

Statistical Analysis of Anisotropic Microstructures in Friction-Extruded Aluminum Alloy AA6082 from Chips

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Abstract

Solid-state materials processing techniques, such as friction extrusion (FE), offer a promising and energy-efficient route for recycling aluminum from secondary raw materials. By combining severe plastic deformation with friction-induced heating, FE enables the consolidation of fragmented feedstock into dense extruded products. In this study, aluminum chips are used as feedstock material, where the high shear deformation at elevated processing temperatures promotes dynamic recrystallization, resulting in complex anisotropic microstructures of the extruded wire. A quantitative analysis is presented of two-dimensional electron backscatter diffraction (EBSD) data acquired from orthogonal sections of a friction-extruded AA6082 alloy. Geometric and crystallographic grain descriptors relevant to material performance are evaluated, and their probability distributions, directional dependencies, and variations associated with feedstock chip size are investigated through a systematic statistical analysis. The results demonstrate the influence of chip sizes on the morphological properties of the resulting microstructures. Beyond the univariate probabilistic characterization of individual grain descriptors, a statistical framework for the parametric representation of multivariate distributions of descriptor vectors is introduced, using so-called copulas. By combining univariate marginal distribution models with dependence structures between descriptors, the framework enables the quantitative reconstruction of multivariate microstructural statistics and provides a basis for statistically informed microstructure reconstruction methodologies.

Key words: grain architecture; friction extrusion; recycling; EBSD; statistical image analysis; multivariate probabilistic modeling; copula

1 Introduction

Aluminum (Al) alloys are among the most widely used lightweight metallic materials owing to their low density, high corrosion resistance, and favorable mechanical properties [1]. These characteristics enable their widespread use across a broad range of applications, including transportation, construction, and mechanical engineering. Despite their technological importance, the production and processing of metals account for a substantial share of global industrial greenhouse gas emissions. Al production, in particular, is highly energy-intensive and therefore contributes significantly to the environmental footprint of the metallurgical sector [2, 3]. At the same time, global Al demand continues to rise [4], increasing the pressure on primary resource extraction and highlighting the need for more sustainable material utilization strategies. In this context, Al production from secondary raw materials has emerged as a key approach to reduce both energy consumption and environmental impact. Compared with primary Al production, recycling can reduce energy requirements by up to 95%, making it one of the most effective pathways toward a more circular and resource-efficient Al industry [5].

Recycling routes for Al machining chips can generally be classified into two categories: conventional remelting-based recycling and solid-state direct conversion approaches [6]. In conventional recycling, chips are remelted, cast into billets, and subsequently reprocessed into new products. However, this route is still associated with significant energy consumption and material losses, while the high surface area to volume ratio of machining chips promotes

oxidation and contamination during melting. Al chips are particularly susceptible to the formation of stable oxide layers, such as Al_2O_3 , and repeated recycling cycles may further lead to impurity accumulation, thereby degrading material quality [7]. To mitigate these challenges, direct conversion approaches eliminate the melting step and instead transform chips directly into usable products through solid-state consolidation. By preserving the material in the solid state, these processes reduce oxidation, improve material yield, and lower energy requirements. Among the available direct conversion techniques, friction extrusion (FE) has emerged as a particularly promising method [8]. FE utilizes frictional heating in combination with severe plastic deformation to consolidate metallic chips into dense extrudates [9] while simultaneously promoting microstructural refinement [10]. As a result, FE has been extensively investigated as a solid-state recycling route for Al alloy scrap, enabling the production of wires [8], rods, and other profiles with mechanical properties that are often comparable or superior to conventionally processed materials [11, 12]. However, the severe plastic deformation and complex material flow inherent to FE also generate pronounced spatial and directional variations in the microstructure, resulting in microstructural anisotropy and potentially direction-dependent material properties [13]. A comprehensive quantitative characterization of these microstructures across sections oriented differently relative to the extrusion direction is therefore essential for understanding, predicting, and optimizing the performance of friction-extruded materials.

However, quantitative comparisons of grain-scale morphological and crystallographic descriptor distributions across orthogonal sections of friction-extruded material remain limited, particularly with respect to the influence of feedstock chip size. Addressing this gap requires techniques capable of resolving both the grain morphology and the crystallographic orientation. Electron backscatter diffraction (EBSD) has become a standard tool for this purpose, providing spatially resolved information on crystallographic orientations, grain boundaries, and grain geometry. Although three-dimensional (3D) EBSD techniques offer a more complete representation of microstructure, their application remains limited by high experimental costs, long acquisition times, and the extensive reconstruction procedures required [14]. As a result, two-dimensional (2D) EBSD measurements remain the primary source of quantitative microstructural data [15].

By segmenting EBSD datasets into individual grains, geometric and crystallographic descriptors can be extracted to provide a quantitative representation of the microstructure. Previous EBSD-based work on conventional extruded aluminum has used grain-size, morphological, and orientation statistics to investigate microstructural evolution [16]. However, analyzing the descriptors separately cannot capture the relationships among them. A combination of grain-size and orientation distribution functions has been used to examine such relationships [17], while copula models provide a way to reveal the dependencies between microstructural descriptors [18].

These analyses enable a systematic comparison of microstructural variability between feedstock conditions, including chip size. For this purpose, parametric stochastic models summarize these distributions and dependencies with only a few interpretable model parameters. Although nonparametric methods such as kernel density estimation can also yield smooth densities, the parametric representation provides a compact basis for subsequent analyses. By capturing variation in grain morphology and crystallographic characteristics, descriptor-based microstructural statistics provide a foundation for establishing quantitative process-structure-property relationships. Such relationships may be investigated directly through the analysis of large experimental datasets or indirectly through stochastic microstructure modeling [19, 20], where experimentally derived statistics are used to generate representative virtual microstructures for subsequent numerical materials testing and property prediction.

Existing studies commonly assess effects of processing and feedstock conditions, including chip size, using individual histograms and mean values [8, 21], which do not capture the full distribution or interactions among microstructural descriptors. Since chip size is inherently variable in recycling applications, a comprehensive statistical analysis is needed to quantify its influence on the resulting anisotropic microstructure and assess the quality of friction-extruded products from heterogeneous scrap. In this study, we analyze orthogonal 2D EBSD sections of friction-extruded AA6082 alloy produced from different feedstock chip sizes. We compare the distributions of grain-scale geometric and crystallographic descriptors across chip sizes and sectioning directions, and fit parametric marginal distributions and bivariate copula models to characterize selected dependencies. Together, these analyses allow us to assess how microstructural anisotropy and the relationships among grain descriptors vary with feedstock chip size. The remainder of this paper is organized as follows. Section 2 describes the FE process, EBSD characterization, microstructure segmentation, and the statistical framework used for descriptor analysis and multivariate modeling. Section 3 presents the statistical characterization of the microstructures, focusing on anisotropy and the influence of feedstock chip size through univariate and multivariate analyses of microstructural descriptors. Finally, in Section 4, the paper concludes with a summary of the main findings and an outlook for future work.

2 Materials and Methods

2.1 Sample preparation and imaging

AA6082 machining chips were generated by dry milling of a clean, T6-tempered AA6082 rod using the milling unit of a LASERTEC 65 DED Hybrid machine. Different milling parameters were used to produce chips of varying sizes. Coolant and lubricant were intentionally avoided to minimize contamination of the recycled feedstock.

The chips were classified into six size categories according to their width: 0.5 mm, 1 mm, 2 mm, 3 mm, 4 mm and 5 mm. The classified chips were subsequently compacted via hydraulic press manually and processed by FE using the FE100 system (Bond technology, USA) to produce wires. Each experiment utilized 160 g of chips. The extrusion setup consisted of a container diameter of 50 mm and a die bore diameter of 1.6 mm, resulting in an extrusion ratio (ER) of 977.

The general experimental methodology is summarized in Figure 1, which illustrates the complete process flow from feedstock preparation to the final wire product, where the classified machining chips are first compacted into a billet and then subjected to FE, where the combined action of axial force and tool rotation generates the frictional heat and severe plastic deformation needed to consolidate the chips into a continuous and dense wire extrudate.

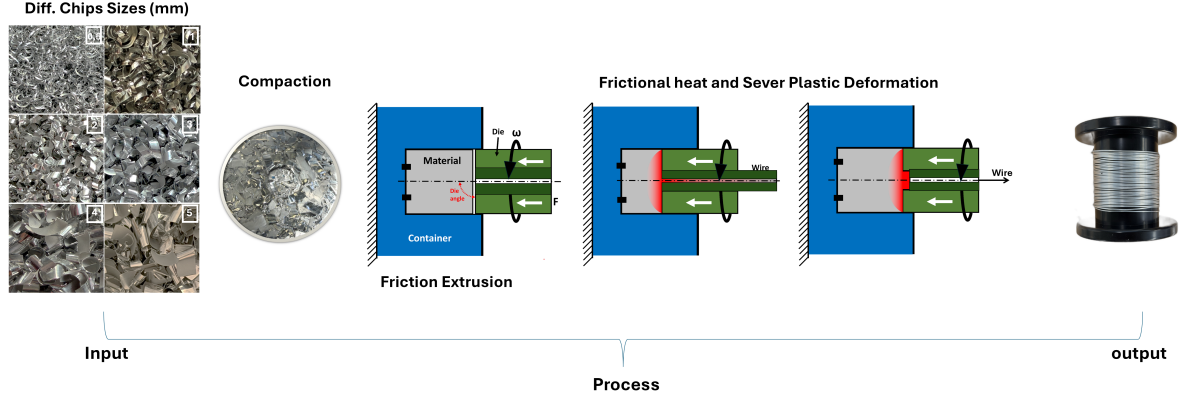


Figure 1: Schematic overview of the experimental methodology for recycling AA6082 machining chips into continuous wire via friction extrusion (FE). Input: machining chips classified into six width categories (0.5–5 mm). Compaction: classified chips consolidated into a billet prior to extrusion. FE process: chip compaction within the container (left), frictional heating and severe plastic deformation induced by the applied axial force F and tool rotational speed ω (middle), and formation of a continuous wire extrudate through the die (right). Output: continuous wire product collected on a spool.

The extrusion process was operated in force control mode, with the spindle-speed sequence programmed as a function of extrusion distance. Before extrusion began, the die was brought into contact with the feedstock during an approach stage at 300 rpm, advancing up to 3 mm under a comparatively low axial force of 50 kN, sufficient to establish contact and initiate consolidation without driving full extrusion. Once extrusion began, a constant axial force of approximately 400 kN was applied while the spindle rotation speed was maintained at 300 rpm for the first 10 mm of die travel and then reduced to 270 rpm for the following 5 mm. The axial force was then reduced to approximately 320 kN coinciding with a further reduction in spindle speed to 80 rpm, over the remaining fixed distance of 25 mm that completed the extrusion. This speed and force profile was established through preliminary trials and was applied consistently across all investigated feedstock chip sizes. Temperature during extrusion was measured with a K-type thermocouple, positioned 1.0 mm beneath the die face and at two-thirds of the radial distance from the center.

Microstructural characterization was performed using EBSD [22] on longitudinal and transverse sections of extruded wires to assess directional microstructural variations. Wire segments of 30 mm in length were selected based on visual inspection, choosing regions with a clean, crack-free surface, and were then mounted in epoxy resin and prepared using standard metallographic procedures and final polishing with colloidal silica. A subsequent 3 h vibratory polishing step was applied to remove residual deformation layers and ensure a surface quality suitable for reliable EBSD measurements.

The prepared samples were characterized using an FEI Quanta 650 field-emission gun scanning electron microscope (FEG-SEM; Thermo Fisher Scientific, Hillsboro, OR, USA), equipped with an EDAX Velocity EBSD camera (EDAX, Mahwah, NJ, USA). EBSD data were acquired and analyzed using OIM Analysis software (EDAX, Mahwah, NJ, USA). EBSD orientation maps were acquired from longitudinal and transverse sections using an accelerating voltage of 20 kV and a step size of 0.2 μm . The acquired orientation datasets were subsequently segmented into individual grains and subgrains to extract geometric and crystallographic descriptors for quantitative microstructural characterization and statistical analysis.

2.2 Microstructural grain descriptors

To statistically quantify the morphology and crystallographic orientation variation of the extrudates, we introduce a set of geometric and crystallographic descriptors. These descriptors are defined for the grains in the extrudate, each of which is represented by a connected set $G \subset W$ within some rectangular sampling window $W \subset \mathbb{R}^2$.

Area-equivalent diameter. The size of a grain G is given by its area-equivalent diameter $d_2(G)$, which is defined as the diameter of a circle with the same area as G , i.e.,

$$d_2(G) = 2\sqrt{\frac{A(G)}{\pi}}, \quad (1)$$

where $A(G)$ denotes the area of G . To compute the quantity $A(G)$ from pixelized image data, it is approximated by the number of pixels assigned to the pixelized representation $G_p \subset G$ of G multiplied by the pixel area.

Aspect ratio. To compute the aspect ratio of a grain $G \subset W$, an ellipse is first fitted to the set G . More precisely, principal component analysis (PCA) [23, 24] applied to G_p yields two orthonormal principal directions $a_1, a_2 \in \mathbb{R}^2$ together with their eigenvalues $\lambda_1 \geq \lambda_2 > 0$, where the vector a_1 points in the direction of the major axis and a_2 points in the direction of the minor axis of the fitted ellipse. The lengths corresponding to the axes are $l_1 = 4\sqrt{\lambda_1}$ and $l_2 = 4\sqrt{\lambda_2}$. The aspect ratio $E(G)$ of G is then given by

$$E(G) = \frac{l_2}{l_1} = \sqrt{\frac{\lambda_2}{\lambda_1}} \in (0, 1]. \quad (2)$$

Note that the case $E(G) = 1$, implying $\lambda_1 = \lambda_2$, corresponds to a circular best fit for the ellipse. In this case, the principal directions are not unique, since any pair of orthonormal directions provides an equivalent PCA solution.

Orientation of the major axis. The orientation of the major axis, denoted by $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2}]$, is defined as the angle between the x-axis and the direction $a_1 = (a_{1x}, a_{1y})$ of the major axis of the best-fitting ellipse, mathematically determined as

$$\theta(G) = \begin{cases} \arctan\left(\frac{a_{1y}}{a_{1x}}\right), & \text{if } a_{1x} \neq 0, \\ \frac{\pi}{2}, & \text{if } a_{1x} = 0. \end{cases} \quad (3)$$

Note that this accounts for the fact that the major axis is undirected. Consequently, a_1 and $-a_1$ represent the same orientation of the major axis. The orientation is therefore π -periodic, such that θ and $\theta + k\pi$, $k \in \mathbb{Z}$, are equivalent. In particular, the endpoints $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ correspond to the same orientation. In the equiaxed case, where $\lambda_1 = \lambda_2$, no major axis exists and its orientation is therefore undefined.

Solidity. The solidity $S(G)$, sometimes also called convexity, of a grain $G \subset W$, is defined as the ratio of the grain area $A(G)$ to the area $A(c(G))$ of its convex hull $c(G) \subset \mathbb{R}^2$, i.e.,

$$S(G) = \frac{A(G)}{A(c(G))} \in (0, 1], \quad (4)$$

where $c(G) = \bigcap \{C \subseteq \mathbb{R}^2 : C \text{ is convex and } G \subseteq C\}$ is the smallest convex set that contains G , which is computed on G_p using the Quickhull algorithm [25], implemented by scikit-image [24]. Note that for a convex grain $G' \subset W$ it holds $A(G') = A(c(G'))$ and, consequently, $S(G') = 1$.

Irregularity. The irregularity of a grain $G \subset W$ is defined based on the radii of its largest inscribed and smallest circumscribed circles. Note that the radius $r_{\text{in}}(G)$ of the largest inscribed circle is given by

$$r_{\text{in}}(G) = \sup \{r > 0 : B_r(x) \subseteq G \text{ for some } x \in G\}, \quad (5)$$

where $B_r(x) = \{y \in \mathbb{R}^2 : \|y - x\| \leq r\}$ denotes the closed disk of radius $r > 0$ centered at $x \in \mathbb{R}^2$, and $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^2$. Numerically, $r_{\text{in}}(G)$ can be estimated by taking the maximum value of the Euclidean distance transform [26] of the binary mask that represents the set G_p . Similarly, the radius of the smallest circumscribed circle is defined as

$$r_{\text{circ}}(G) = \inf \{r > 0 : G \subseteq B_r(x) \text{ for some } x \in \mathbb{R}^2\}. \quad (6)$$

This radius is numerically computed on G_p , where the circle center is initialized at the grain centroid and iteratively moved halfway toward the point farthest from the current center. The estimated circumscribed radius is then determined as $\max_{x \in G_p} \|c_{\text{circ}} - x\|$, where $c_{\text{circ}} \in \mathbb{R}^2$ denotes the final center. Since $r_{\text{in}}(G) \leq r_{\text{circ}}(G)$, for the irregularity $I(G)$ defined as

$$I(G) = \frac{r_{\text{in}}(G)}{r_{\text{circ}}(G)}, \quad (7)$$

it holds that $I(G) \in (0, 1]$. Values close to one indicate grains whose shape is close to circular, whereas smaller values correspond to increasingly elongated or irregular geometries. Since the largest inscribed circle is sensitive to local constrictions, this descriptor is particularly affected by narrow necks, indentations, or holes. For example, a perfectly circular grain containing a small central hole has an irregularity such that $I(G) \approx \frac{1}{2}$.

Grain orientation spread. The grain orientation spread $\text{GOS}(G)$ quantifies the average crystallographic deviation of the orientations within a grain $G \subset W$ from its mean orientation. It is defined as follows. Suppose that G is a grain represented by $n \in \mathbb{N}$ sampled pixels with associated orientations $g_1, \dots, g_n \in \text{SO}(3)$, where $\mathbb{N} = \{1, 2, 3, \dots\}$ is the set of natural numbers and $\text{SO}(3) = \{R \in \mathbb{R}^{3 \times 3} : R^T R = I, \det R = 1\}$ is the three-dimensional rotation group, with I denoting the identity matrix. Let $\bar{g}(G) \in \text{SO}(3)$ be the mean orientation of the grain, defined as the Fréchet mean on the crystallographic orientation space [27]. The misorientation angle $\Delta(g_i, \bar{g}(G))$ between a pixel orientation g_i and the mean orientation $\bar{g}(G)$ is then defined as

$$\Delta(g_i, \bar{g}(G)) = \min_{S \in \mathcal{S}} \phi(g_i^{-1} \bar{g}(G) S), \quad (8)$$

where $\mathcal{S}$ denotes the relevant crystal symmetry group, $\phi(\cdot)$ is the angle associated with a rotation, and $g_i^{-1} \in \text{SO}(3)$ denotes the inverse of g_i . For more details, the reader is referred to [28]. The grain orientation spread $\text{GOS}(G)$ is then defined as the mean misorientation angle of all pixel orientations with respect to the mean grain orientation, i.e.,

$$\text{GOS}(G) = \frac{1}{n} \sum_{i=1}^n \Delta(g_i, \bar{g}(G)). \quad (9)$$

Hence, the quantity $\text{GOS}(G)$ provides a measure of the intragranular orientation variation, with low values indicating nearly uniform crystal orientation and higher values reflecting stronger orientation gradients resulting from geometrically necessary dislocations that occur after plastic deformation.

2.3 Kernel density estimation

The values of many microstructural descriptors, such as the area-equivalent diameter, solidity, aspect ratio, or grain orientation spread, can be interpreted as realizations of an underlying random variable X . Given a finite sample of observed descriptor values $x_1, x_2, \dots, x_k \in \mathbb{R}$ of k grains for some $k \in \mathbb{N}$, the goal is to estimate the (unknown) probability density function $f_X : \mathbb{R} \rightarrow [0, \infty)$ from which these observations were generated.

A simple and non-parametric approach for estimating probability densities is the use of histograms. However, histograms strongly depend on the choice of bin width and bin placement and lead to discontinuous density estimates. Therefore, we employ kernel density estimation (KDE) [29], which provides smooth non-parametric estimates of the underlying probability density $f_X : \mathbb{R} \rightarrow [0, \infty)$. In particular, for KDE, we consider a kernel function $K : \mathbb{R} \rightarrow [0, \infty)$ that satisfies

$$K(x) \geq 0 \text{ for each } x \in \mathbb{R}, \quad \text{and} \quad \int_{\mathbb{R}} K(x) dx = 1. \quad (10)$$

Given a bandwidth $h > 0$, the kernel density estimate $\hat{f}_h : \mathbb{R} \rightarrow [0, \infty)$ of f_X is then defined as

$$\hat{f}_h(x) = \frac{1}{kh} \sum_{i=1}^k K\left(\frac{x - x_i}{h}\right) \quad \text{for each } x \in \mathbb{R}. \quad (11)$$

The estimate $\hat{f}_h$ given in Eq. (11) causes a certain smoothing, which can be interpreted as placing a localized probability density around each observation x_i , where the bandwidth h controls the degree of smoothing. In this paper, Gaussian kernel density estimates are employed using the Gaussian kernel function $K_G : \mathbb{R} \rightarrow (0, \infty)$ given by

$$K_G(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \quad \text{for each } x \in \mathbb{R}. \quad (12)$$

Substituting this kernel into Eq. (11) yields

$$\hat{f}_h(x) = \frac{1}{kh\sqrt{2\pi}} \sum_{i=1}^k \exp\left(-\frac{(x - x_i)^2}{2h^2}\right) \quad \text{for each } x \in \mathbb{R}. \quad (13)$$

Consequently, each sample value contributes a Gaussian density centered at the observation location.

Note that the kernel density estimate introduced in Eq. (11) can be extended to the d -dimensional case for any $d \in \mathbb{N}$, where $x_1, x_2, \dots, x_k \in \mathbb{R}^d$. Then, the estimate $\hat{f}_h$ given in Eq. (11) generalizes to the estimate $\hat{f}_H : \mathbb{R}^d \rightarrow [0, \infty)$ such that

$$\hat{f}_H(x) = \frac{1}{k\sqrt{|H|}} \sum_{i=1}^k K_d\left(H^{-\frac{1}{2}}(x - x_i)\right) \quad \text{for each } x \in \mathbb{R}^d, \quad (14)$$

where $H \in \mathbb{R}^{d \times d}$ is the (positive definite) bandwidth matrix and $K_d : \mathbb{R}^d \rightarrow [0, \infty)$ is a multivariate kernel function. However, the sample size required for reliable multivariate KDE increases rapidly with dimensionality [29].

2.4 Copulas

The sampled values of two different grain descriptors can be interpreted as realizations of some random variables X_1 and X_2 with probability density functions $f_{X_1} : \mathbb{R} \rightarrow [0, \infty)$ and $f_{X_2} : \mathbb{R} \rightarrow [0, \infty)$, respectively. If X_1 and X_2 are independent, then the joint density $f_{(X_1, X_2)} : \mathbb{R}^2 \rightarrow [0, \infty)$ of the two-dimensional random vector (X_1, X_2) is given by $f_{(X_1, X_2)}(x_1, x_2) = f_{X_1}(x_1)f_{X_2}(x_2)$ for any $x_1, x_2 \in \mathbb{R}$. However, in general, the assumption of independence is not justified by grain morphology data, which often show clear correlations. To account for such dependencies while preserving the (marginal) distributions of each individual descriptor, we employ the concept of copulas, and, in particular, a copula-based representation formula of multivariate distribution functions. Moreover, to fit copulas to data, we consider a specific parametric copula family, the so-called Archimedean copulas [30].

2.4.1 Copula-based representation of multivariate distribution functions

For some $d \geq 2$, consider d grain descriptors whose values can be represented by some random variables $X_1, \dots, X_d$ with probability density functions $f_{X_i} : \mathbb{R} \rightarrow [0, \infty)$ and cumulative distribution functions $F_{X_i} : \mathbb{R} \rightarrow [0, 1]$ defined as $F_{X_i}(x_i) = \int_{-\infty}^{x_i} f_{X_i}(t) dt$ for any $x_i \in \mathbb{R}$ and $i \in \{1, \dots, d\}$. Thus, $F_{X_i}(x_i)$ is the probability that the values of the random variable X_i do not exceed the level x_i , i.e., $F_{X_i}(x_i) = \mathbb{P}(X_i \leq x_i)$ for any $x_i \in \mathbb{R}$ and $i \in \{1, \dots, d\}$. Furthermore, consider the joint probability density function $f_{(X_1, \dots, X_d)} : \mathbb{R}^d \rightarrow [0, \infty)$ of the random vector $(X_1, \dots, X_d)$ and its (joint) cumulative distribution function $F_{(X_1, \dots, X_d)} : \mathbb{R}^d \rightarrow [0, 1]$ which is defined as $F_{(X_1, \dots, X_d)}(x_1, \dots, x_d) = \int_{-\infty}^{x_1} \dots \int_{-\infty}^{x_d} f_{(X_1, \dots, X_d)}(t_1, \dots, t_d) dt_d \dots dt_1$ for any $(x_1, \dots, x_d) \in \mathbb{R}^d$, i.e., $F_{(X_1, \dots, X_d)}(x_1, \dots, x_d) = \mathbb{P}(X_1 \leq x_1, \dots, X_d \leq x_d)$ for any $(x_1, \dots, x_d) \in \mathbb{R}^d$.

Sklar's theorem [30] states that the values of any multivariate cumulative distribution function $F_{(X_1, \dots, X_d)}$ with marginals $F_{X_1}, \dots, F_{X_d}$ can be represented by some copula $C : [0, 1]^d \rightarrow [0, 1]$, i.e.,

$$F_{(X_1, \dots, X_d)}(x_1, \dots, x_d) = C(F_{X_1}(x_1), \dots, F_{X_d}(x_d)) \quad \text{for each } (x_1, \dots, x_d) \in \mathbb{R}^d. \quad (15)$$

The advantage of the representation formula given in Eq. (15) is that the complexity of the multivariate cumulative distribution function $F_{(X_1, \dots, X_d)} : \mathbb{R}^d \rightarrow [0, 1]$ is broken down into two (simpler) components: the univariate distribution functions $F_{X_1}, \dots, F_{X_d} : \mathbb{R} \rightarrow [0, 1]$ that represent the behavior of the individual random descriptors $X_1, \dots, X_d$, and a copula $C : [0, 1]^d \rightarrow [0, 1]$, which is a specific multivariate cumulative distribution function that represents the dependence structure of $X_1, \dots, X_d$, where

$$C(\underbrace{1, \dots, 1}_{(i-1)\text{-times}}, x_i, \underbrace{1, \dots, 1}_{(d-i)\text{-times}}) = x_i, \quad (16)$$

for any $x_i \in [0, 1]$ and $i \in \{1, \dots, d\}$, i.e., the copula $C : [0, 1]^d \rightarrow [0, 1]$ is a multivariate cumulative distribution function whose marginal distributions are equal to the standard uniform distribution on the unit interval $[0, 1]$. Note that this is consistent with the fact that for each $i \in \{1, \dots, d\}$, the marginal distribution function F_{X_i} transforms X_i into the random variable $U_i = F_{X_i}(X_i)$, which is uniformly distributed on $[0, 1]$ [30].

Moreover, from Eq. (15) it follows that the values of the joint density function $f_{(X_1, \dots, X_d)} : \mathbb{R}^d \rightarrow [0, \infty)$ of the random vector $(X_1, \dots, X_d)$ can be given as

$$f_{(X_1, \dots, X_d)}(x_1, \dots, x_d) = c(F_{X_1}(x_1), \dots, F_{X_d}(x_d)) f_{X_1}(x_1) \dots f_{X_d}(x_d) \quad \text{for each } (x_1, \dots, x_d) \in \mathbb{R}^d, \quad (17)$$

where $c : [0, 1]^d \rightarrow [0, \infty)$ is the probability density of C . If $X_1, \dots, X_n$ are independent, then $c(u_1, \dots, u_d) = 1$ for each $(u_1, \dots, u_d) \in [0, 1]^d$ and, in this case, Eq. (17) simplifies to $f_{(X_1, \dots, X_d)}(x_1, \dots, x_d) = f_{X_1}(x_1) \dots f_{X_d}(x_d)$ for each $(x_1, \dots, x_d) \in \mathbb{R}^d$.

2.4.2 Archimedean copulas

To parametrically model the joint distribution function $F_{(X_1, \dots, X_d)}$ of some random vector $(X_1, \dots, X_d)$ with parametric marginal distribution functions $F_{X_1}, \dots, F_{X_d}$, using the representation formula given in Eq. (15), it is sufficient to additionally specify the corresponding copula C . Note that a wide variety of parametric copula families exists, but many of them do not admit a closed-form analytical representation.

In this paper, we restrict our attention to the class of Archimedean copulas that are a suitable tool for multivariate modeling of grain descriptor distributions [18, 31]. Their definition is based on Archimedean generators [30], which are continuous, strictly decreasing, and convex functions $\varphi : [0, 1] \rightarrow [0, \infty]$ with $\varphi(0) = \infty$ and $\varphi(1) = 0$. The copula $C : [0, 1]^d \rightarrow [0, 1]$ is then given by

$$C(u_1, \dots, u_d) = \varphi^{-1}(\varphi(u_1) + \dots + \varphi(u_d)) \quad (18)$$

for each $u = (u_1, \dots, u_d) \in [0, 1]^d$, where φ^{-1} denotes the inverse function of φ .

To parametrically model the copula density c considered in Eq. (17), we employ a parametric family of Archimedean generators $\{\varphi_\gamma : \gamma \in \Gamma\}$, where Γ denotes the admissible parameter space. Each family of such generators induces a corresponding parametric family $\{c_\gamma : \gamma \in \Gamma\}$ of copula densities, where

$$c_\gamma(u_1, \dots, u_d) = \frac{\partial^d}{\partial u_1 \dots \partial u_d} \varphi_\gamma^{-1}(\varphi_\gamma(u_1) + \dots + \varphi_\gamma(u_d)) \quad \text{for any } u_1, \dots, u_d \in [0, 1] \text{ and } \gamma \in \Gamma. \quad (19)$$

Examples of Archimedean copulas are the Gumbel, Frank, Clayton, and Joe copulas [30], where the corresponding families of Archimedean generators $\{\varphi_\gamma : \gamma \in \Gamma\}$ and their parameter spaces are listed in Table 1.

Copula family	Generator $\varphi_\gamma(u)$	Parameter range
Gumbel	$(-\log u)^\gamma$	$\gamma \in \Gamma = [1, \infty)$
Frank	$-\log \left(\frac{\exp(-\gamma u) - 1}{\exp(-\gamma) - 1} \right)$	$\gamma \in \Gamma = \mathbb{R} \setminus \{0\}$
Clayton	$u^{-\gamma} - 1$	$\gamma \in \Gamma = (0, \infty)$
Joe	$-\log(1 - (1 - u)^\gamma)$	$\gamma \in \Gamma = [1, \infty)$

Table 1: Generators of Archimedean copulas.

For each family $\{c_\gamma : \gamma \in \Gamma\}$ of parametric copula densities given in Eq. (19), we also consider rotated variants obtained by reflecting any subset of the coordinates about the center of the unit hypercube $[0, 1]^d$. If $d = 2$, then the four variants correspond to rotations by 0° , 90° , 180° , and 270° . In general, for $d \geq 2$, these variants are indexed by $r = (r_1, \dots, r_d) \in \{0, 1\}^d$. For each $r \in \{0, 1\}^d$, let $R_r : [0, 1]^d \rightarrow [0, 1]^d$ denote the corresponding coordinate-reflection map, defined componentwise by

$$(R_r(u))_j = \begin{cases} u_j, & r_j = 0, \\ 1 - u_j, & r_j = 1, \end{cases} \quad (20)$$

for any $u = (u_1, \dots, u_d) \in [0, 1]^d$ and $j \in \{1, \dots, d\}$. The resulting family $\{c_{(\gamma, r)} : \gamma \in \Gamma\}$ of rotated copula densities is then given by $c_{(\gamma, r)}(u) = c_\gamma(R_r(u))$ for each $u = (u_1, \dots, u_d) \in [0, 1]^d$. Substituting these copula densities into Eq. (17) results in the parametric family $\{f_{(\gamma, r)}, (\gamma, r) \in \Gamma \times \{0, 1\}^d\}$ of multivariate probability densities $f_{(\gamma, r)} : \mathbb{R}^d \rightarrow [0, \infty)$, where

$$f_{(\gamma, r)}(x_1, \dots, x_d) = c_{(\gamma, r)}(F_{X_1}(x_1), \dots, F_{X_d}(x_d)) f_{X_1}(x_1) \dots f_{X_d}(x_d), \quad (21)$$

for any $(x_1, \dots, x_d) \in \mathbb{R}^d$ and $(\gamma, r) \in \Gamma \times \{0, 1\}^d$.

2.4.3 Maximum-likelihood estimation

The parametric copula approach stated in Section 2.4.2 yields a flexible class of candidate models that account for both different dependence strengths and asymmetric dependence structures induced by rotations, where we select the best fitting copula model from the family $\{c_{(\gamma, r)}, (\gamma, r) \in \Gamma \times \{0, 1\}^d\}$ based on a suitable model selection criterion.

For this purpose, given a sample of k observations $\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(k)} \in \mathbb{R}^d$ of some d -dimensional descriptor vector $(X_1, \dots, X_d)$ and a fixed rotation $r \in \{0, 1\}^d$, the corresponding log-likelihood function $\log L(\cdot \mid \mathbf{x}^{(1)}, \dots, \mathbf{x}^{(k)}, r) : \Gamma \rightarrow [-\infty, \infty)$ is defined as

$$\log L(\gamma \mid \mathbf{x}^{(1)}, \dots, \mathbf{x}^{(k)}, r) = \sum_{i=1}^k \log f_{(\gamma, r)}(\mathbf{x}^{(i)}), \quad (22)$$

for each $\gamma \in \Gamma$. Maximizing the log-likelihood function given in Eq. (22) over Γ yields the maximum-likelihood estimate

$$\hat{\gamma} = \arg \max_{\gamma \in \Gamma} \log L(\gamma \mid \mathbf{x}^{(1)}, \dots, \mathbf{x}^{(k)}, r). \quad (23)$$

Note that $\hat{\gamma}$ is the parameter value that provides the best fit to the observed sample $(\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(k)}) \in \mathbb{R}^{kd}$ for given model type and rotation. The value of $\hat{\gamma}$ was determined by numerically solving the maximization problem considered in Eq. (23) using Brent's optimization method [32]. This estimation procedure is carried out for all types of copulas and rotation angles considered in the present paper. Then, among all fitted models, the combination of those copula type and rotation that produce the highest value of the likelihood function is selected as the best fitting model.

Compared to non-parametric approaches such as d -dimensional kernel density estimation, copula models provide a compact and interpretable representation of multivariate probability distributions. By modeling the

marginal distributions and the dependence structure separately, they allow changes in individual descriptors to be distinguished from changes in their statistical dependence. Furthermore, each marginal distribution can be represented by a parametric model whose support matches the natural range of the corresponding descriptor values (belonging often to the unit interval $[0, 1]$), thus avoiding the boundary corrections typically required for kernel density estimation of bounded variables.

Besides the four types of Archimedean copulas mentioned above, i.e. Gumbel, Frank, Clayton, and Joe copulas, four parametric families of univariate marginal distributions are considered. Their selection depends on the range and empirical shape of the respective descriptor values. The distributions of descriptors whose values are restricted to the unit interval (e.g., solidity) are modeled using the beta distribution whose probability density is defined as

$$f_{\text{Beta}}(x; \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}, \quad 0 < x < 1, \quad \alpha, \beta > 0, \quad (24)$$

where $\Gamma : (0, \infty) \rightarrow (0, \infty)$ denotes the gamma function. Depending on its parameters, the beta distribution can represent symmetric, skewed, uniform-like, and U-shaped densities [33].

Strictly positive and right-skewed descriptors like the area-equivalent diameter are modeled using either a lognormal or a Burr type XII distribution. A positive random variable X follows a lognormal distribution if its probability density is given by

$$f_{\text{LN}}(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{(\log x - \mu)^2}{2\sigma^2}\right], \quad x > 0, \quad \mu \in \mathbb{R}, \quad \sigma > 0. \quad (25)$$

Equivalently, the transformed random variable $\log X$ follows a normal distribution with mean μ and standard deviation σ [33]. The Burr XII distribution provides additional flexibility for strongly skewed or heavy-tailed positive descriptors. Using two shape parameters $c, \eta > 0$ and a scale parameter $\lambda > 0$, its density is given by

$$f_{\text{Burr}}(x; c, \eta, \lambda) = \frac{c\eta}{\lambda} \left(\frac{x}{\lambda}\right)^{c-1} \left[1 + \left(\frac{x}{\lambda}\right)^c\right]^{-(\eta+1)}, \quad x > 0. \quad (26)$$

Note that the shape parameters c, η allow the distribution to accommodate a wide range of tail weights and density shapes [34].

Angular descriptors, such as the orientation of the major grain axis, require a circular rather than a linear probability model, since an angle of $\frac{-\pi}{2}$ is equivalent to an angle of $\frac{\pi}{2}$, see Section 2.2. Therefore, they are represented by the von-Mises distribution, whose density is defined as

$$f_{\text{vM}}(\theta; \mu, \kappa) = \frac{\exp[\kappa \cos(\theta - \mu)]}{2\pi I_0(\kappa)}, \quad 0 \leq \theta < 2\pi, \quad (27)$$

where $\mu \in [0, 2\pi)$ is the mean direction, $\kappa \geq 0$ is the concentration parameter and $I_0 : [0, \infty) \rightarrow [1, \infty)$ is the modified Bessel function of the first kind and order zero [33]. The case $\kappa = 0$ corresponds to the uniform distribution on the circle, whereas increasing κ produces a stronger concentration around μ . For axial orientations, for which θ and $\theta + \pi$ represent the same direction, the model is fitted to the doubled angles 2θ .

Similar to the procedure considered in Eqs. (22) and (23), the parameters of each admissible family of univariate marginal distributions are estimated by the maximum-likelihood method. The fitted family is selected according to its likelihood or an information criterion [35]. The fitted marginal cumulative distribution functions $\hat{F}_{X_1}, \dots, \hat{F}_{X_d}$ are subsequently used to transform the observation $\mathbf{x}^{(j)} = (x_1^{(j)}, \dots, x_d^{(j)}) \in \mathbb{R}^d$ into the pseudo-observation $\hat{\mathbf{u}}^{(j)} = (\hat{F}_{X_1}(x_1^{(j)}), \dots, \hat{F}_{X_d}(x_d^{(j)})) \in [0, 1]^d$ for each $j \in \{1, \dots, k\}$. Provided that the fitted marginal models are appropriate, each transformed component is approximately uniformly distributed on $[0, 1]$. The resulting sample $\hat{\mathbf{u}}^{(1)}, \dots, \hat{\mathbf{u}}^{(k)}$ is then supplied to the copula model to estimate the dependence structure independently of the marginal distributions.

3 Results and discussion

3.1 Crystallographic and textural characterization

2D maps of the microstructure of the consolidated and extruded wires described in Section 2.1 are shown in Figure 2. For clarity, only the microstructures obtained from the chip sizes of 0.5 mm, 3 mm, and 5 mm are presented here, while the corresponding maps for 1 mm, 2 mm, and 4 mm feedstock are provided in Figure A1 in the Appendix. The reference direction of the inverse pole figure (IPF) color key is chosen parallel to the extrusion direction (ED) in both the transverse and longitudinal direction. A pronounced [100] texture is evident from the predominately red IPF coloring in both longitudinal and transverse sections, indicating a preferred crystallographic orientation along the ED.

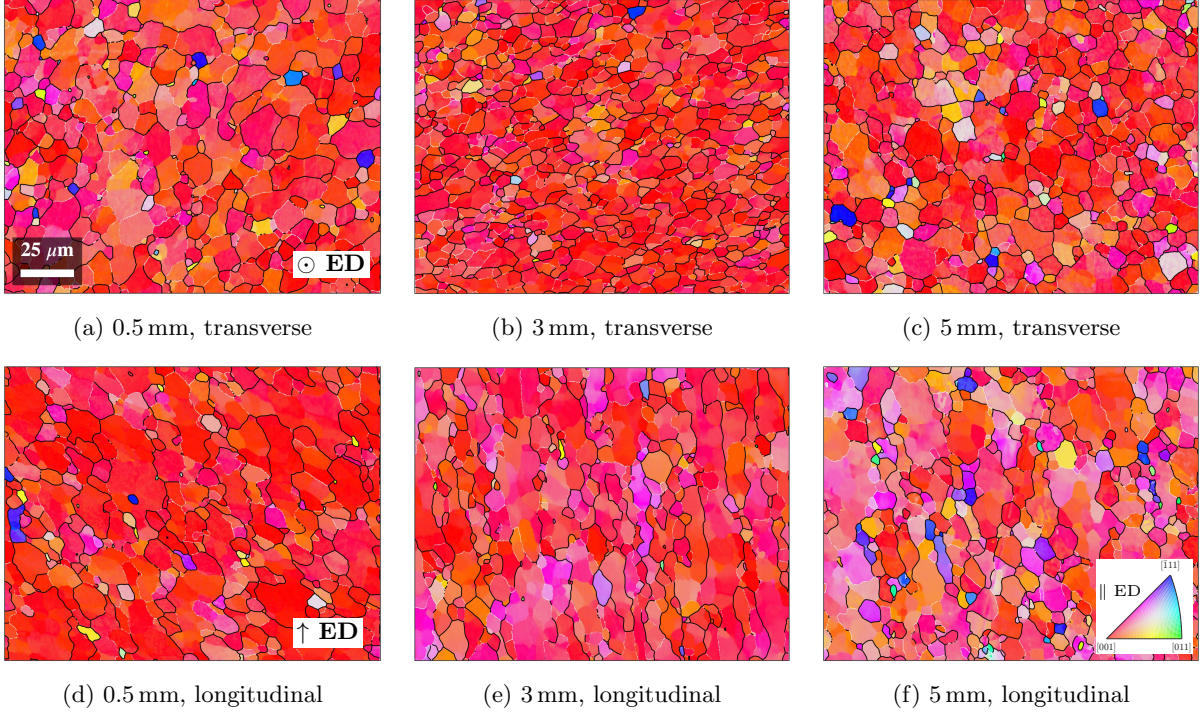


Figure 2: IPF orientation maps colored parallel to the extrusion direction in both transverse and longitudinal sections (ED out-of-plane for transverse sections, ED vertical for longitudinal sections) with high-angle grain boundaries ($\geq 15^\circ$) overlaid. Feedstock chip size increases left to right.

In addition to the strong texture, the microstructures shown in Figure 2 exhibit predominantly equiaxed grains in the wire cross sections, whereas the longitudinal sections reveal elongated grain morphologies aligned with the ED. This contrast reflects the anisotropic deformation conditions and the material flow during FE. Furthermore, a refined grain structure is observed throughout all specimens, consistent with extensive dynamic recrystallization induced by severe plastic deformation and frictional heating. While the overall microstructural appearance remains similar across the investigated feedstock chip sizes, subtle differences in grain morphology can be identified. In particular, the longitudinal sections of the intermediate chip size (3 mm) exhibit a more pronounced grain elongation, whereas the microstructures produced from the smallest and largest chips appear to be comparatively more equiaxed. This behaviour may be related to differences in chip packing and deformation mode during extrusion. The smallest chips pack densely and distribute strain across numerous interfaces, while the largest chips deform largely as bulk material. Both conditions favor uniform deformation and an equiaxed grain structure. Intermediate chips, in contrast, may pack less uniformly and tend to align and shear along their interfaces, promoting localized, directional material flow, and a more elongated grain structure.

Figure 3 provides a more detailed view of the grain-boundary network by distinguishing low-angle grain boundaries (LAGBs, $2\text{--}15^\circ$ misorientation, red) from high-angle grain boundaries (HAGBs, $\geq 15^\circ$ misorientation, black). Across all investigated chip sizes, HAGBs constitute the dominant boundary type and define the overall grain structure, indicating extensive recrystallization during FE. In contrast, LAGBs are distributed inside the grains, reflecting the presence of subgrain structures formed during recovery and recrystallization. Similarly to the IPF maps, directional differences are evident between the transverse and longitudinal sections. While the transverse sections exhibit predominantly equiaxed grain structures, the longitudinal sections reveal elongated grains and grain-boundary networks aligned with the ED. This alignment is particularly pronounced for the material processed from 3 mm chips, suggesting enhanced microstructural anisotropy compared to the 0.5 mm and 5 mm feedstock conditions. Corresponding plots for chip sizes of 1 mm, 2 mm, and 4 mm are provided in the Appendix, see Figure A2.

The IPFs corresponding to the ED are shown in Figure 4. All investigated conditions exhibit a pronounced texture with a strong concentration of orientations in the vicinity of the $[001]$ corner, indicating the presence of a dominant $\langle 001 \rangle \parallel \text{ED}$ fiber texture. However, the texture intensity decreases systematically with increasing feedstock chip size. For the material produced from 0.5 mm chips, the orientation distribution is strongly localized around the $[001]$ direction, reaching maximum intensities of approximately 12 mrd. In contrast, the distributions obtained from the 3 mm and 5 mm feedstocks become progressively broader and less intense, indicating a reduction in crystallographic alignment. The broadening of the orientation distribution suggests an increase in orientation dispersion and a weaker preferential crystallographic grain orientation for larger chip sizes. Nevertheless, the $\langle 001 \rangle \parallel \text{ED}$ fiber texture remains the dominant texture component for all investigated conditions.

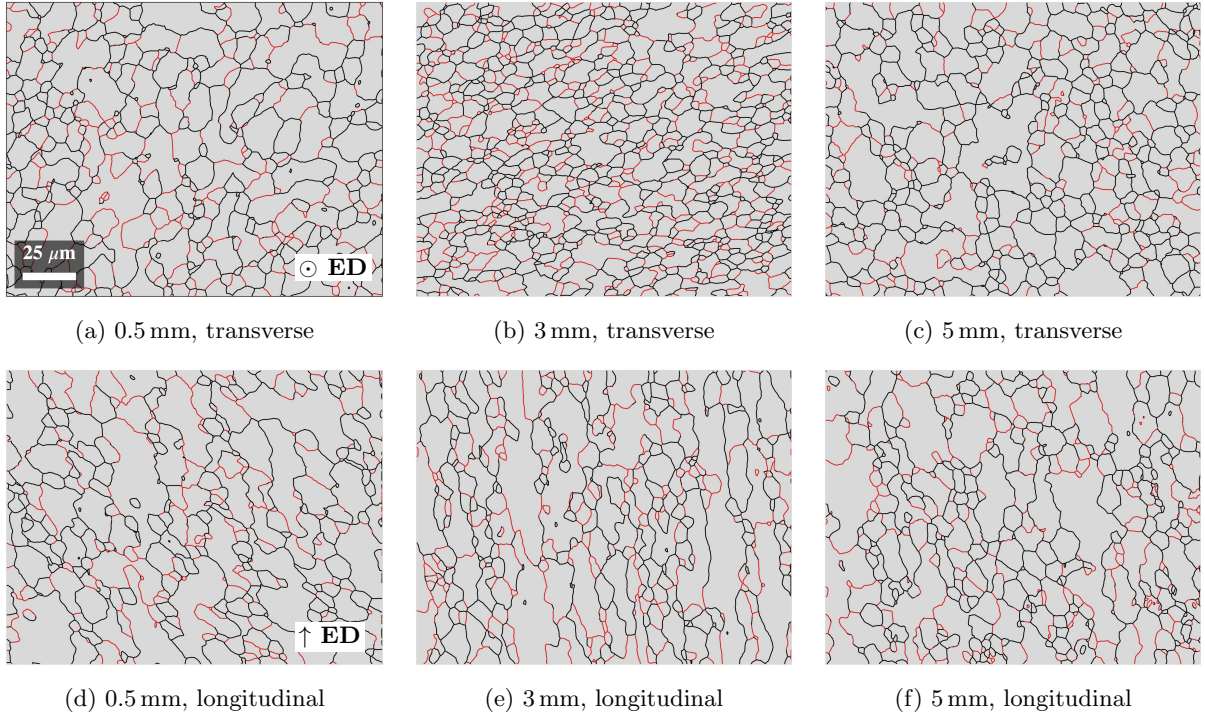


Figure 3: Grain-boundary character showing low-angle boundaries ($2\text{--}15^\circ$, red) and high-angle boundaries ($\geq 15^\circ$, black) for feedstock chip sizes of 0.5 mm, 3 mm and 5 mm. Cross sections are shown on the top and longitudinal sections on the bottom.

The weakening of the fiber texture with increasing chip size, observed for the three representative chip sizes shown in Figure 4, may be attributed to greater initial crystallographic heterogeneity in the larger chip size. This heterogeneity may not be fully homogenized during deformation and dynamic recrystallization in a single extrusion pass, resulting in a broader final orientation distribution than that obtained from smaller chips. This can be understood in terms of the strain required to reorient material into the dominant fiber texture: achieving a strong $\langle 001 \rangle \parallel \text{ED}$ texture requires that the original crystallographic orientations inherited from the parent rod be substantially rotated and aligned through slip and dynamic recrystallization during extrusion. Smaller chips consist of finer initial crystallographic domains, so the strain imposed during a single extrusion pass is distributed over many small domains, allowing each one to be more completely reoriented. Larger chips, in contrast, retain coarser initial domains, so the same total strain must reorient a larger volume of material per domain, leaving a greater share of the original orientation spread intact and producing the broader, less intense texture observed for larger chip sizes.

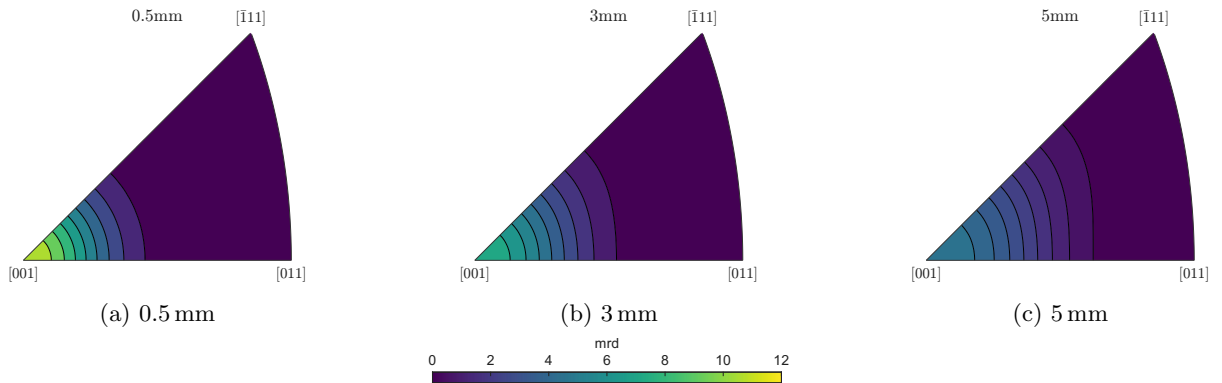


Figure 4: IPFs of the extrusion direction (area-weighted grain-mean orientations, shared mrd scale), showing the $\langle 001 \rangle \parallel \text{ED}$ fibre texture weakening with increasing feedstock chip size.

Figure 5 shows the spatial distribution of the GOS, which provides a measure of the intragranular orientation variation and is used as an indicator of recrystallization. Grains with GOS values below 2° , classified as recrystallized, are outlined in blue. For all investigated feedstock conditions, the microstructures predominantly consist of grains exhibiting low GOS values, indicating that FE promotes extensive recrystallization throughout

the material. Regions with elevated GOS are generally localized and occur only in a limited number of grains, suggesting the presence of residual deformation or incomplete recrystallization.

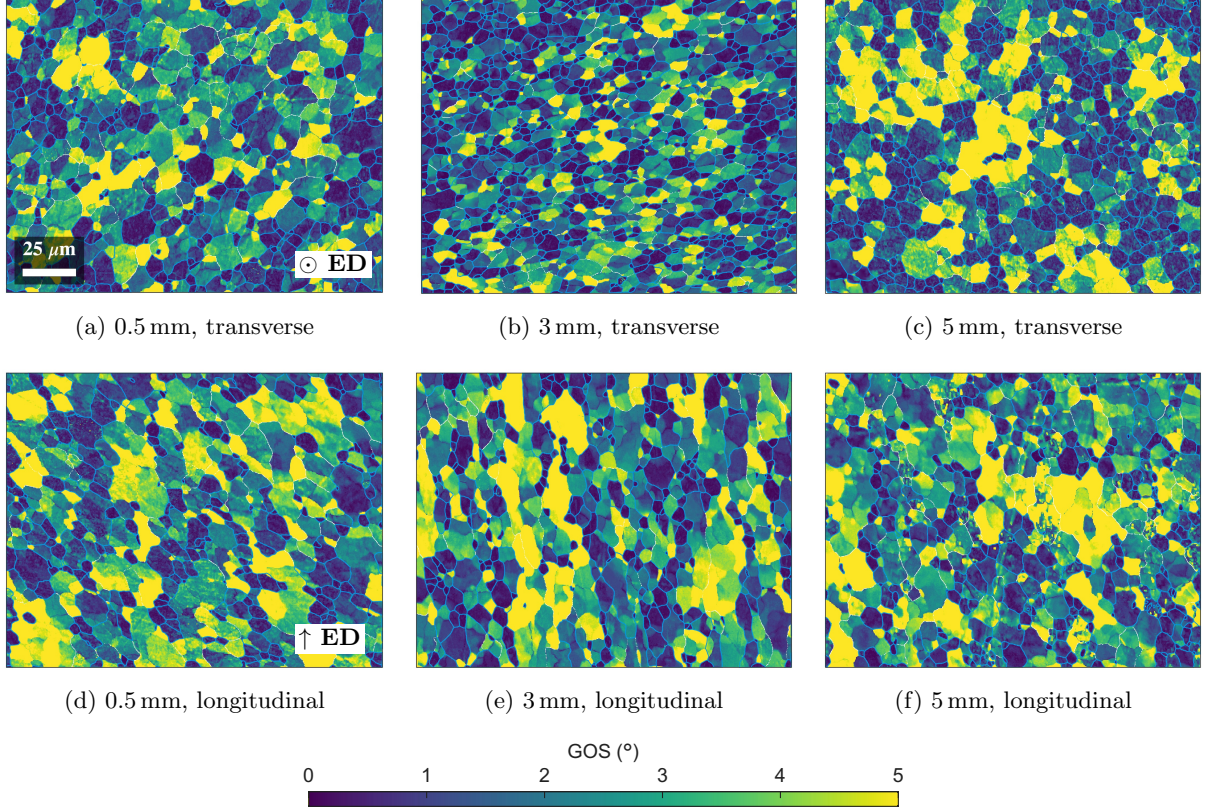


Figure 5: Grain orientation spread (shared 0–5° scale); recrystallized grains (GOS < 2°) outlined in blue.

Differences between the investigated chip sizes are also apparent in Figure 5. The microstructure produced from 3 mm chips exhibits a larger fraction of elongated grains in the longitudinal section, consistent with the anisotropic grain morphology observed in the IPF and grain-boundary maps. In contrast, the 0.5 mm and 5 mm feedstock conditions display a more homogeneous distribution of the GOS values and a greater proportion of equiaxed recrystallized grains. Overall, the GOS maps indicate that the FE process results in a highly recrystallized microstructure irrespective of feedstock chip size, while subtle variations in grain morphology and local orientation gradients remain evident and warrant further quantitative analysis. Additional plots illustrating the GOS for chip sizes of 1 mm, 2 mm, and 4 mm can be found in the Appendix in Figure A3.

3.2 Statistical analysis of microstructural grain descriptors

A key aspect of quantitative microstructure characterization is the definition of grain boundaries through a misorientation threshold. In conventional grain reconstruction, a threshold of 15° is commonly used to identify HAGBs, resulting in grains that represent regions of approximately uniform crystallographic orientation. Although this definition is widely established, it can conceal important features of the deformation substructure. As illustrated in Figure 2, the resulting grains can be comparatively large and contain substantial internal orientation gradients separated by LAGBs. Consequently, the information associated with subgrain structures and intragranular misorientation is not explicitly captured in the resulting grain descriptor statistics.

In contrast, adopting a lower misorientation threshold of 2° enables the resolution of LAGB networks and thereby reveals the internal crystallographic subdivision of grains (see Figure 3). These subgrain structures arise from the accumulation and rearrangement of dislocations during deformation and subsequent recovery processes, leading to local orientation gradients within the material [36]. As a result, subgrain-scale representations provide a more detailed description of the microstructure than conventional grain reconstructions based on a HAGB criterion, retaining information about internal deformation state and crystallographic heterogeneity.

Furthermore, reducing the misorientation threshold from the HAGB to the LAGB criterion increases the number of segmented microstructural features available for analysis. HAGB-defined grains are subdivided into multiple subgrains, thus increasing the number of available observations. Additionally, the subdivision reduces the loss of information associated with boundary-intersecting features. While a HAGB-defined grain may intersect with the boundary of the observation window and therefore be excluded from the analysis, some of its constituent

subgrains may be fully contained within the observable frame and can be retained. As a result, the lower misorientation threshold allows for the characterization of finer-scale microstructural features and increases the amount of available statistical samples.

Prior to statistical analysis, grains intersecting the boundary of the observation window are excluded. Since their complete geometry is not observable, geometric descriptors such as the area equivalent diameter cannot be determined without introducing additional assumptions. On the other hand, this procedure may introduce a certain sampling bias, as larger grains have a higher probability of intersecting the boundary of the observation window and are therefore more likely to be omitted from the analysis.

Across all EBSD images, including both sectioning directions and all investigated chip sizes, 429 out of the 1752 HAGB grains (24.49%) and 945 of the 6646 LAGB grains (14.22%) were excluded because they intersected the boundary of the observation window. Thus, reducing the misorientation threshold from 15° to 2° decreased the exclusion rate by 10.27 percentage points. The effect becomes more pronounced when evaluated in terms of area. For the HAGB reconstruction, 73.15% of the total grain area belongs to boundary-intersecting grains and is therefore excluded from the analysis. In contrast, only 19.64% of the total area is discarded for the LAGB reconstruction. This substantial reduction demonstrates that the finer segmentation significantly mitigates the loss of microstructural information caused by the boundedness of the observation window while simultaneously increasing the number of statistically analyzable features.

3.2.1 Influence of the grain-boundary misorientation threshold

Figure 6 compares grain descriptor distributions obtained using misorientation thresholds of 2° and 15° for the microstructure produced from 3 mm chips, which is taken as a representative example. Overall, reducing the misorientation threshold has only a moderate influence on most geometric descriptor distributions. However, as expected, the refinement of HAGB-defined grains into multiple subgrains increases the fraction of microstructural features with small area-equivalent diameters, resulting in a shift of the corresponding distribution towards lower values. This effect is less pronounced in the longitudinal section, where the retained LAGB-defined features exhibit slightly larger characteristic sizes than the retained HAGB-defined grains. At first sight, this observation appears counterintuitive because lowering the misorientation threshold subdivides existing grains. However, this trend is primarily a consequence of boundary truncation effects rather than a true increase in grain size. As discussed previously, a considerable fraction of large HAGB grains intersects the observation-window boundary and is therefore excluded from the analysis. The remaining HAGB grain population is consequently biased towards smaller grain sizes, whereas the finer LAGB segmentation reduces this bias by retaining a larger proportion of fully observable features.

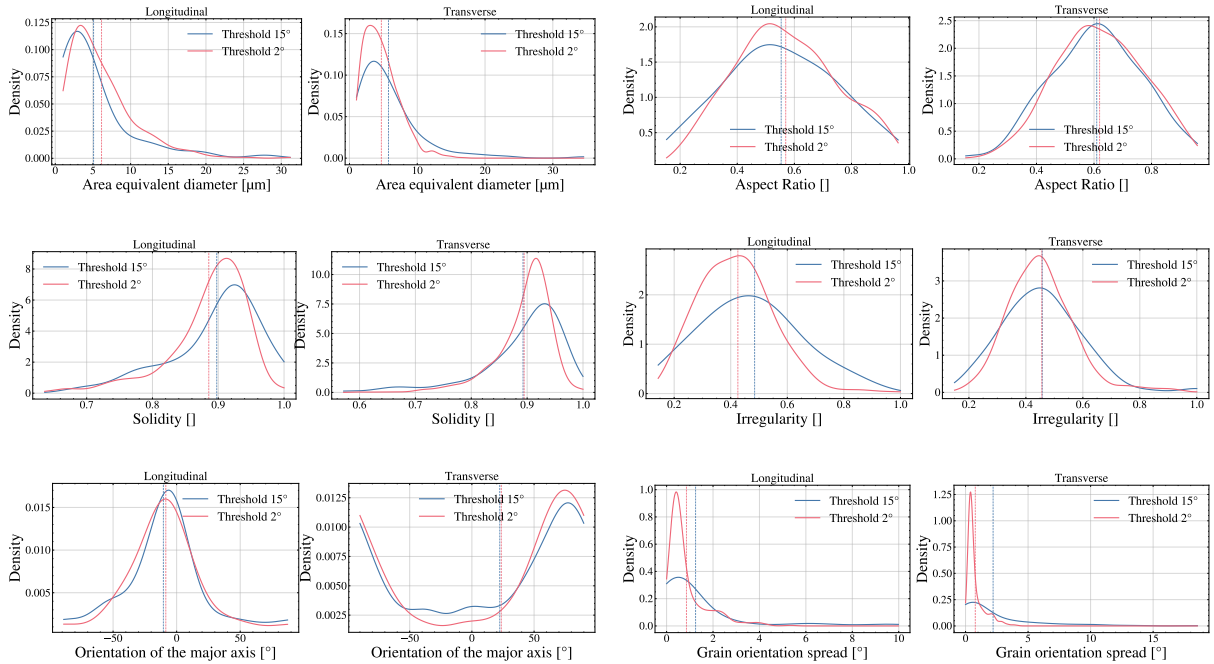


Figure 6: Kernel density estimates for various microstructural grain descriptors for longitudinal and transverse sections obtained from a wire of the friction extrusion process with chip size 3 mm for 2° (red) and 15° (blue) misorientation threshold. The mean values of the density functions are displayed with vertical lines.

A more pronounced effect is observed for the GOS. The GOS distribution shifts towards lower values when

the 2° threshold is used, reflecting the fact that newly resolved subgrains contain smaller intragranular orientation variations than the corresponding parent grains. In contrast, descriptors that primarily characterize the grain shape, including aspect ratio, solidity, irregularity, and orientation of the major axis, are only weakly affected by the change in the reconstruction threshold. This indicates that the refinement mainly influences the internal crystallographic subdivision of the microstructure while preserving its overall geometric characteristics.

3.2.2 Quantitative assessment of anisotropy

The FE process combines severe plastic deformation, friction-induced heating, and directional material flow, resulting in heterogeneous local deformation conditions throughout the extrudate. These processing conditions can give rise to anisotropic microstructures characterized by direction-dependent grain morphologies and crystallographic features. A quantitative assessment of such anisotropy is essential both for the development of realistic 3D microstructure reconstruction methodologies and for understanding potential links between microstructural anisotropy and direction-dependent material properties.

Figure 7 reveals noticeable differences in the distributions of several microstructural descriptors between the two sectioning directions. For the area-equivalent diameter, the transverse sections exhibit a higher proportion of grains with diameters below $10\mu\text{m}$, indicating a tendency toward finer observable features compared to the longitudinal sections. A similar trend is observed for the GOS, whose distribution is shifted toward lower values in the transverse direction. As will be shown later (Section 3.3, Figure 10), GOS is strongly correlated with grain size, suggesting that both observations are related to the same underlying microstructural characteristics.

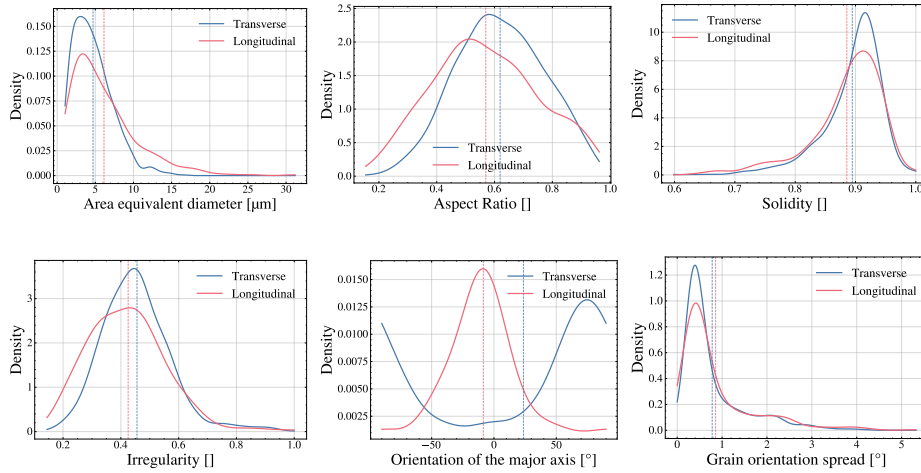


Figure 7: Kernel density estimates for various microstructural grain descriptors for longitudinal (red) and transverse (blue) sections of friction extruded wire based on 3 mm chip. The mean values of the density functions are displayed with vertical lines.

Differences are also observed for shape-related descriptors, including aspect ratio, solidity, and irregularity. The aspect ratio distribution shifts towards smaller values in the longitudinal sections, with quartiles of $Q_1/Q_2/Q_3 = [0.44/0.56/0.69]$, compared with $[0.51/0.61/0.73]$ in the transverse sections. This shift indicates a larger proportion of elongated grains in the longitudinal sections. Similarly, the solidity quartiles are slightly smaller in the longitudinal sections ($[0.86/0.90/0.93]$) than in the transverse sections ($[0.87/0.91/0.93]$). The longitudinal sections also exhibit lower irregularity, with corresponding quartiles of $[0.32/0.41/0.50]$ and $[0.37/0.45/0.52]$, respectively.

Both longitudinal and transverse sections show non-uniform distributions of major-axis orientation. In the longitudinal section, the orientation is concentrated near -10° relative to the chosen image axis, indicating a preferred major-axis orientation in the extrusion direction. In the analyzed region of the transverse section, a peak occurs near 75° . However, because the transverse orientation depends on the position around the wire axis, this angle characterizes only the sampled region and should not be interpreted as a preferred orientation of the entire cross section.

3.2.3 Analysis of microstructural descriptors across different chip sizes

As discussed in Section 3.1, the wire texture weakens with increasing chip size (Figure 4). To determine whether this crystallographic trend is accompanied by systematic changes in grain geometry, Figure 8 compares the kernel density estimates of the investigated microstructural grain descriptors for the different chip sizes. Overall,

the distributions overlap substantially, and no consistent monotonic relationship between chip size and the microstructural grain descriptors is apparent. Thus, within the investigated range, the chip size influences the crystallographic texture more strongly than the grain geometry.

The microstructure produced from the 3 mm chips constitutes an exception to this general observation. Compared to the other chip sizes, it is characterized by smaller grain sizes in the transverse direction, more elongated grains, a stronger preferred orientation of the major axis, and a noticeably lower GOS, which can also be qualitatively seen in Figure 5.

In the transverse section, the lower GOS of the 3 mm condition is consistent with its smaller grain size, since GOS and grain size are positively correlated (Figure 10, Table 4). In the longitudinal section, however, the grain sizes of the 3 mm condition are comparable to those of the other chip sizes (Table 2), while its GOS is still the lowest, indicating a lower intragranular orientation spread that cannot be attributed to grain size alone. During FE, the feedstock is subjected to severe torsional shear at the die interface, where chip interfaces are rapidly consolidated and the grain structure is continuously renewed by dynamic recrystallization. A chip-size effect on the final microstructure therefore requires a feedstock feature that persists through this deformation. Two candidates are plausible. First, the oxide originating from the chip surfaces is fragmented but retained, and its spatial arrangement depends on chip size. For 0.5 mm chips, the numerous interfaces may result in a fine and homogeneous oxide dispersion, whereas for 5 mm chips, the few interfaces contribute comparatively little oxide, and the material deforms largely as bulk. In both cases, grain boundary migration is expected to be affected uniformly, consistent with the more homogeneous GOS distributions and equiaxed grains observed in Figure 5. For 3 mm chips, in contrast, shear may localize along a limited number of interfaces, concentrating the oxide into stringers aligned with the flow direction, which could restrict grain growth perpendicular to the ED and promote finer, elongated grains. The locally increased strain along these interfaces may additionally promote dynamic recrystallization, consistent with the lower GOS observed for the 3 mm condition in the longitudinal section. Second, the packing of the compacted billet varies with chip size. While the smallest chips pack densely and the largest chips form coherent bulk regions, intermediate chips may pack less uniformly, resulting in less homogeneous consolidation and local deformation conditions at the die interface. Clarifying the relative contribution of these mechanisms will be the subject of future work.

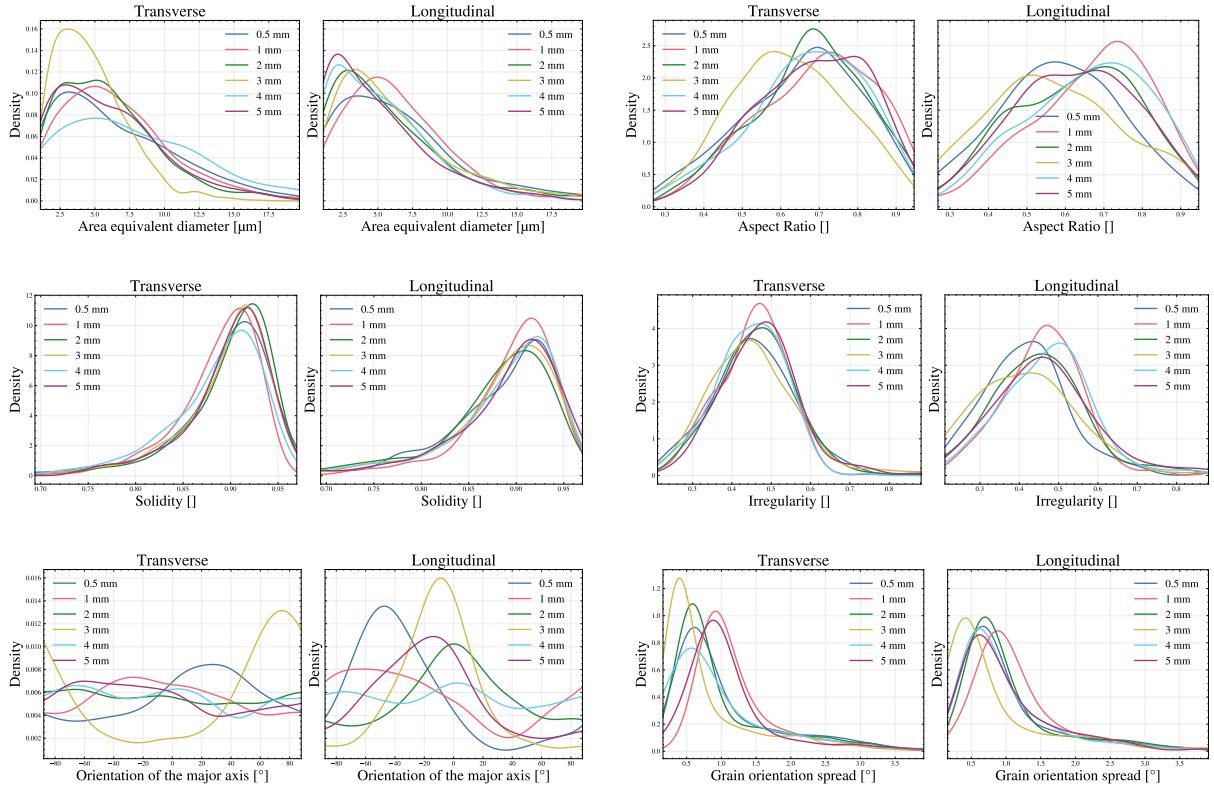


Figure 8: Kernel density estimates for various microstructural grain descriptors and for transverse (left) and longitudinal (right) sections, the influence of different chip sizes

3.3 Parametric copula-based modeling of descriptor vectors

A statistical description of the microstructure of wires must capture not only the marginal distributions of the individual grain descriptors but also the dependencies between them. Although multivariate KDE can provide a

flexible non-parametric representation of joint distributions, its application becomes increasingly data-intensive and computationally demanding with increasing dimensionality. Moreover, KDE-based representations require the underlying sample data to be retained and provide only limited interpretability in terms of the dependence between individual descriptors. On the other hand, parametric copula models offer a compact alternative by decomposing a multivariate distribution into its marginal distributions and a dependence structure; see Eq. (15) in Section 2.4.1. This separation permits different marginal models to be combined with copula families representing different types of dependence, including asymmetric and tail dependence [37]. Copula-based models also facilitate efficient sampling from the fitted joint distribution. These properties make copulas particularly suitable for the statistical representation of multidimensional grain descriptor vectors and provide a potential basis for statistical 2D-to-3D reconstruction approaches that extend existing methods, such as those implemented, e.g., in DREAM.3D [38], by explicitly accounting for dependencies between multiple descriptors [39].

3.3.1 Parametric models of univariate marginal distributions

In a first step, the univariate marginal distributions of the individual grain descriptors are modeled parametrically, where the parameters of the pre-selected families of parametric distributions are determined by maximum-likelihood estimation, as stated in Section 2.4.3. This provides a compact and interpretable representation, yields smooth estimates of the marginal densities and distribution functions, and enables efficient evaluation and sampling within the subsequent copula model. The best fitting types of distributions and the corresponding parameter estimates, obtained for the chip sizes of 0.5 mm, 3 mm and 5 mm, are summarized in Table 2 and visualized in Figure 9.

An advantage of the parametric modeling approach is that the fitted parameters describe interpretable changes in distributional location, spread, and concentration. For example, the fitted area-equivalent diameters show that the transverse 3 mm condition has both the lowest location parameter $\mu = 1.40$ and the lowest dispersion parameter $\sigma = 0.475$. Thus, its fitted distribution is shifted toward smaller values and is narrower than those for the other chip sizes. The von Mises concentration parameter κ is also largest for 3 mm chips in both sectioning directions, indicating the most pronounced fitted preferred major-axis orientation. In longitudinal sections, the fitted preferred orientation of the major axis μ varies with chip size, from -43.7° for 0.5 mm chips to -10.9° and -21.9° for 3 mm and 5 mm chips, respectively.

Together with the fitting of parametric copulas, as stated below, a fully parametric formulation is then used to obtain a compact representation of the multivariate distribution of grain descriptor vectors that can be evaluated and sampled without retaining the original observations. However, in principle, the subsequent copula-based model could also be formulated semi-parametrically by combining a parametric copula with non-parametric marginal distributions [40].

Descr.	Dir.	Marginal	Params.	0.5 mm	3 mm	5 mm
$d_2(G)$	t	Lognormal	(μ, σ)	(1.62, 0.759)	(1.40, 0.475)	(1.54, 0.733)
	l			(1.61, 0.768)	(1.59, 0.686)	(1.34, 0.775)
$GOS(G)$	t	Burr Type XII	(c, η, λ)	(6.5, 0.24, 0.47)	(2.9, 0.58, 0.40)	(2.3, 1.8, 1.3)
	l			(4.9, 0.36, 0.54)	(4.2, 0.34, 0.33)	(1.3, 4274, 844)
$S(G)$	t	Beta	(α, β)	(24.6, 2.96)	(36.9, 4.31)	(21.0, 2.33)
	l			(23.7, 3.13)	(30.6, 3.95)	(18.9, 2.34)
$I(G)$	t	Beta	(α, β)	(8.25, 9.74)	(7.07, 8.45)	(10.3, 11.5)
	l			(6.55, 9.06)	(5.33, 7.35)	(5.95, 6.80)
$E(G)$	t	Beta	(α, β)	(5.06, 2.64)	(5.45, 3.31)	(5.40, 2.56)
	l			(4.65, 3.35)	(3.52, 2.67)	(4.81, 2.82)
$\theta(G)$	t	von Mises	(κ, μ)	(0.509, 28.7)	(1.27, 74.7)	(0.296, -37.8)
	l			(1.47, -43.7)	(1.69, -10.9)	(0.968, -21.9)

Table 2: Parameters of the fitted marginal distributions for the chip sizes of 0.5 mm, 3 mm and 5 mm and the two cutting directions (transverse, t , and longitudinal, l). The von Mises distribution was fitted using angles expressed in degrees rather than radians; consequently, its location parameter μ is reported in degrees, whereas the concentration parameter κ remains dimensionless.

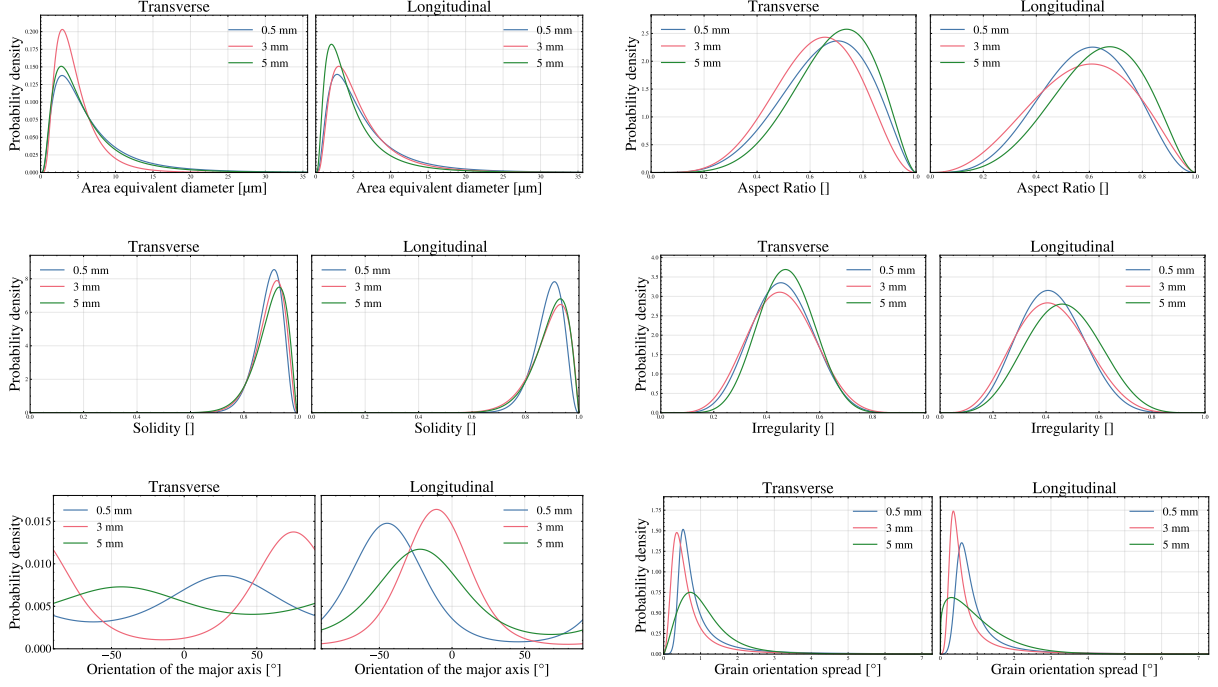


Figure 9: Parametric fits of the probability density functions of microstructural grain descriptors for transverse (left) and longitudinal (right) sections, the influence of different chip sizes. The blue lines correspond to a chip size of 0.5 mm, the red lines to a chip size of 3 mm and the green lines to a chip size of 5 mm

3.3.2 Parametric modeling of bivariate distributions of descriptor pairs

Building upon the fitted marginal distributions, copulas are employed to describe the statistical dependencies between the individual descriptors. This is particularly relevant because such dependencies can be nonlinear and may therefore not be adequately characterized by linear correlation coefficients alone. In the present paper, the bivariate joint distributions of pairwise selected grain descriptors are considered, with particular emphasis on descriptor combinations involving the GOS. Nevertheless, the framework is not inherently restricted to the bivariate case. Higher-dimensional dependence structures can, for example, be constructed using vine copulas [37, 41], which represent multivariate dependencies through a sequence of bivariate and conditional bivariate copulas. Consequently, the proposed framework provides a systematic route from the statistical characterization of individual descriptors to a compact multivariate representation of grain descriptor vectors.

Des. 1	Des. 2	Dir.	0.5 mm	3 mm	5 mm
$d_2(G)$	$GOS(G)$	t	Gumbel 0°, 1.993	Gumbel 0°, 1.708	Gumbel 0°, 1.756
		l	Clayton 180°, 1.423	Clayton 180°, 1.407	Gumbel 0°, 1.572
$E(G)$	$GOS(G)$	t	Frank 0°, -1.018	Joe 90°, 1.181	Clayton 270°, 0.1019
		l	Joe 90°, 1.143	Clayton 270°, 0.6221	Clayton 270°, 0.2680
$S(G)$	$GOS(G)$	t	Frank 0°, -2.145	Gumbel 270°, 1.487	Frank 0°, -2.433
		l	Gumbel 90°, 1.365	Frank 0°, -3.599	Frank 0°, -3.041
$I(G)$	$GOS(G)$	t	Clayton 270°, 0.9182	Gumbel 90°, 1.449	Frank 0°, -3.553
		l	Gumbel 90°, 1.360	Frank 0°, -4.230	Frank 0°, -3.960

Table 3: Overview of best fitting copulas for the chip sizes of 0.5 mm, 3 mm and 5 mm, and for the two cutting directions (transverse, t , and longitudinal, l), where the type and the value of the parameter γ of the best fitting copula are given.

To fit bivariate distributions, the original observations are first transformed into the uniform domain using their respective (marginal) cumulative distribution functions; see Section 2.4.1. Subsequently, Clayton, Gumbel, Frank and Joe copulas, along with their rotations of 0°, 90°, 180° and 270°, are considered to identify a suitable

dependence structure. For each candidate copula, the copula parameter is determined by maximum-likelihood estimation. The fitted copula models are then evaluated based on their likelihood, and the copula providing the highest likelihood is selected as the best-fitting dependence model; see Table 3.

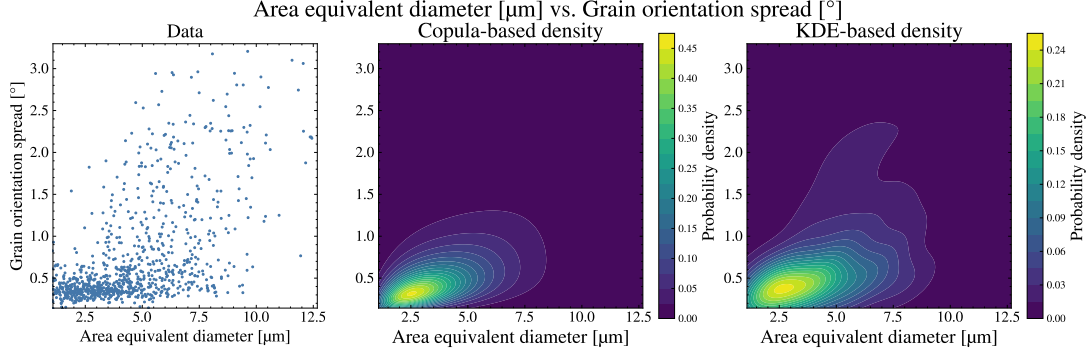


Figure 10: Relationship between area-equivalent diameter and GOS for the transverse section produced from 3 mm chips: observed descriptor pairs (left), fitted copula-based joint density (center), and 2D KDE (right).

Figure 10 illustrates the relationship between the area-equivalent diameter and GOS, comparing the observed descriptor pair with the fitted parametric joint density and a bivariate KDE for the transverse section produced from 3 mm chips. Note that the fitted copula-based model reproduces the overall positive association between the two descriptors while using only the parameters of the marginal distributions and the selected copula. On the other hand, the 2D KDE provides a flexible non-parametric reference but is more sensitive to sample size and outliers.

Figure 11 compares the fitted copula-based bivariate densities of GOS and area-equivalent diameter d_2 , aspect ratio E , and solidity S across three chip sizes and both sectioning directions. The contours suggest a positive dependence between d_2 and GOS, little apparent dependence between E and GOS, and a negative dependence between S and GOS. For the 5 mm chip size, GOS appears to increase more steeply with d_2 on the original descriptor scales.

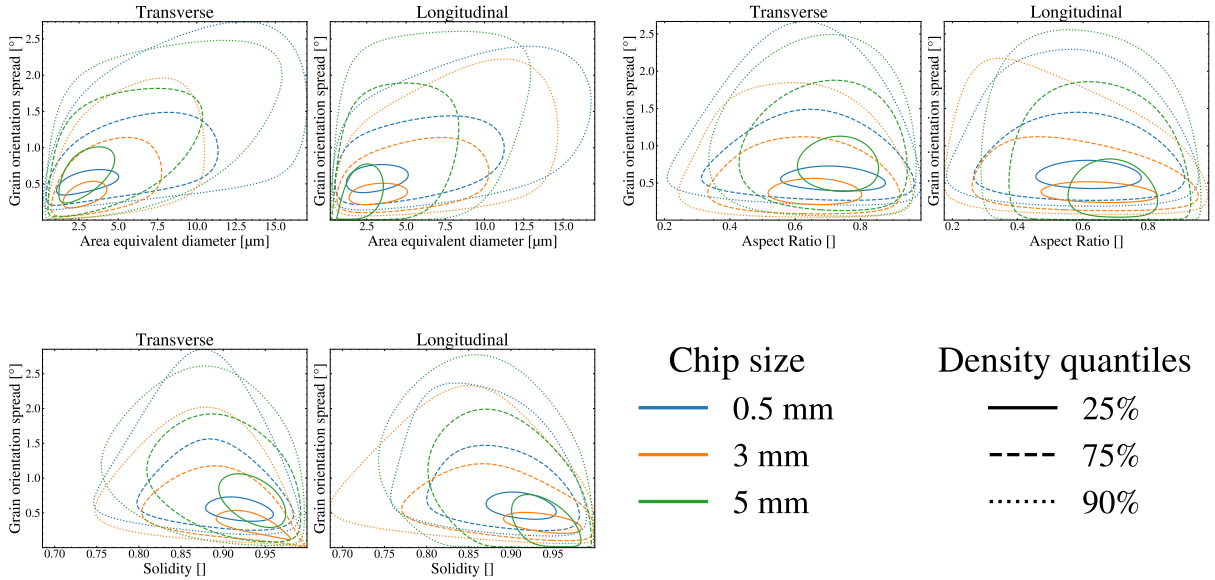


Figure 11: Copula-based bivariate probability densities for three pairs of grain descriptors, shown for three chip sizes and both sectioning directions. Colors distinguish the chip sizes, while the different line styles represent isodensity contours bounding highest-density regions containing 25 %, 75 %, and 90 % of the fitted probability mass.

To compare the fitted copula families on a common scale, we report Kendall's τ implied by each fitted copula parameter and rotation [37]. Table 4 shows that there is a positive dependence between d_2 and GOS in all six conditions ($\tau \in [0.36, 0.50]$). The fitted copulas for (S, GOS) yield a negative dependence, while the dependence between E and GOS is comparatively weak. For each descriptor pair, the sign of Kendall's τ remains the same across chip sizes, although the magnitude varies. The negative dependence between S and GOS is strongest

for 3 mm chips in both sectioning directions. The apparently steeper trend in (d_2, GOS) suggested by the fitted joint density for 5 mm chips does not imply stronger rank dependence, it is mostly caused by the marginal distributions. Considering both the joint densities and Kendall's τ therefore helps to distinguish changes in individual descriptors from changes in how they co-vary.

Descriptor pair	Dir.	0.5 mm	3 mm	5 mm
(d_2, GOS)	t	0.50	0.41	0.43
	l	0.42	0.41	0.36
(E, GOS)	t	-0.11	-0.09	-0.05
	l	-0.08	-0.24	-0.12
(S, GOS)	t	-0.23	-0.33	-0.26
	l	-0.27	-0.36	-0.31

Table 4: Kendall's τ implied by the fitted copulas for three descriptor pairs and both sectioning directions (transverse, t ; longitudinal, l).

These results highlight the value of the proposed statistical framework beyond simple descriptive statistics. The multivariate analysis shows that GOS is statistically related to several morphological descriptors and that the strengths of these relationships vary moderately among chip sizes. Together with the differences observed in the marginal distributions, these results indicate that chip size is associated with the individual grain characteristics and how these characteristics co-vary. For example, separating the marginal distributions from the dependence structure shows that the lower GOS of the 3 mm condition in the longitudinal section cannot be attributed to grain size alone, since its grain sizes are comparable to those of the other chip sizes while the dependence between grain size and GOS remains similar.

Note that the fitted bivariate distributions provide an initial understanding of the dependencies between multiple grain descriptors. Extending the parametric copula-based models considered in the present paper to higher dimensions would allow more than two grain descriptors to be sampled jointly while preserving their marginal distributions and statistical dependencies. Such a multivariate representation could supply statistical target distributions or input parameters for future stochastic microstructure reconstruction [38, 42].

4 Conclusion

In this work, microstructures of friction-extruded AA6082 chips of different sizes were quantitatively characterized using 2D EBSD measurements obtained from two perpendicular sections. The analysis revealed pronounced microstructural anisotropy, with the longitudinal sections exhibiting larger and more elongated grains compared to the transverse sections, as well as a distinct preferred orientation of the major axis, reflecting the directional material flow imposed by severe plastic deformation during extrusion. Furthermore, the influence of the machining chip size on the resulting microstructure was investigated. While the $\langle 001 \rangle \parallel \text{ED}$ fiber texture weakened with increasing chip size, grain geometry also varied with chip size but did not follow a corresponding monotonic trend. The 3 mm condition was particularly distinct, exhibiting the lowest GOS values among all investigated conditions and more elongated grains. In the transverse direction, the 3 mm chip size also produced the smallest grain size. This illustrates the benefit of the proposed statistical analysis: by separating the marginal distributions from the dependence structure, it shows that the lower GOS in the longitudinal direction cannot be attributed to grain size alone, a distinction that, together with the systematic dependencies between grain descriptors identified in Section 3.3, would not have been visible from univariate summary statistics or simple histograms alone.

To enable a more comprehensive statistical representation of friction-extruded microstructures, a multivariate parametric stochastic modeling approach based on copulas was proposed. Within this framework, the marginal distribution of each grain descriptor is modeled separately, while the copula describes the dependence structure between the descriptors. By incorporating dependencies between microstructural descriptors, this framework provides a foundation for extending existing 2D-to-3D reconstruction methods to generate more realistic grain microstructures. For such a reconstruction, correlated descriptor vectors sampled from the fitted model could be assigned to individual synthetic grains and used as input for existing reconstruction frameworks such as DREAM.3D and DRAGen [38, 42].

Future work will focus on developing a statistical 2D-to-3D reconstruction methodology that generates 3D microstructures from 2D EBSD measurements while preserving experimentally observed morphological and crystallographic characteristics. Furthermore, the reconstruction approach will be extended using an interpretable parametric stochastic model, whose parameters quantify the relevant microstructural characteristics and their dependence structure [39]. Interpolation of these parameters between experimentally characterized microstructures will then enable the generation of statistically consistent virtual 3D microstructures representing inter-

mediate, previously unobserved material states. The reconstructed microstructures will subsequently serve as input geometries for numerical simulations to determine effective material properties and establish complete processing–structure–property relationships. Such a framework is an important step toward inverse design and optimization of friction-extruded materials.

CRedit authorship contribution statement

Léon F. Schröder: Methodology, Formal analysis, Writing - Original Draft. **Ali R. Safi:** Conceptualization, Writing - Original Draft. **Lukas Fuchs:** Conceptualization, Writing - Review & Editing. **Usman Aziz:** Resources, Investigation, Writing - Original Draft. **Uceu F. H. Suhuddin:** Resources, Investigation, Writing - Review & Editing. **Benjamin Klusemann:** Funding acquisition, Project administration, Supervision, Writing - Review & Editing. **Volker Schmidt:** Funding acquisition, Project administration, Supervision, Writing - Review & Editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

- [1] J.-F. Nie. Physical metallurgy of light alloys. In D. E. Laughlin and K. Hono, editors, *Physical Metallurgy*, 2009–2156. Elsevier, 2014.
- [2] G. Saevarsdottir, H. Kvande and B. J. Welch. Aluminum production in the times of climate change: The global challenge to reduce the carbon footprint and prevent carbon leakage. *JOM*, 72:296–308, 2020.
- [3] M. Gautam, B. Pandey and M. Agrawal. Carbon footprint of aluminum production: Emissions and mitigation. In S. S. Muthu, editor, *Environmental Carbon Footprints*, 197–228. Butterworth-Heinemann, 2018.
- [4] X. Yi, Y. Lu and G. He. Aluminum demand and low carbon development scenarios for major countries by 2050. *Journal of Cleaner Production*, 475:143647, 2024.
- [5] T. G. Gutowski, S. Sahni, J. M. Allwood, M. F. Ashby and E. Worrell. The energy required to produce materials: constraints on energy-intensity improvements, parameters of demand. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 371:20120003, 2013.
- [6] J. R. Dufloy, A. E. Tekkaya, M. Haase, T. Welo, K. Vanmeensel, K. Kellens, W. Dewulf and D. Paraskevas. Environmental assessment of solid state recycling routes for aluminium alloys: Can solid state processes significantly reduce the environmental impact of aluminium recycling? *CIRP Annals*, 64:37–40, 2015.
- [7] D. Raabe, D. Ponge, P. J. Uggowitzer, M. Roscher, M. Paolantonio, C. Liu, H. Antrekowitsch, E. Kozeschnik, D. Seidmann, B. Gault, F. De Geuser, A. Deschamps, C. Hutchinson, C. Liu, Z. Li, P. Prangnell, J. Robson, P. Shanthraj, S. Vakili, C. Sinclair, L. Bourgeois and S. Pogatscher. Making sustainable aluminum by recycling scrap: The science of “dirty” alloys. *Progress in Materials Science*, 128:100947, 2022.
- [8] W. Tang and A. P. Reynolds. Production of wire via friction extrusion of aluminum alloy machining chips. *Journal of Materials Processing Technology*, 210:2231–2237, 2010.
- [9] D. Baffari, G. Buffa, G. Ingarao, A. Masnata and L. Fratini. Aluminium sheet metal scrap recycling through friction consolidation. *Procedia Manufacturing*, 29:560–566, 2019.
- [10] U. F. Suhuddin, L. Rath, R. M. Halak and B. Klusemann. Microstructure evolution and texture development during production of homogeneous fine-grained aluminum wire by friction extrusion. *Materials Characterization*, 205:113252, 2023.
- [11] R. M. Halak, L. Rath, U. F. Suhuddin, J. F. dos Santos and B. Klusemann. Changes in processing characteristics and microstructural evolution during friction extrusion of aluminum. *International Journal of Material Forming*, 15:24, 2022.

- [12] D. Baffari, A. P. Reynolds, A. Masnata, L. Fratini and G. Ingarao. Friction stir extrusion to recycle aluminum alloys scraps: Energy efficiency characterization. *Journal of Manufacturing Processes*, 43:63–69, 2019.
- [13] U. Aziz, U. F. Suhuddin, L. Rath, L. F. Schröder, V. Schmidt and B. Klusemann. Investigation of microstructure evolution during AA6082 chips recycling through friction consolidation. *Key Engineering Materials*, 1051:41–48, 2026.
- [14] M. A. Groeber, B. K. Haley, M. D. Uchic, D. M. Dimiduk and S. Ghosh. 3D reconstruction and characterization of polycrystalline microstructures using a FIB–SEM system. *Materials Characterization*, 57:259–273, 2006.
- [15] M. Groeber, S. Ghosh, M. D. Uchic and D. M. Dimiduk. A framework for automated analysis and simulation of 3D polycrystalline microstructures.: Part 1: Statistical characterization. *Acta Materialia*, 56:1257–1273, 2008.
- [16] T. Kayser, B. Klusemann, H.-G. Lambers, H. Maier and B. Svendsen. Characterization of grain microstructure development in the aluminum alloy EN AW-6060 during extrusion. *Materials Science and Engineering: A*, 527(24-25): 6568–6573, 2010.
- [17] J. Galán López and L. A. Kestens. A multivariate grain size and orientation distribution function: Derivation from electron backscatter diffraction data and applications. *Applied Crystallography*, 54(1):148–162, 2021.
- [18] O. Furat, T. Leißner, K. Bachmann, J. Gutzmer, U. Peuker and V. Schmidt. Stochastic modeling of multidimensional particle properties using parametric copulas. *Microscopy and Microanalysis*, 25(3):720–734, 2019.
- [19] B. Prifling, D. Westhoff, D. Schmidt, H. Markoetter, I. Manke, V. Knoblauch and V. Schmidt. Parametric microstructure modeling of compressed cathode materials for Li-ion batteries. *Computational Materials Science*, 169:109083, 2019.
- [20] B. Prifling, M. Röding, P. Townsend, M. Neumann and V. Schmidt. Large-scale statistical learning for mass transport prediction in porous materials using 90,000 artificially generated microstructures. *Frontiers in Materials*, 8:786502, 2021.
- [21] X. Li, D. Baffari and A. Reynolds. Friction stir consolidation of aluminum machining chips. *The International Journal of Advanced Manufacturing Technology*, 94(5):2031–2042, 2018.
- [22] A. J. Schwartz, M. Kumar, B. L. Adams and D. P. Field. *Electron Backscatter Diffraction in Materials Science*, volume 2. Springer, 2009.
- [23] T. Hastie, R. Tibshirani and J. H. Friedman. *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*, volume 2. Springer, 2009.
- [24] S. van der Walt, J. L. Schönberger, J. Nunez-Iglesias, F. Boulogne, J. D. Warner, N. Yager, E. Gouillart and T. Yu. scikit-image: Image processing in Python. *PeerJ*, 2:e453, 2014.
- [25] C. B. Barber, D. P. Dobkin and H. Huhdanpaa. The quickhull algorithm for convex hulls. *ACM Transactions on Mathematical Software*, 22:469–483, 1996.
- [26] C. R. Maurer, R. Qi and V. Raghavan. A linear time algorithm for computing exact euclidean distance transforms of binary images in arbitrary dimensions. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 25:265–270, 2003.
- [27] F. Bachmann, R. Hielscher, P. E. Jupp, W. Pantleon, H. Schaeben and E. Wegert. Inferential statistics of electron backscatter diffraction data from within individual crystalline grains. *Applied Crystallography*, 43:1338–1355, 2010.
- [28] O. Engler, S. Zaefferer and V. Randle. *Introduction to Texture Analysis: Macrotexture, Microtexture, and Orientation Mapping*. CRC Press, 2024.
- [29] D. W. Scott. *Multivariate Density Estimation: Theory, Practice, and Visualization*. John Wiley & Sons, 2015.
- [30] R. B. Nelsen. *An Introduction to Copulas*. Springer, 2006.
- [31] R. Ditscherlein, O. Furat, M. de Langlard, J. Martins de Souza e Silva, J. Sygusch, M. Rudolph, T. Leißner, V. Schmidt and U. A. Peuker. Multiscale tomographic analysis for micron-sized particulate samples. *Microscopy and Microanalysis*, 26(4):676–688, 2020.
- [32] R. P. Brent. *Algorithms for Minimization without Derivatives*. Courier Corporation, 2013.
- [33] C. Forbes, M. Evans, N. Hastings and B. Peacock. *Statistical Distributions*. John Wiley & Sons, 2011.
- [34] C. Kleiber and S. Kotz. *Statistical Size Distributions in Economics and Actuarial Sciences*. John Wiley & Sons, 2003.
- [35] K. P. Burnham and D. R. Anderson. *Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach*. Springer, 2 edition, 2002.
- [36] J. Lv, J.-H. Zheng, V. A. Yardley, Z. Shi and J. Lin. A review of microstructural evolution and modelling of aluminium alloys under hot forming conditions. *Metals*, 10(11):1516, 2020.
- [37] H. Joe. *Dependence Modeling with Copulas*. CRC Press, 2014.
- [38] M. A. Groeber and M. A. Jackson. DREAM. 3D: A digital representation environment for the analysis of microstructure

in 3D. *Integrating Materials and Manufacturing Innovation*, 3:56–72, 2014.

- [39] L. F. Schröder, A. R. Safi, L. Fuchs, M. Osenberg, I. Manke, B. Klusemann and V. Schmidt. Tessellation-based 3D reconstruction of anisotropic polycrystalline microstructures with crystallographic orientations from 2D EBSD images. 2026. Working paper (under preparation).
- [40] T. Bouezmarni and J. V. Rombouts. Semiparametric multivariate density estimation for positive data using copulas. *Computational Statistics & Data Analysis*, 53(6):2040–2054, 2009.
- [41] O. Furat, T. Kirstein, T. Leißner, K. Bachmann, J. Gutzmer, U. A. Peuker and V. Schmidt. Multidimensional characterization of particle morphology and mineralogical composition using CT data and R-vine copulas. *Minerals Engineering*, 206:108520, 2024.
- [42] M. Henrich, N. Fehlemann, F. Bexter, M. Neite, L. Kong, F. Shen, M. Könemann, M. Dölz and S. Münstermann. DRAGen – A deep learning supported RVE generator framework for complex microstructure models. *Heliyon*, 9(8): e19003, 2023.

Appendix

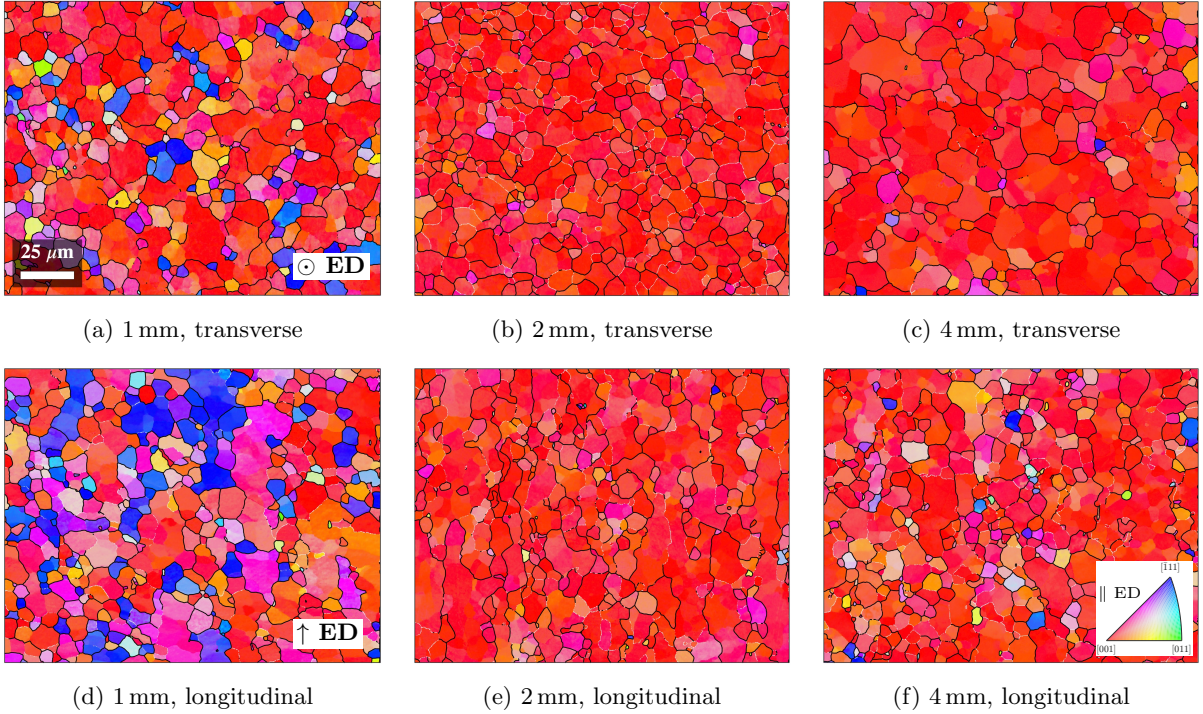
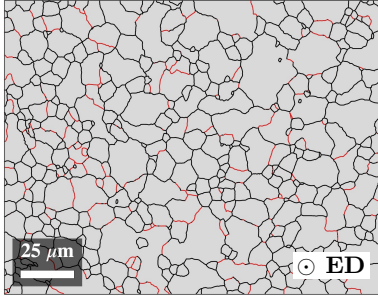
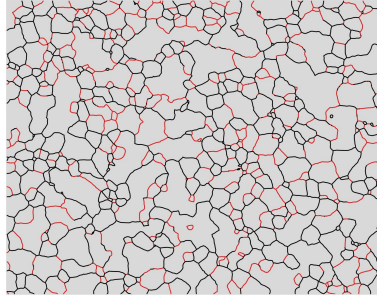


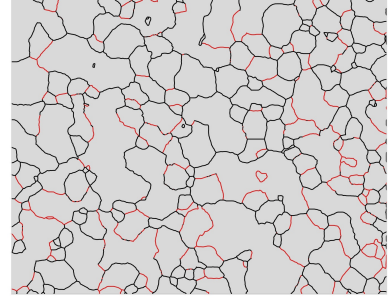
Figure A1: IPF orientation maps coloured parallel to the extrusion direction (ED out-of-plane for transverse sections, ED vertical for longitudinal sections) with high-angle grain boundaries ($\geq 15^\circ$) overlaid. Feedstock chip size increases left to right.



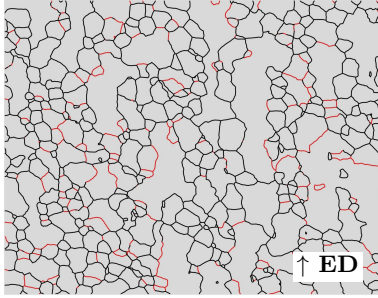
(a) 1 mm, transverse



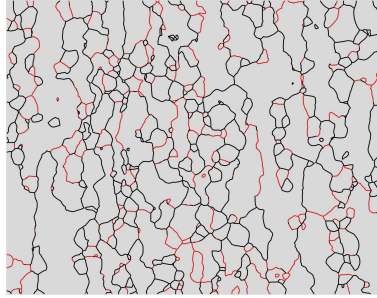
(b) 2 mm, transverse



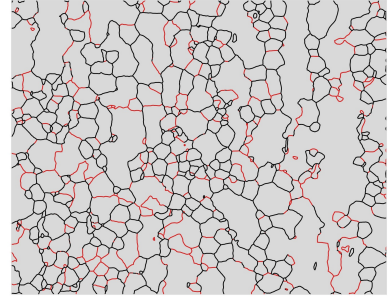
(c) 4 mm, transverse



(d) 1 mm, longitudinal



(e) 2 mm, longitudinal



(f) 4 mm, longitudinal

Figure A2: Grain-boundary character showing low-angle boundaries ($2-15^\circ$, red) and high-angle boundaries ($\geq 15^\circ$, black) for feedstock chip sizes of 1 mm, 2 mm and 4 mm. Transverse sections are shown on the top and longitudinal sections on the bottom.

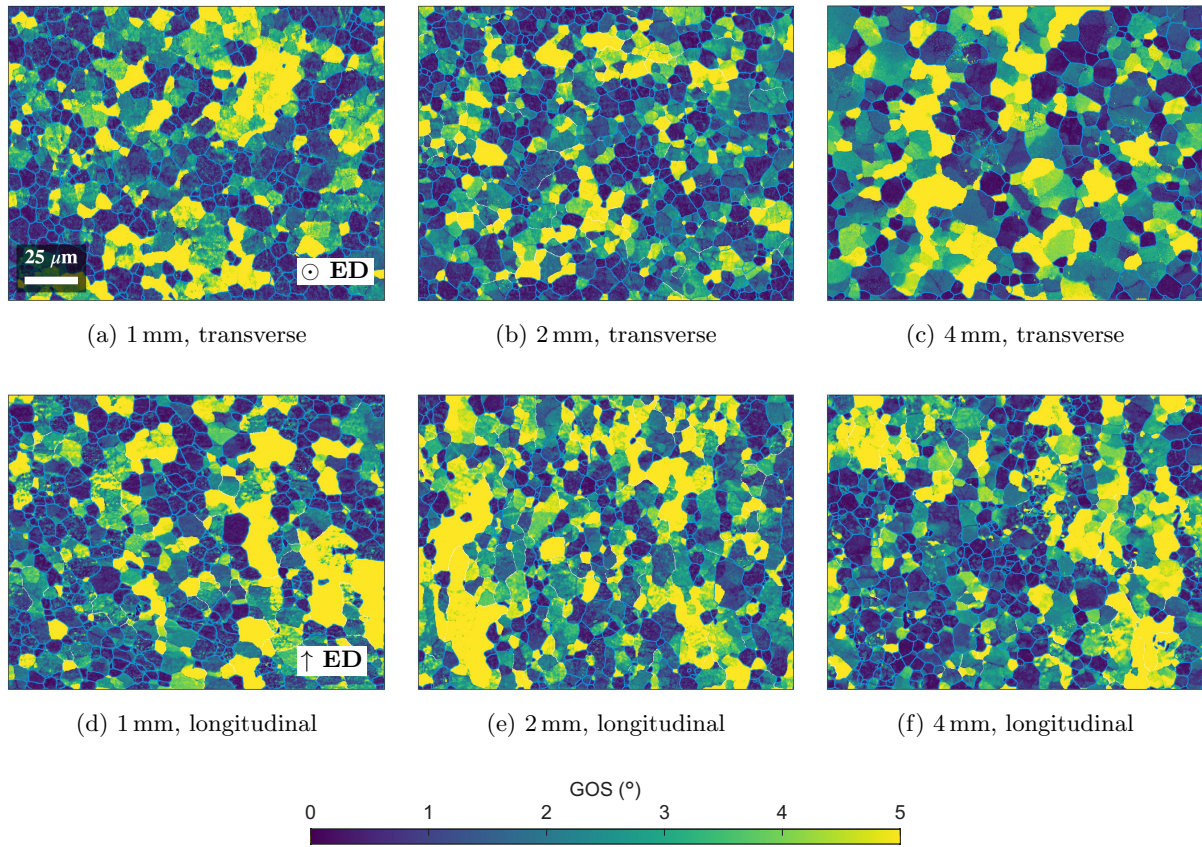


Figure A3: Grain orientation spread (shared 0–5° scale); recrystallised grains (GOS < 2°) outlined in blue. Feedstock chip sizes of 1, 2, and 4 mm are shown. Transverse sections are displayed on the top and longitudinal sections on the bottom.