancer cells. They transformed greyscale SEM images into their binary form by using a threshold value corresponding to the mean greyscale value of the image. These images contain predominantly the keratin filament structures. Further, to reduce the noise in the filament networks, they carried out skeletonization of the images and pruning of the resulting linear structures. Subsequently, they analysed the spatial patterns of the structures present with spatial statistics tools to understand how the topology of the keratin filament networks of untreated cancer cells differ from the treated ones. With this in mind, we investigated whether or not we could generate spatial point patterns from processed reflection seismic data to use them to quantify the geometrical structure in the data with point process characteristics.

Final binary images of reflection seismic data used for interpretation have usually undergone a series of data processing steps, including seismic migration and coherency filtering and/or seismic skeletonization. Migration attempts to determine the true sub-surface locations of the reflectors, and coherency-filtering (Neidell and Taner, 1971; Milkereit and Spencer, 1989; van der Baan and Paul, 2000) and seismic skeletonization enhance the signal to noise ratio of the data. Such binary images have been analyzed using classical statistics tools with a view to understand the geometrical patterns present in the data (Hurich, 1996; Cook *et al*. 1997, Li *et al*. 1997; Vasudevan & Cook, 1998; Rea & Knight, 1998; Hurich & Kocurko, 2000; Eaton & Vasudevan, 2004; Vasudevan *et al*. 2005). The processed binary images of the reflection seismic data are analogous to the processed binary images of the keratin filaments in that both are comprised of short and long linear segments, albeit the former has short and long curvilinear segments. Also, they have characteristic geometric patterns. Although the underlying processes causing the patterns in images of keratin filaments and reflection data are distinctly and totally different, extracting and analyzing the spatial point patterns from binary images of reflection seismic data is new, and hence, undertaken here.

Once spatial point patterns are extracted from binary images, the results can be subjected to modern analytical tools to arrive at a variety of summary statistics that may provide new information (Ripley, 1976, 1977, 1981; Cressie, 1993; Stoyan & Stoyan, 1994; Stoyan *et al*. 1995; Diggle, 2003; Beil *et al*. 2005a, 2005b, 2006; Fleischer *et al*. 2006a, 2006b; Stoyan, 2006). The chosen seismic line segment (for the location map, see Figure 1) has been previously interpreted to provide an image of a Proterozoic (ca. 1.7-0.7 Ga) Basin (Cook *et al*. 1999; Cook & Erdmer, 2005). While the reflection profile has all the earmarks of a deep basin, there are internal variations within the fabric of the basin that are not well understood. Quantifying these variations with spatial point pattern analysis is one of the objectives of this work. Furthermore, to understand the effect of the presence of noise in the data on the final conclusions, binary images from a stack profile without coherency filtering and/or migration are considered.

First, we begin by providing a brief description of point processes and approaches to modelling. Subsequently, we describe the data sets used in this study, the extracted point patterns, and present the results of the point pattern analysis and Monte Carlo tests on complete spatial randomness.

2 POINT PROCESSES

Spatial point patterns extracted from a data set describe locations of points observed in the space, in this case two-dimensional (2-D) space. These patterns are compared to those that are derived from a complete spatial random (CSR) process using modelling 'experiments'. For each model, point pattern characteristics are computed and used as 'fingerprints' to understand the model. In the final analysis, point pattern characteristics from known models are compared with those of the real data. Following are a few model descriptions.

2.1 Model Descriptions

For a complete spatial random process, each point is an "event" and events corresponding to such a process are independent. The number of events in any fixed observation window is Poisson distributed. Here, we use three different point process models corresponding to Poisson point process, Matern hard core point process, and Matern cluster point process. For each case, the definitions and estimators of point process characteristics that we used such as the pair correlation function and the *L*-

function (defined below) are given in Appendix A. The realizations of the different point process models mentioned above and the parameters used for the simulations are summarized in Figure 2. The choice of parameters here is arbitrary and is to demonstrate how the point patterns are represented by the three different point process models.

2.1.1 Poisson point process

The Poisson point process is used as a reference model. In a Poisson point process, points are independently scattered, implying complete spatial randomness. As a result, only a single parameter, the intensity λ (the mean number of points per unit area in the 2-D space), is necessary to characterize a (homogeneous) Poisson point process (Stoyan *et al.* 1995).

2.1.2 Matern hard core point process

For the Matern hard core point process the points are located in a more regular way than in the case of the Poisson point process. A possible principle for hard core processes is done by thinning the points of a Poisson point process (Figure 3a; Stoyan *et al*. 1995). This can be accomplished by considering a specified hard core distance. This means that all points which have a certain minimum distance to their nearest neighbour are retained. This may be interpreted to imply certain repulsion effects between the points. Two parameters describe a Matern hard core point process: the intensity, λ of the underlying Poisson point process and the hard core distance, *D*. Then, the intensity, λ_{MH} of the Matern hard core point process is given by (Stoyan *et al*. 1995, p.164)

$$\lambda_{MH} = \frac{1 - e^{(-\lambda D^2 \pi)}}{D^2 \pi}.$$

Such a hard core point process is shown in Figure 2b.

2.1.3 Matern cluster point process

In the case of the Matern cluster point process the points are attracted to each other. For the characterization of a Matern cluster point process, three parameters are necessary; the intensity, λ_p of the parent point process, which is given by a (homogeneous) Poisson point process, the radius R of the circular cluster areas and the intensity, λ_c, of the child point process within these circles around the parent points. The child point process is again a Poisson point process (Figure 3b). The intensity of the Matern cluster point process, λ_{MC} is given by

$$\lambda_{MC} = R^2 \pi \lambda_p \lambda_c .$$

2.2 Point process characteristics

Mathematical methods to determine differences or common grounds of point patterns are based on point process characteristics. Point process characteristics such as the pair correlation function and the L-function are widely used in the statistical analysis of spatial point patterns. They are second-order functions of the inter-point distance, r. Their functional values indicate the kind of interaction between points prevailing at a certain distance. The interaction, by definition, here refers to attraction (clustering) or repulsion. For certain distances, neither attraction nor repulsion may be present. They offer the possibility to obtain not only qualitative knowledge about the spatial structure of such point pattern, but to quantify them for specific regions of point pair distances.

2.2.1 Pair correlation function

The pair correlation function, $g(r)$ gives information about the (relative) frequency of point pairs with a certain distance, r. In the case of complete spatial randomness, the pair correlation function satisfies $g_{Poisson}(r) \equiv 1$ for all r. The inequality $g(r) > 1$ indicates clustering of point pairs with distance, r, in relation to complete spatial randomness, where $g(r) < 1$ indicates repulsion.

For a Matern cluster point process an analytical formula for the theoretical pair correlation function, g_{MC}, is known. The function, g_{MC}, has the shape

$$g_{MC}(r) = 1 + \begin{cases} \dfrac{2}{\pi^2 R^2 \lambda_p}\left[\arccos\dfrac{r}{2R} - \dfrac{r}{2R}\sqrt{1-\dfrac{r^2}{4R^2}}\right] & \text{for } 0 \le r \le 2R \\ 0 & \text{for } r > 2R. \end{cases}$$

and is plotted in Figure 3c.

2.2.2 *L*-function

The *L*-function is a scaled version of Ripley's *K*-function. Ripley's *K*-function (or the reduced second moment function) is defined by the mean number of points within distance, *r* from an arbitrary point of the point process divided by the intensity, λ . In its place, a modified form of Ripley's *K*-function, the so-called *L*-function is used here (see Appendix A). By definition, the *L*-function is given as

$$L(r) = \sqrt{\frac{K(r)}{\pi}}\ .$$

In the case of complete spatial randomness the *L*-function satisfies $L_{Poisson}(r) = r$, i.e. $L_{Poisson}(r) - r \equiv 0$. Because of this, the slope of the estimated difference $L(r) - r$ is useful for analytical purposes, where a positive (negative) slope indicates clustering (repulsion) of point pairs with distance, *r*.

In the case of a Matern cluster point process the formula for the theoretical *K*-function is given as

$$K_{MC}(r) = \pi r^2 + \frac{1}{\lambda_p} \begin{cases} 2 + \dfrac{1}{\pi}[(8z^2 - 4)\arccos z - 2\arcsin z \\ \qquad + 4z\sqrt{(1-z^2)^3} - 6z\sqrt{(1-z^2)}\ ] & \text{for } r \le 2R \\ 1 & \text{for } r > 2R. \end{cases}$$

where z is defined as $\frac{r}{2R}$. A plot of the theoretical $L_{MC}(r) - r$ function is shown in Figure 3c.

For the realizations of the different point process models in Figure 2, the pair correlation function $g(r)$ and the $L(r) - r$ functions are estimated and displayed in Figure 4. From an interpretation point of view, the $g(r)$ function for the Poisson point process (Figure 4a(i)) shows an undulating structure around $g(r) = 1$. The slight hills and valleys above and below the constant, $g(r) = 1$ indicate clustering or repulsion over a certain domain of r values respectively. For the Matern cluster point process model, the $g(r)$ curve, as in (Figure 4a(iii)), shows clustering over a wide range of r values. For large r values, $g(r)$ values are negative suggesting repulsion. The Matern hard core point process for $g(r)$ reveals a strong repulsive behaviour over a certain range of r values before settling into an undulating behaviour, similar to the behaviour of the Poisson point process model (Figure 4a(ii)). In the case of the $L(r) - r$ curves (Figure 4b), $L(r) - r = 0$ for a given r demarcates clustering from repulsion. For the Poisson point process model, the $L(r) - r$ curve in Figure 4b(i) is hovering around 0, as the theory suggests. For the Matern hard core process model used, the $L(r) - r > 0$ over a wide-range of r values, implying clustering (Figure 4b(iii)).

3 DESCRIPTION OF THE DATA SETS

The seismic data were acquired in 1996 as part of Lithoprobe's SNORCLE (Slave Northern Cordillera Lithospheric Evolution) transect (see Figure 1 for the study area). The acquisition and processing parameters were described in Cook *et al*. (1999); the data presented here are the final coherency filtered results, one a coherency-filtered stack, and the other a coherency-filtered, migrated, and coherency-filtered stack. Interpretations of the latter profile were also described by Cook *et al*. (1999); the focus here is on one segment of the data that crosses a buried Proterozoic basin (called the Fort Simpson basin by Cook *et al*. 1999) in which the subsurface layering (probably sedimentary in origin) deepens and thickens westward (Figures 5a and 5b). The base of the basin is marked with a grey dashed line in Figure 5a.

The Fort Simpson Basin is formed as a result of extension of the lithosphere following the development of the Wopmay Orogen at about 1.84 Ga. The basin is imaged as a west facing monocline with at least 20 km of subsurface relief. The base of the monocline is delineated by reflection that dips about 20 degrees to 30 degrees westward from about 1.0 s at the east end to about 8.0 s (about 20 km) near the west end of the profile where it flattens. In both profiles (Figures 5a and 5b), the layering of the Fort Simpson Basin along its eastern ramp is representative of a typical sedimentary basin.

The thickness of the crust decreases systematically with the westward increasing thickness of the Fort Simpson Basin layers. This is almost certainly related to the extensional thinning of the basement during basin development. This base of the crust (reflection Moho) rises eastward beneath the basin and is therefore at least as old as the Fort Simpson Basin. The probable lateral (north-south) extent of this basin, or basins, is suggested by interpretation of potential field anomalies to be at least 1200 km (Cook *et al*. 1998), and yet there is almost no surface expression (e.g. outcrop) of layers associated with it. This is a major reason that application of new pattern recognition techniques may assist in enhancing our knowledge of it.

The choices of the coherency-filtered stack along with the coherency-filtered, migrated, and coherency-filtered stack are made to understand differences in point process characteristics arising out of differences in processing schemes. The main difference in processing schemes is the post-stack migration step. Also, to examine differences in basin characteristics of the data, we looked at six segments of the data, two from the coherency-filtered stack data and four from the coherency-filtered, migrated, and coherency-filtered stack (areas indicated in rectangular boxes in Figures 5a and 5b). The segments are from a shallow and a deep part of the basin with differences in the structures, as evidenced in the bitmaps used for seismic interpretation. For both the shallow and deep parts of the migrated stack, we considered an additional segment to account for lateral variations in the migrated structure. The size of the windows is kept large to accommodate distinctly different reflectivity characteristics but small enough to be able to do analysis of the entire image frame. The choice of location of the windows is dictated by the marked differences in reflectivity patterns both at the shallow and deep sections of the data.

Further, to elucidate what impact the noise level in the data would have on the point pattern analysis, stack of the data without any coherency-filtering and migration is included in this study.

4 DATA ANALYSIS

We carried out the analysis of the six segments of data (Figure 6) by examining objects in binary images. We define an object as a set of pixels which are connected to each other (Figure 7). The line segment shown in this figure is an object. After some testing with 4-pixel and 8-pixel neighbourhoods, we used the 8-pixel neighbourhood, i.e., pixels on the left and right side, directly below and above and diagonally connected pixels (Beil *et a*l. 2005b). The centre of gravity of the object leads to a "point". We generated point patterns by building the centres of gravity of the detected objects in a given binary image. In this paper, the extracted points do not carry appropriate weights corresponding to the size of the object. Although the measures taken here are restrictive, the points thus defined are adequate in this exploratory point pattern analysis.

In order to analyze the point patterns we used point process characteristics. In the initial phase of this work, we assumed stationarity and isotropy of the underlying point process and hence, were not subjected to usual statistical tests. In other words, the point processes were assumed to be invariant under translation (stationarity condition) and invariant under rotation (isotropy condition) for the segments of data investigated here. While we assumed the point patterns to be stationary, we verified the isotropy assumption by examining the angular distribution of point pairs within all of the six segments chosen for study here (see Section 5.1). In this study, we focused on two functions for point pattern analysis, the pair correlation function and the L-function. Formal definitions of these functions, examples of models for specific point processes, as well as descriptions of estimators used here are provided in Section 2 and Appendix A.

Following the definitions of the pair correlation function and the L-function, the investigated data from six segments show certain patterns that appear not to conform to a complete spatial random (CSR) style. However, any rejection of CSR requires hypothesis testing. To this end, we conducted a series of Monte Carlo tests (Appendix B) to test the null-hypothesis that the obtained point pattern was a realization of a Poisson point process. Furthermore, we computed r-wise confidence intervals (Appendix C) for the pair correlation function and the $L(r) - r$ function of the Poisson point process with the same (estimated) intensity. For numerical computations done in this work, we used the GeoStoch library of software (Mayer et al. 2004; http://www.geostoch.de).

5 RESULTS

5.1 Point patterns

For all objects in the images shown in Figure 6, we used the procedure outlined in the previous section to compute their centres of gravity. Each coherency-filtered segment is looked upon as an object. The centre of gravity of the object, which is a point, is determined by using the 8-point neighbourhood. Regardless of the length and curvature of the coherency-filtered segment, there is only one point associated with it. The 2-D space is filled with points from such an extraction procedure. The resulting point patterns are displayed in Figures 8a to 8f. These are the spatial point patterns that we subject to detailed point pattern analysis.

The angular distributions for the two regions with each region represented by three segments are shown as rose diagrams in Figures 9a to 9f. Although there are indications of elements of anisotropy with the shallower region represented by 6a, 6b and 6c, we assumed all of them to satisfy the isotropy condition in this work.

5.2 Intensity

The estimated intensities of the point patterns are shown in Table 1. From the table, it is evident that the intensities are different for the two regions bearing six segments of data. However, within each region, the two different post-stack processing schemes offer intensity values that are close to each other.

5.3 Estimated pair correlation function

The bandwidth h plays an important role in estimating the pair correlation function $g(r)$ which uses an Epanechnikov kernel (see Appendix A for details) for smoothing.

We used the bandwidth $h = c\lambda^{-\frac{1}{2}}$ with the smoothing parameter, $c = 0.15$, based on the suggestions in Stoyan & Stoyan (1994). A value of c=0.1 refers to weaker smoothing of $g(r)$ and a value of c=0.2 stronger smoothing of $g(r)$. The graphs of the estimated pair correlation functions $\hat{g}(r)$ are shown in Figure 10a. A cursory glance of the Figure 10a suggests that the three graphs for region 1 (graphs i, ii and iii) are significantly different from the three graphs for region 2 (graphs iv to vi). The relative frequency of point pairs with distances $r \in (2,20)$ is much larger in region 1 than in region 2. Thus the locations of the objects are more clustered in region 1 than those in region 2.

From Figure 10a, it appears that there is a small hardcore distance of a few pixels between the objects due to their size. Then, all the curves in Figure 10a show an increase above a level of 1. This indicates the presence of a cluster effect for the centres of gravity in the images. As r increases the graphs converge slowly against the theoretical graph of the pair correlation function corresponding to a situation of complete spatial randomness. For region 1, there seems to be no interaction between the points with distances $r > 25$, whereas for region 2, no interaction between the points sets in for distances $r > 10$.

5.4 Estimated *L*-function

The graphs of the estimated function, $\hat{L}(r) - r$, are shown in Figure 10b. They suggest that there is a clear difference between region 1 and region 2 in terms of the behaviour of the L-function as a function of r. Graphs (iv) to (vi) of Figure 10b indicate that the L-function curves have a positive slope for small r values, pointing to deviation from complete spatial randomness. For large r values, the L-function curves converge towards zero, with the exception of Figure 10b(vi), suggesting the presence of complete spatial randomness. The graph of region 2b (vi in Figure 10b) always runs above level zero for large values of r which needs further investigation.

All graphs of Figure 10b show the same qualitative behaviour as those of Figure 10a. First, there is a hardcore distance of about 1.5 pixels between the objects, since $\hat{L}(r) - r = -r$ for $0 \le r \le 1.5$. Then the $\hat{L}(r) - r$ curves rise above level zero which shows the attraction effects for these distances. As r value continues to increase, all curves tend to go towards a value of 0 for $\hat{L}(r) - r$, indicating the onset of complete spatial randomness. For larger distances with $r > 50$ the computed values of $\hat{g}(r)$ and $\hat{L}(r) - r$ are less reliable, i.e. less meaningful, because only a few point pairs with such a distance exist due to the sizes of the images. The *L*-function curve in Figure 10b(vi) always runs above level zero. This is interesting because the spatial pattern of the data shown in Figure 7f of the segment of the data chosen is in the neighbourhood of spatial point patterns shown in Figures 7d and 7e, and one would expect a similar *L*-function behaviour for the point patterns represented in Figures 7c, 7d, and 7e. Hence, the observation made with Figure 10b(vi) defies explanation and needs further study.

Also related to the processing question is whether or not the point process statistics computed for point patterns derived from the stack profile which has undergone neither coherency-filtering nor migration nor any other post-stack processing, would differ significantly from the results described in sections 5.2 to 5.3. In response to this question, we selected a shallow and a deep segment from the stack profile (Figures 11a and 11b) in the vicinity of the segments analyzed earlier (segments 5a and 5d from Figure 5). The point patterns extracted from the binary images for Figures 11a and 11b are shown in Figures 11c and 11d respectively. The estimated pair-correlation function, $g(r)$, and $L(r) - r$ function have shapes (Figure 11e and 11f) similar to what are observed for segments 5a and 5d, as noted in 10a(i) and 10b(i), and 10a(iv) and 10b(iv). However, there is a reduction in the highest value for both $g(r)$ and $L(r) - r$ functions. Also, the range of '*r*' values over which the clustering occurs is altered.

The estimated intensities of the point patterns for both the shallow and deep segments of the stack data (Table 1) are roughly the same, unlike the results observed in the case of the coherency-filtered and migrated stacks.

5.5 Monte Carlo tests on CSR

Spatial point patterns observed in data are customarily null-hypothesis tested for complete spatial randomness (Besag & Diggle, 1977; Gignoux *et al*. 1999; Diggle, 2003; Lucio *et al*. 2004; Lancaster & Downes, 2004), i.e. whether or not the points in the observed spatial point patterns derived from the data are distributed independent of each other and conditionally independent. We performed the Monte Carlo (rank) test on CSR for all images, using the pair correlation function and the *L*-function, with a choice of 5% as the significance level (see Appendix B). We used *r* values from 0.5 to 100 with an increment of 0.5. We summarize the results in Table 2 for the pair correlation function and *L*-function. The null-hypothesis is rejected in all cases, except for the *L*-function of the point patterns shown in Figures 8d, 11c and 11d, although the ranks are very close to the rejection region over a certain range of 'r' values. The rejection of null-hypothesis suggests that the points in the point patterns given in Figure 8 are not completely spatially random.

5.6 Confidence Intervals

In addition to the Monte Carlo tests, we computed *r*-wise confidence intervals, i.e. 5% upper and 5% lower bounds (see Appendix C). The point pattern can be regarded as a realization of a Poisson point process if the curve of the estimated characteristic runs inside the bands; otherwise, it is less likely that the point pattern is completely spatially random. We computed the confidence intervals for the $L(r) - r$ function and the pair correlation function, $g(r)$. Figures 12 and 13 indicate that the point patterns observed in image segments are not random. It is important to note, however, that the point pattern characteristics for regions 1 and 2, are not inside the confidence interval bands for small point pair distances, *r*.

Figure 14 illustrates the computed r-wise confidence intervals for both $g(r)$ and $L(r) - r$ functions for the stack case. Figures 14a and 14b support the point patterns for the shallow segment to favour strong clustering over a range of '*r*' values. However, the results for the noisier, deeper part, shown in Figures 14c and 14d, indicate very weak clustering.

6 DISCUSSION

For the two point pattern characteristics that we investigated, we note clear differences in results not only in the two processing schemes done to the reflection profiles but also in the two regions from where the data originated (see Figures 8 and 10). The differences in the processing schemes manifest themselves in the values of the pair correlation function for small r values. Coherency-filtered, migrated, coherency-filtered stacks appear to have larger values of $g(r)$ and $L(r) - r$ for small r values than coherency-filtered stacks (Figure 10). This observation should not be surprising since seismic migration acts as a spatial filter of the data. Most importantly, we observe that the spatial point patterns from region 1, regardless of the type of processing applied to it, lead to $g(r)$ values greater than 1 for small to intermediate r values (Figures 10a(i) to 10a(iii)) , and a positive slope for $L(r)$ values for small to intermediate r values (Figures 10b(i) to 10a(iii). This observation is in line with strong sediment layering noted in the shallow part of the data. However, this is not found to be completely true with the data from region 2 (Figures 10a(iv) to 10a(vi) and 10b(iv) to 10b(vi)). Low values of the point process characteristics suggest the attenuation or removal of the signature of the sedimentary layering at depths where the region 2 is present.

The presence of noise in the data with the stack example reveals marked differences in the point pattern characteristics between the shallow and the deep zone, similar to what is observed with the coherency-filtered data. The difference between the stack and the coherency-filtered data lies in the strength of the frequency of the $g(r)$ and $L(r) - r$ functions with values larger for the coherency-filtered data than for the stack data.

Since the present approach based on point process characteristics appears to discern the shallower and more stratified segments from the deeper segments, one might expect that a mosaic of a "single-attribute" that summarizes the point process characteristic for the entire binary image of the coherency-filtered, migrated, and coherency-filtered data of the Fort Simpson Basin is useful. Since the sum of the difference between the estimated L-function and the CSR result (see Appendix B) is used for hypothesis testing, it would be an appropriate single statistical measure. The sum here is done over a selected range of values of r. We used a window procedure which goes through the step of computing the single measure, the L-function attribute, for each window of spatial point patterns of pre-determined size, and repeating the procedure for overlapping windows to cover the entire binary image. The L-function attribute map thus derived is shown in Figure 15. This attribute map delineates the differences in stratigraphy in different parts of the Fort Simpson Basin.

7 CONCLUSIONS

The results of the present study address the stated objectives of comparing data with different processing schemes and of comparing point patterns from different regions:

(1) The differences between the two regions are much greater than the differences between the same region which has undergone two different types of processing. Furthermore, the spatial structure of object locations is maintained through this process. The intensity, the pair correlation function and the *L*-function show similar characteristics for the same region with different processing schemes, although they reveal marked differences between the regions. The presence of noise in the data, as is the case with the stack profile, does not alter the main conclusions drawn here.

(2) The point patterns built by the centres of gravity are not completely randomly distributed where the clustering effects for small point pair distances r are stronger for region 1 than for region 2. In both regions the slight hardcore effect for very small distances might result from the size of the objects and the discretization.

In summary, preliminary results of spatial point pattern analysis of deep crustal reflection seismic data look promising. They also suggest additional studies on the point process models such as Gibbs point process model (references) for the two regions investigated here. Furthermore, all point patterns in two regions should be tested for isotropy. Differences in anisotropy between the two regions, if any, will provide basis for additional research to quantify the stratigraphic structure within the deep sedimentary basin.

The results presented here are constrained by the spatial and temporal frequencies of the data. Also, for further generalization of the work, it would be necessary and important to include in the point extraction procedure not only the size of the object but also the amplitude information.

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Appendix A. Point processes

Let $X = \{X_n\}_{n\in N}$ be a point process in $\mathbb{R}^2$, i.e., $\{X_n\}$ can be interpreted as a sequence of random locations, i.e. random vectors with values, in the Euclidean plane. Let $X(W) = \#\{n : X_n \in W\}$ denote the number of points X_n of X located in a window $W \subset \mathbb{R}^2$.

Important properties of a point process X are, for example, the following ones.

- The point process X is called stationary if X and the shifted point process $X_y = \{y + X_n\}_{n\in N}$ have the same distribution for any $y \in \mathbb{R}^2$, i.e. X is invariant under translations.
- The point process X is called isotropic if X and the rotated point process $R_\alpha X = \{R_\alpha X_n\}_{n\in N}$ have the same distribution for any rotation $R_\alpha \in [0,2\pi]$ around the origin.

A.1 Point process models

A.1.1 Poisson point process

A (stationary and isotropic) point process X is called a Poisson point process with intensity λ if

- the random variable $X(W)$ is Poisson distributed with expectation $\lambda\,|W|$ for any window $W \subset \mathbb{R}^2$, where $|W|$ denotes the area of W and
- the random variables $X(W_1),\ldots,X(W_n)$ are independent for any sequence of pair-wise disjoint windows $W_1,\ldots,W_n$.

A.1.2. Matern cluster point process

A (stationary and isotropic) cluster point process X consists of a Poisson point process $X_p = \{X_n\}_{n\in N}$ of parent points and an appertained family of child point processes $\{X_{1m}\}_{m\in\{1,\ldots k_1\}}, \{X_{2m}\}_{m\in\{1,\ldots,k_2\}},\ldots$ which are (independent and identically distributed) Poisson point processes in the ball with radius R around the origin. Then the point process $X = \{X_{nm} + X_n\}_{m\in\{1,\ldots,k_n\},n\in N}$ is called a Matern cluster point process.

A.1.3. Matern hard core point process

A Matern hard core point process is obtained in the following way from a Poisson point process $X = \{X_n\}_{n\in \mathbb{N}}$ with intensity λ . First, the points of X are equipped with independent and uniformly on $[0,1]$ distributed random labels $\{M_n\}$. Then, the point X_n of X is deleted if

$$\min\{| X_n - X_m |: m \neq n\} \leq D$$

and if

$$M_n < \min\{M_m :| X_n - X_m |\leq D\}.$$

Otherwise, X_n is retained. The point process consisting of all retained points is the hard core point process considered in this paper.

A.2 Point process characteristics and their estimators

Several important characteristics of point processes are introduced in the following.

A.2.1 Intensity measure

The intensity measure Λ is defined as

$$\Lambda(W) = \mathbb{E}X(W)$$

for a given set W, where $\mathbb{E}$ denotes expectation. Hence $\Lambda(W)$ is the mean number of points in W. Often it is possible to express the intensity measure Λ in terms of a density, i.e. an intensity function $\lambda(x)$, where

$$\Lambda(W) = \int_W \lambda(x)dx .$$

In the stationary case it suffices to regard an intensity λ which does not depend on location $x \in W$ and such that $0 < \lambda < \infty$, since then

$$\Lambda(W) = \lambda\,|W|.$$

A natural estimator for λ is given by

$$\hat{\lambda} = \frac{X(W)}{|W|}.$$

We also remark that an appropriate estimator for λ^2 is given by

$$\hat{\lambda}^2 = \frac{X(W)(X(W)-1)}{|W|^2}. \qquad \text{(A-1)}$$

In the following let the point process X be stationary and isotropic.

A.2.2 Pair correlation function

In order to define the pair correlation function, the second factorial moment measure and the product density of second order have to be introduced first. Let W_1 and W_2 be two sets. The second factorial moment measure $\alpha^{(2)}$ of X is defined by

$$\alpha^{(2)}(W_1 \times W_2) = E\Big(\sum_{i \neq j} \mathbf{1}_{W_1}(X_i)\mathbf{1}_{W_2}(X_j)\Big),$$

where the sum extends over all pairs of points $X_i, X_j \in X$ with $i \neq j$ and $\mathbf{1}_W$ denotes the indicator function of the set W, i.e. $\mathbf{1}_W(x) = 1$ if $x \in W$ and $\mathbf{1}_W(x) = 0$ if $x \notin W$. Often $\alpha^{(2)}$ can be expressed by the usage of a density function $\rho^{(2)}$ as follows

$$\alpha^{(2)}(W_1 \times W_2) = \int_{W_1}\int_{W_2} \rho^{(2)}(x_1, x_2)\,dx_1 dx_2,$$

where $\rho^{(2)}$ is called the product density of second order. If two discs C_1 and C_2 are regarded that have infinitesimal areas dF_1, dF_2 and midpoints x_1, x_2 respectively, the probability for having in each disc at least one point of X is approximately equal to $\rho^{(2)}(x_1, x_2)dF_1 dF_2$. Notice that in the stationary and isotropic case $\rho^{(2)}(x_1, x_2)$ can be replaced by $\rho^{(2)}(r)$, where $r = |x_1 - x_2|$. As an estimator for $\rho^{(2)}(r)$,

$$\hat{\rho}^{(2)}(r) = \frac{1}{2\pi r} \sum_{X_i, X_j \in W, i \neq j} \frac{k_h(r - |X_i - X_j|)}{|W_{X_i} \cap W_{X_j}|} \qquad \text{(A-2)}$$

is used (Stoyan & Stoyan, 2000), where $W_{X_j} = \{x + X_j : x \in W\}$ is the window W translated by the point X_j and $k_h(x)$ denotes the Epanechnikov kernel

$$k_h(x) = \frac{3}{4h}\left(1 - \frac{x^2}{h^2}\right)\mathbf{1}_{(-h,h)}(x)$$

with bandwidth h. Notice that the expression $|W_{X_i} \cap W_{X_j}|$ in equation A-2 is considered for purposes of edge correction. The product density $\rho^{(2)}$ is used to obtain the pair correlation function $g(r)$ as

$$g(r) = \frac{\rho^{(2)}(r)}{\lambda^2}.$$

The pair correlation function can be estimated by the usage of estimators for $\rho^{(2)}(r)$ and λ^2 respectively, i.e.

$$\hat{g}(r) = \frac{\hat{\rho}^{(2)}(r)}{\hat{\lambda}^2},$$

where $\hat{\lambda}^2$ and $\hat{\rho}^{(2)}(r)$ are given in equations A-1 and A-2, respectively.

A.2.3 L-function

The L-function is a scaled version of Ripley's K function which is defined such that $\lambda K(r)$ is the expected number of points of the stationary point process X within a disc $b(X_n, r)$ centered at a randomly chosen point X_n of X which itself is not counted. As a formal definition one gets

$$\lambda K(r) = \mathbb{E} \sum_{X_n \in W} \frac{X(b(X_n, r)) - 1}{\lambda\, |W|} \quad .$$

A possible estimator for $K(r)$ is given by

$$\hat{K}(r) = \frac{k(r)}{\hat{\lambda}^{(2)}},$$

where

$$k(r) = \sum_{X_i, X_j \in W, i \neq j} \mathbb{1}_{b(o,r)}(X_i - X_j) \times \frac{1}{|W_{X_i} \cap W_{X_j}|} \, .$$

Again, the expression $|W_{X_i} \cap W_{X_j}|$ in the denominator of the latter sum is considered for edge correction. The L-function is then defined by

$$L(r) = \sqrt{\frac{K(r)}{\pi}}$$

and can be estimated by

$$\hat{L}(r) = \sqrt{\frac{\hat{K}(r)}{\pi}} \, .$$

Appendix B. Monte Carlo tests

The null-hypothesis of the Monte Carlo test is that the observed point pattern can be considered as a realization of a certain point process. The test procedure is the following: A significance level α has to be chosen. Frequent choices for α are 1% or 5%. Then the distance value *d* between a characteristic of the point pattern and the corresponding theoretical characteristic has to be calculated, e.g.

$$d = \sum_{r=0}^{\frac{1}{2}\min\{a,b\}} (\hat{L}(r) - L_{theoretical}(r))^2 ,$$

where *a* and *b* are the width and length of the window, W. Other point process characteristics can also be used. In the next step n realizations of the point process model under the null-hypothesis have to be generated. For $\alpha = 5\%$ it is customary to consider 99 realizations ($\alpha = 1\%$, $n = 999$). For each simulated point pattern, the distance values $d_1,...,d_n$ have to be calculated. Thus we obtain $n+1$ distance values $d, d_1, d_2,..., d_n$. These have to be arranged in ascending order and the position *i* of *d* has to be determined. Then the null-hypothesis is rejected if $i \in R_\alpha$, where $R_{0.05} = [96,100]$ and $R_{0.01} = [991,1000]$.

In this analysis we tested the null-hypothesis that the point pattern can be regarded as a realization of a Poisson point process with certain intensity, where we used the estimated intensity of the point pattern.

Appendix C. Confidence intervals of CSR

For the construction of *r*-wise 90% confidence intervals of CSR, we need 100 realizations of a Poisson point process with the same intensity. For each realization we estimate the point process characteristics, the $L(r) - r$ function and the pair correlation function. Then we arrange the 100 values for each r in ascending order and take the values at the 5th and 95th position. These constitute the confidence interval for each r value.

List of Tables

Table 1. Number of points (objects), areas |W| and $\hat{\lambda}$ of the images and estimated intensities from Figures 8 and 11

Description of the data	No. points	|W|	$\hat{\lambda}$
CF data, region 1 (Fig. 8a)	406	101404	0.00400
CF, M, CF data, region 1a (Fig. 8b)	336	101808	0.00330
CF, M, CF data, region 1b (Fig 8c)	399	102010	0.00391
CF data, region 2 (Fig. 8d)	899	103412	0.00869
CF, M, CF data, region 2a (Fig. 8e)	702	101808	0.00690
CF, M, CF data, region 2b (Fig. 8f)	677	101808	0.00665
S, shallow region (Fig. 11c)	814	101808	0.00799
S, deep region (Fig. 11d)	835	101808	0.00820

CF: Coherency-filtered; M: Migrated; S: Stack with no coherency-filtering.

Table 2. Results of the Monte Carlo tests on Complete Spatial Randomness

Image	Function	Rank	Reject null-hypothesis
CF data, region 1 (Fig. 8a)	$g(r)$	100	Y
	$L(r)$	98	Y
CF, M, CF data, region 1a (Fig. 8b)	$g(r)$	100	Y
	$L(r)$	100	Y
CF, M, CF data, region 1b (Fig. 8c)	$g(r)$	100	Y
	$L(r)$	100	Y
CF data, region 2 (Fig. 8d)	$g(r)$	100	Y
	$L(r)$	90	N
CF, M, CF data, region 2a (Fig. 8e)	$g(r)$	100	Y
	$L(r)$	98	Y
CF, M, CF data, region 2b (Fig. 8f)	$g(r)$	100	Y
	$L(r)$	99	Y
S, shallow region (Fig. 11c)	$g(r)$	100	Y
	$L(r)$	94	N
S, deep region (Fig. 11d)	$g(r)$	100	Y
	$L(r)$	78	N

CF: Coherency-filtered; M: Migrated; S: Stack with no coherency-filtering.

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Figure 1. Location Map of the study area. The gray-shaded part of deep crustal reflection seismic line 1 acquired under the Lithoprobe program is interpreted to be Fort Simpson Basin.

Figure 2. Realizations of point processes in sampling windows of size 100 x 100. a) Poisson point process with intensity, $\lambda = 0.01$; b) Matern hard core point process with D=10 and λ = 0.01; c) Matern cluster point process with R=10, $\lambda_p = 0.003$ and $\lambda_c = 0.1$.

Figure 3. a) Construction principle of hard core point processes, where the configuration of all (crosses and circles) points is sampled from a Poisson point process and thereafter, the crosses are deleted. Note that the Poisson point process has to be realized in a larger sampling window to avoid edge effects; b) Construction principle of Matern cluster point processes where the big points are sampled from the parent point process and the small points from the child point process. The realization is then obtained by regarding only the small points; c) Graphs for the theoretical pair correlation function $g_{MC}(r)$ and the theoretical $L_{MC}(r) - r$ function for the Matern cluster point process. Parameters used for theoretical $g_{MC}(r)$ and $L_{MC}(r)-r$ for the Matern cluster point process are R=10, λ_p=0.003, and λ_c=0.1.

Figure 4. Estimated point process characteristics, $\hat{g}(r)$ and $\hat{L}(r) - r$, for the realizations of point processes shown in Figure 2. i) Poisson point process; ii) Matern hard core point process; iii) Matern cluster point process.

Figure 5. Data used for spatial point pattern analysis. a) Coherency-filtered stack of a deep crustal reflection profile of SNORCLE line 1. Segments a and d enclosed in rectangular boxes represent structures with differing degrees of stratification; b) Coherency-filtered, migrated, and coherency-filtered stack of SNORCLE line 1. Segments b and c enclosed in rectangular boxes are in the neighbourhood of segment a shown in Figure 5a and segments e and f enclosed in rectangular boxes are in the neighbourhood of segment d shown in Figure 5a. The base of the basin is marked with a shaded, dash line in Figure 5a.

Figure 6. Segments of the data in Figure 5 used for a detailed spatial point pattern analysis. a) Segment from 5a where the stratified structure is intact (region 1); b) First segment from 5b in the same vicinity as the segment from 5a (region 1); c) Second segment from 5b in the neighbourhood of 6b; d) Segment from 5a where the stratified structure is considerably perturbed (region 2); e) First segment from 5b in the neighbourhood of 6d (region 2); e) Second segment from 5b in the neighbourhood of 6d (region 2). The reason for choosing two segments from the shallower (Figure 5a) and from the deeper (Figure 5b) parts of leading to 6b, 6c and 6e, 6f is to examine the effects of seismic migration.

Figure 7. Centre of gravity of an object. For a line segment or an object shown in a bitmap, using the 8-point neighbourhood connectivity, one extracts a point or the centre of gravity which is shown in gray colour. The source of a bitmap can be either a stack, or a migrated stack or a coherency-filtered migrated stack. In this study, for one line segment or object, no matter how long or short it is, there can only be one point.

Figure 8. Point patterns built by the centres of gravity of the objects in region 1. a) Segment 6a; b) Segment 6b; c) Segment 6c. Point patterns built by the centers of gravity of the objects in region 2. d) Segment 6d; e) Segment 6e; f) Segment 6f.

Figure 9. Angular distribution of point pairs in circular point patterns built in region 1. a) Segment 8a; b) Segment 8b; c) Segment 8c. Angular distribution of point pairs in circular point patterns in region 2. d) Segment 8d; e) Segment 8e; f) Segment 8f.

Figure 10. a) Estimated pair correlation functions $\hat{g}(r)$ with bandwidth $h = 0.15\lambda^{-1/2}$ for (i) Segment 8a; (ii) Segment 8b; (iii) Segment 8c; (iv) Segment 8d; (v) Segment 8e; (vi) Segment 8f. The line $g(r) \equiv 1$ is the theoretical pair correlation function in the case of complete spatial randomness; b) Estimated functions $\hat{L}(r) - r$ for the same six data sets as used in the estimated pair correlation function. (i) Segment 8a; (ii) Segment 8b; (iii) Segment 8c; (iv) Segment 8d; (v) Segment 8e; (vi) Segment 8f.

Figure 11. Point pattern analysis tests on a stack profile. The segments chosen for analysis are in the vicinity of the segments marked for the coherency-filtered stack. a) Shallow segment of the data; b) Deep segment of the data; c) Point pattern extracted from a); d) Point pattern extracted from b); e) $\hat{g}(r)$ for both the shallow (i) and deep (ii) segments; f) $\hat{L}(r) - r$ function for both the shallow (i) and deep (ii) segments. Symbols '+' and '◊' in graphs (e) and (f) refer to the shallow and deep segments respectively.

Figure 12. Monte Carlo tests on CSR based on pointwise confidence intervals for the point patterns of region 1. a) $\hat{g}(r)$ for Segment 8a; b) $\hat{L}(r) - r$ function for Segment 8a; c) $\hat{g}(r)$ function for Segment 8b; d) $\hat{L}(r) - r$ function for Segment 8b; e) $\hat{g}(r)$ for Segment 8c; f) $\hat{L}(r) - r$ function for Segment 8c.

Figure 13. Monte Carlo tests on CSR based on pointwise confidence intervals for the point patterns of regions 2. a) $\hat{g}(r)$ for Segment 8d; b) $\hat{L}(r) - r$ function for Segment 8d; c) $\hat{g}(r)$ for Segment 8e; d) $\hat{L}(r) - r$ function for Segment 8e; e) $\hat{g}(r)$ for Segment 8f; f) $\hat{L}(r) - r$ function for Segment 8f.

Figure 14. Monte Carlo tests on CSR based on pointwise confidence intervals for the point patterns of the shallow and deep segments of the shallow and deep segments of the stack shown in Figures 11c) and 11d). a) $\hat{g}(r)$ for Segement 11c; b) $\hat{L}(r) - r$ for Segment 11c; c) $\hat{g}(r)$ for Segement 11d; d) $\hat{L}(r) - r$ for Segment 11d.

Figure 15. a) Interpreted section of the coherency-filtered, migrated, and coherency-filtered stack of the Fort Simpson profile, based on Cook et al.(1999); b) *L*-function attribute display of the coherency-filtered, migrated, and coherency-filtered stack of the Fort Simpson Basin profile. This is a mosaic of the sum of the difference between the estimated *L*-function and the CSR result over *r* for all moving windows with each window of size 250 x 250 pixels and with an overlap of 200 pixels in both directions. In the grayscale bar, W (white) refers to small deviation from CSR, B (black) refers to high deviation from CSR, and R (red) zones where there are too few points to compute numerically stable L-function.