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Practice 3: Real Functions

Exercise 7

Infer the Theorem of B. Levi from the Lemma of Fatou.

Exercise 8

Let (f_n), $f_n > 0$ be a sequence in $L(X)$ with $f_n(x) \to f(x)$ as $n \to \infty$ for all $x \in X$. Furthermore let $(F_n) \subset L(X)$ be a sequence with the property that there exists $F \in L(X)$, $F > 0$ such that

$$I(|F - F_n|) \to 0 \; as \; n \to \infty$$

and $|f_n(x)| \leq F_n(x)$ for all $x \in X$. Show that

$$\lim_{n \to \infty} I(f_n) = I(f).$$

Hint: (1) Examine the convergence of the function $(F - f_n)^+$. (2) Estimate the integral $I(F - f_n)^-$.