Universität Ulm SS08
Institut für Analysis
Prof. Dr. Friedmar Schulz David Kaiser

Practice 1: Real Functions

Exercise 1

We introduce the Function

$$\Phi(t) := \begin{cases} 0, & t \leq 0 \\ t, & t \geq 0 \end{cases}$$

which is continuous and weakly monotonically increasing. Furthermore we define

$$f^+(x) := \Phi(f(x)) = \max(f(x), 0), x \in X.$$

Prove the following properties of the prescription $f \longmapsto f^+$ for all $x \in X$:

(i) $f(x) \leq f^+(x)$,

(ii) $f(x) \leq g(x) \Rightarrow f^+(x) \leq g^+(x)$,

(iii) $f(x) \leq g(x) \Rightarrow g^+(x) - f^+(x) \leq g(x) - f(x)$,

(iv) $f_n(x) \to f(x) \Rightarrow f_n^+(x) \to f^+(x)$,

(v) $f_n(x) \downarrow f(x) \Rightarrow f_n^+(x) \downarrow f^+(x)$,

(vi) $f_n(x) \uparrow f(x) \Rightarrow f_n^+(x) \uparrow f^+(x)$.

Exercise 2

(1) Let the symbols $V(X)$, $-V(X)$ and M be the set of functions defined in the script. If $f \in V \cap (-V)$ holds true, we find sequences $(f_n)_{n\in\mathbb{N}}$ and $(g_n)_{n\in\mathbb{N}}$ in M, which fulfill $f_n \uparrow f$ and $g_n \downarrow f$. Show that $I(f) \in \mathbb{R}$, $g_n - f_n \downarrow 0$ and

$$0 = \lim_{n\to\infty} I(g_n - f_n) = \lim_{n\to\infty} I(g_n) - \lim_{n\to\infty} I(f_n).$$

(2) Prove that from $f \in V$ it follows that $f^+ \in V$ and that $f \in -V$ implies $f^- \in V$.

Exercise 3

Prove that

$$a \leq b \Longleftrightarrow b - a \geq 0$$

for all $a, b \in \bar{\mathbb{R}}$ with $\bar{\mathbb{R}} = \mathbb{R} \cup \{+\infty\} \cup \{-\infty\}$.