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SS08

Practice 4: Real Functions

Exercise 9

Let $M = C^0(I)$, $I \subset \mathbb{R}$ be a compact and nondegenerate interval and let $f \in L(I)$. Show that f^+ is integrable.

Exercise 10

Let $\mathcal{A}$ be a σ-Algebra of subsets $A \subset I$ of a set $I \subset \mathbb{R}$. Show that

(i) $\emptyset \in \mathcal{A}$

(ii) $(A_i)_{i\in\mathbb{N}}$ in $\mathcal{A} \Rightarrow \bigcap_{i=1}^{\infty} A_i \in \mathcal{A}$.

Exercise 11

Let $S(I)$ be the set of all finite measurable sets in $I \subset \mathbb{R}$. Prove that every open and every closed set $A \subset I$ belongs to $S(I)$.

Hint: (1) Show that infinite half-intervals are measurable. (2) Show that finite intervals are measurable. (3) Approximate an open set by a countable union of intervals.