Challenges of Pricing Interruptible Contracts in Electricity Markets with High Mean Reversion

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Overview

- The MRJD model
  - ◇ seasonality
  - ◇ high mean-reversion
  - ◇ double exponential jumps
  - ◇ the forward
- Implications of high speed of mean reversion
  - ◇ forward curves and surfaces
  - ◇ range of applicability
- A closed-form solution for European-style options
- Application: An interruptible bermudan contract
- Preliminary results
- Conclusions and future research

The model (I)

As presented in Cartea & Figueroa (2005)

$$\ln S_t = g(t) + Y_t,$$

where: $g(t) := \ln G(t)$, $G(t)$ is a deterministic seasonality function and

$$dY_t = -\alpha Y_t dt + \sigma(t) dZ + \ln J dq.$$

We assume this time, as in Kou (2002), that $\mathcal{Y} = \ln J$ has an asymmetric double exponential distribution with density

$$f_{\mathcal{Y}}(y) = p\eta_1 e^{-\eta_1 y}\mathbf{1}_{y\geq 0} + q\eta_2 e^{\eta_2 y}\mathbf{1}_{y<0},$$

where $\eta_1 > 1$, $\eta_2 > 0$, and p,$q \geq 0$, $p + q = 1$, represents the probabilities of upward and downward jumps.

The model (II)

The dynamics of the SDE under $\mathcal{P}$ are given by

$$dS_t = \alpha(\rho(t) - \ln S_t)S_t dt + \sigma(t)S_t dZ + S_t(J-1)dq,$$

where the time dependent mean reverting level is given by

$$\rho(t) = \frac{1}{\alpha}\left(\frac{dg(t)}{dt} + \frac{1}{2}\sigma^2(t)\right) + g(t).$$

The forward

We price the forward under a $\mathcal{Q}$ measure by incoorporating a market price of risk per unit of volatility, λ, and calibrating it from forward data, obtaining

$$\begin{aligned} F(t,T') &= \mathbb{E}_t^{\mathcal{Q}}\left[S'_T|\mathcal{F}_t\right]; \\ F(t,T') &= G(T)\left(\frac{S(t)}{G(t)}\right)^{h_t} e^{\frac{\sigma^2}{4\alpha}(1-h_t^2)-\frac{\lambda\sigma}{\alpha}(1-h_t)} \left(\frac{\eta_2+h_t}{\eta_2+1}\right)^{\frac{ql}{\alpha}} \left(\frac{h_t-\eta_1}{1-\eta_1}\right)^{\frac{pl}{\alpha}}, \end{aligned}$$

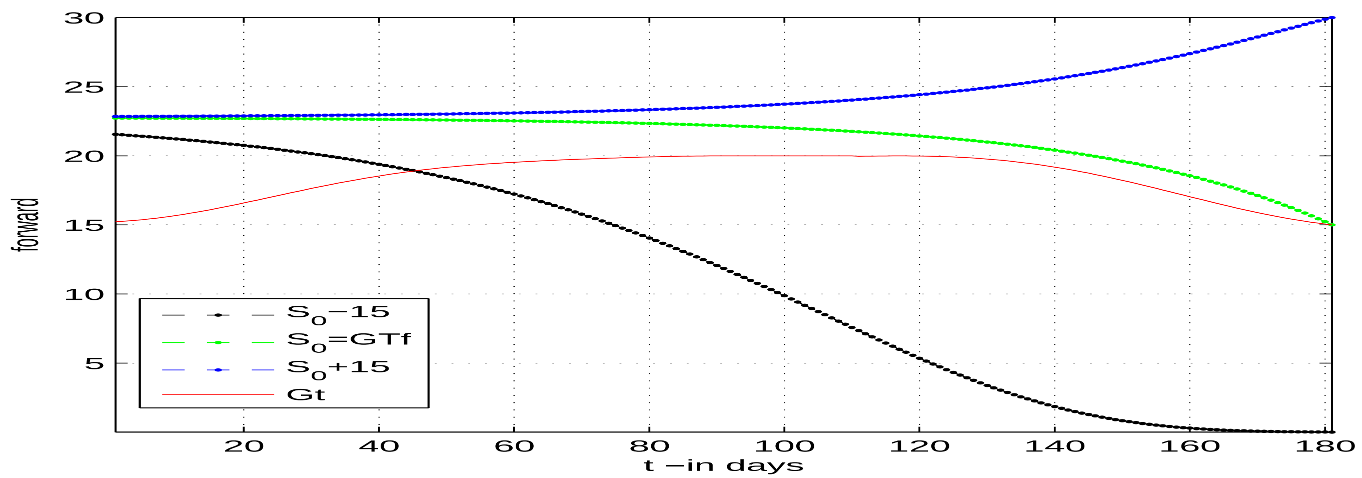
where $h_t := e^{-\alpha(T'-t)}$ and $\eta_1 > 1$, $\eta_2 > 0$.

Implications of a high mean reversion

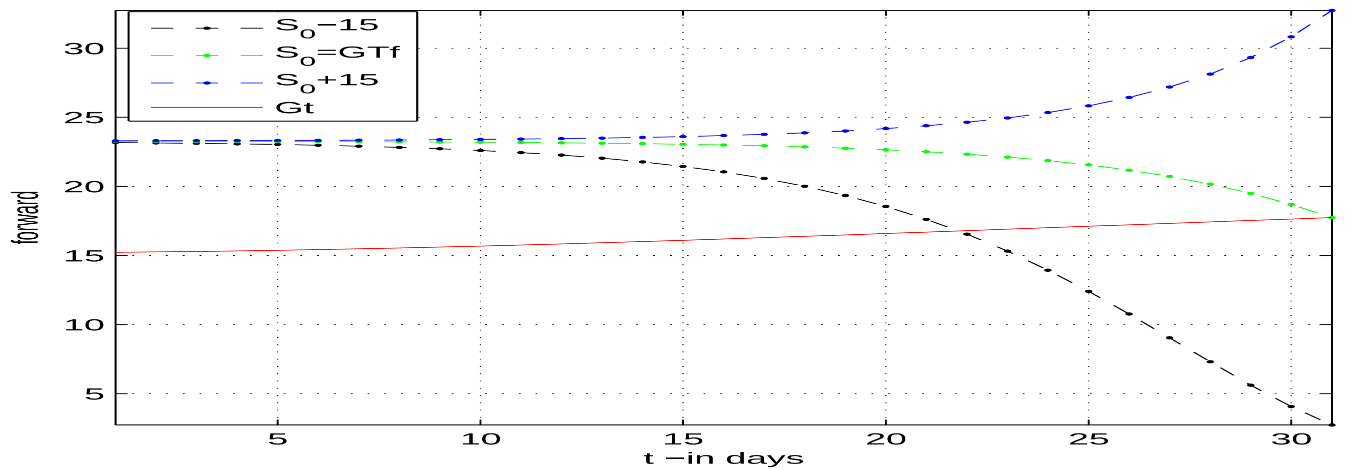
- A high mean reversion ($\alpha \approx 100$ annualized, 3 days) implies that option pricing will be driven mainly by:
  - ◇ seasonality;
  - ◇ frequency of jumps;
  - ◇ more importantly, <u>when</u> are these jumps more likely to occur.
- A high α imposes a restriction on the range of applicability of these models when pricing on forwards,
  - ◇ for the market of England & Wales we observe variation on the forwards for up to 30 days between T and T'.

Forward curves (I)

Forward with T' fixed and varying t, $\alpha\downarrow$

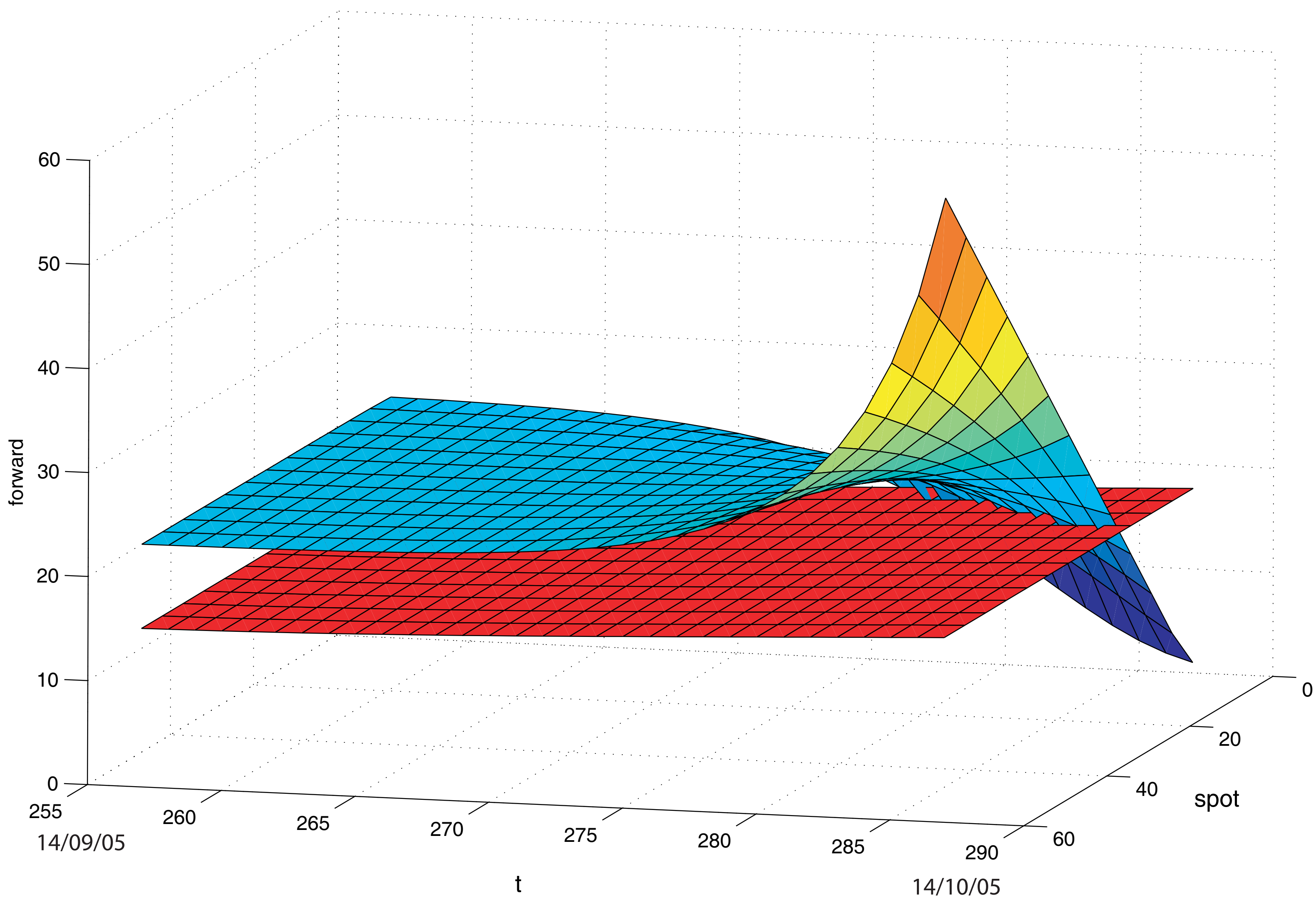


Forward with T' fixed and varying t, $\alpha\uparrow$



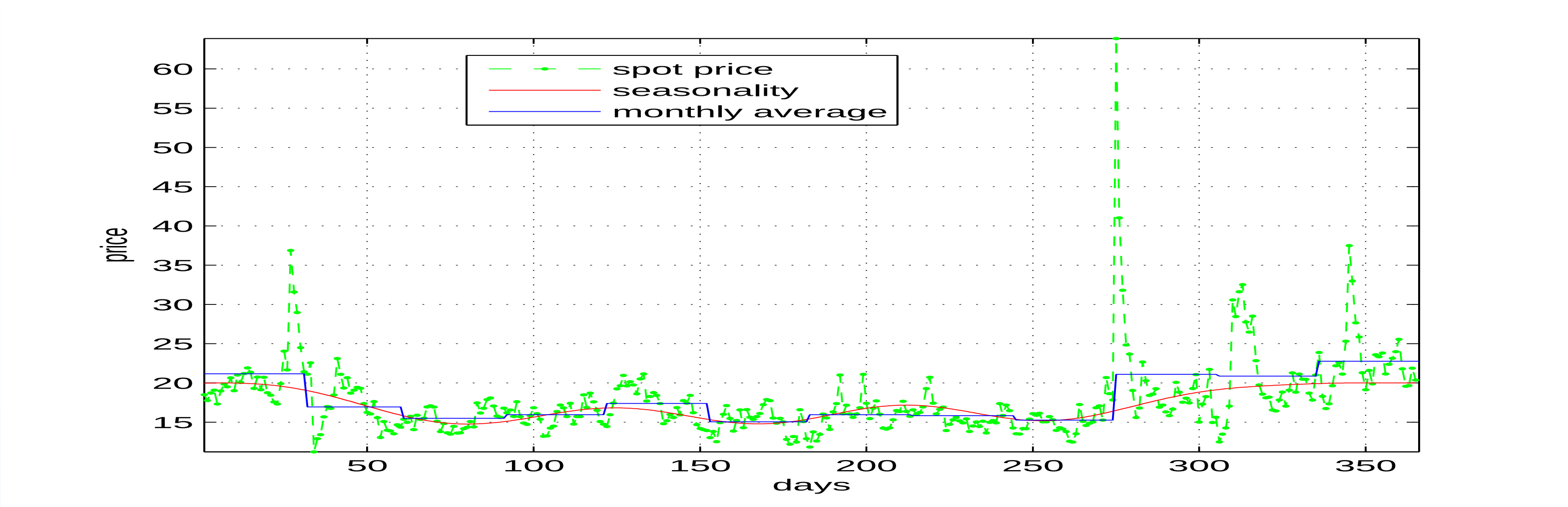
Forward curves (II)

Forward surface with T' fixed and varying t, $\alpha\uparrow$

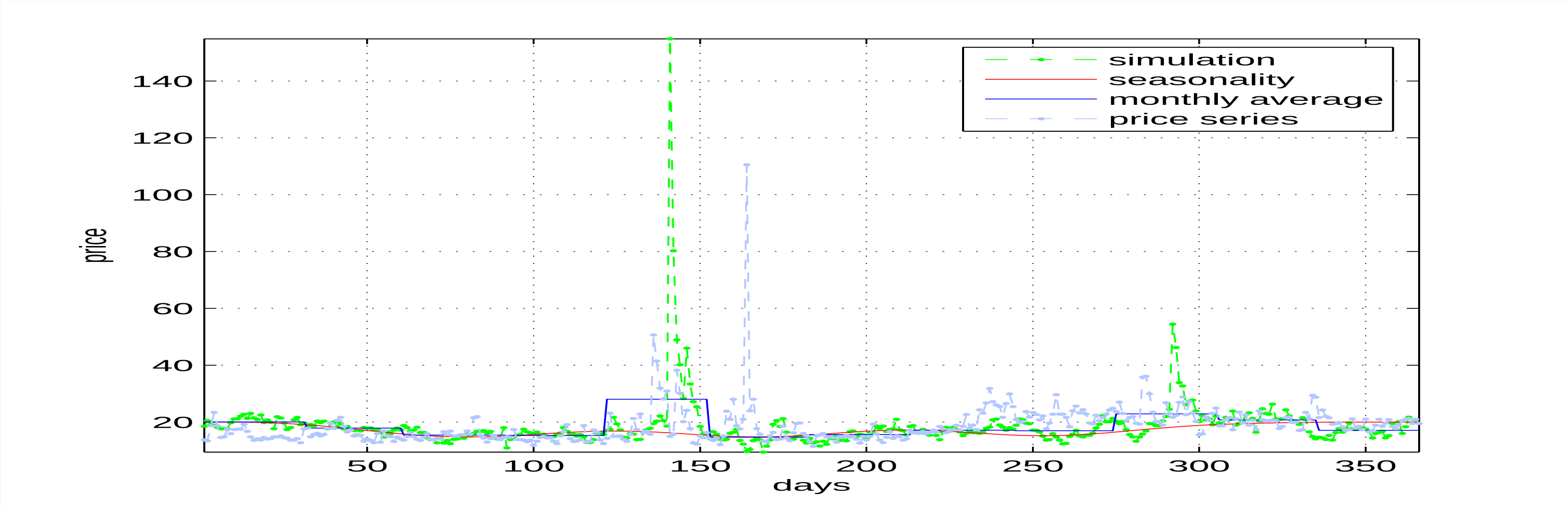


Spot price and simulations

Simulated and calibrated electricity prices for 1 year



Simulated and calibrated electricity prices for 1 year



Closed form solution for the European Call –CFT (I)

Let $F_T^{T'}\left(F_t^{T'}\right)$ be the forward at time T maturing at time T' conditional on the forward at time t maturing at time T', $\hat{V}_T$ the transformed payoff of the call option and $\Psi(-\xi)$ the characteristic function, defined as $\Psi(-\xi) := \mathbb{E}_t\left[e^{-i\xi \ln\left(F_T^{T'}\right)}\right]$. Then the price of a T-maturity European call option written on the underlying $F_t^{T'}$ with strike price K is given by

$$V(F_t^{T'}, t) = \frac{e^{-r(T-t)}}{2\pi}\int_{-\infty+ib}^{\infty+ib} \hat{V}_T \Psi(-\xi) d\xi; \qquad \max(1, \alpha) < b < \beta,$$

where $\xi := a + ib$, and α, β, a, $b \in \mathbb{R}$.

The integration can be performed along any closed-curve within the contour defined by $\mathcal{A} = \{b\colon \max(1, \alpha) < b < \beta\}$.

Closed form solution for the European Call –CFT (II)

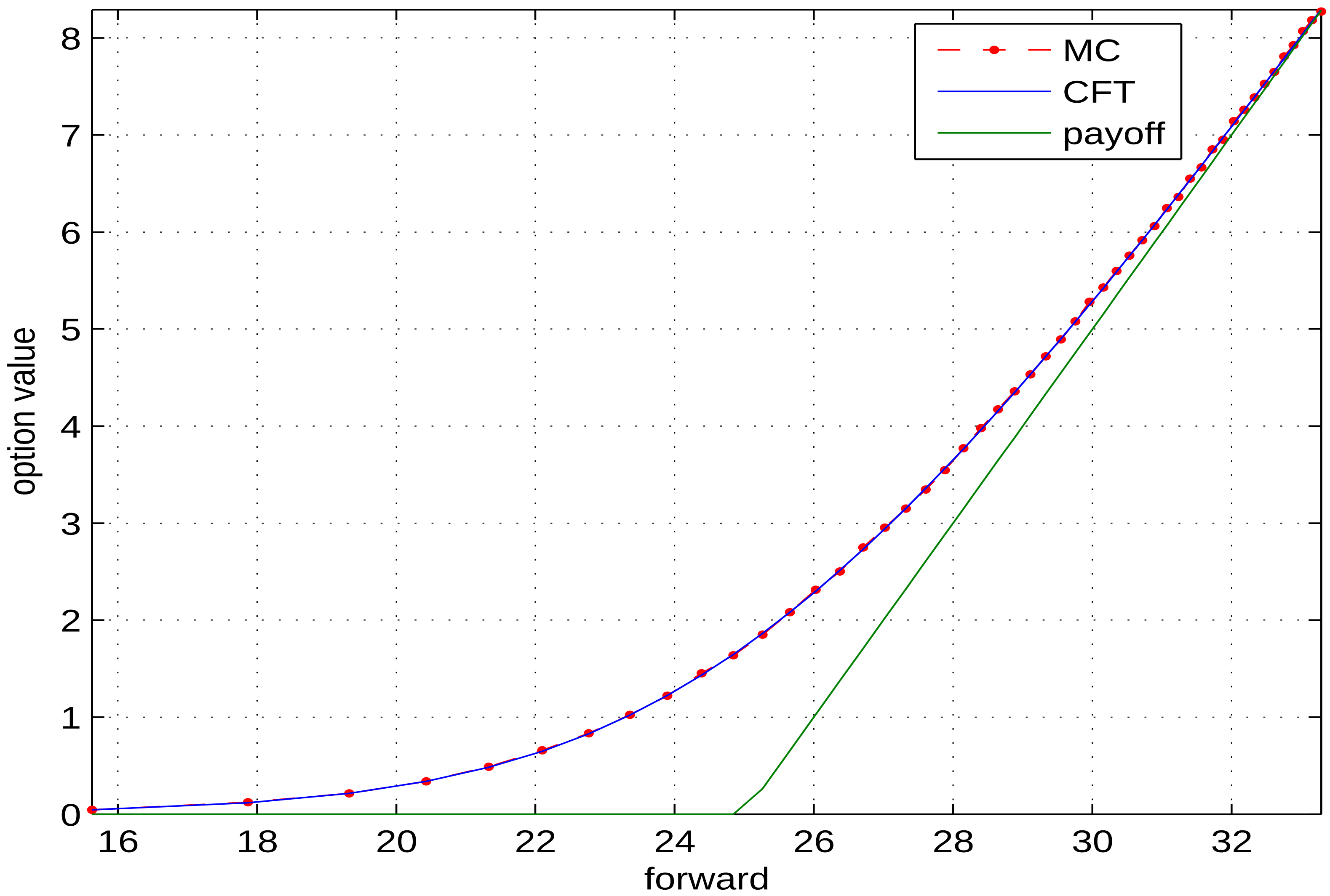
For the proposed model we obtain

$$
\begin{aligned}
V(F_t^{T'}, t) &= \frac{e^{-r(T-t)}}{2\pi} \int_{-\infty+ib}^{\infty+ib} \hat{V}_T \Psi^{CF}(-\xi) d\xi, \qquad \max(1, \alpha) < b < \beta; \\
\hat{V}_T &:= \frac{-K^{1+i\xi}}{\xi^2 - i\xi}; \\
\Psi^{CF}(-\xi) &:= e^{-i\xi \ln(F_t^{T'}) + \left(i\xi - \xi^2\right) \frac{\sigma^2}{4\alpha}\left(H_t^2 - h_t^2\right)} \\
&\times \left(\frac{\eta_2 + h_t}{\eta_2 + H_t}\right)^{\frac{i\xi q l}{\alpha}} \left(\frac{h_t - \eta_1}{H_t - \eta_1}\right)^{\frac{i\xi p l}{\alpha}} \left(\frac{\eta_2 + \hat{h}_t}{\eta_2 + \hat{H}_t}\right)^{\frac{ql}{\alpha}} \left(\frac{\hat{h}_t - \eta_1}{\hat{H}_t - \eta_1}\right)^{\frac{pl}{\alpha}}
\end{aligned}
$$

where $h_t := e^{-\alpha(T'-t)}$, $H_t := e^{-\alpha(T'-T)}$, $\hat{h}_t := -i\xi h_t$ and $\hat{H}_t := -i\xi H_t$.

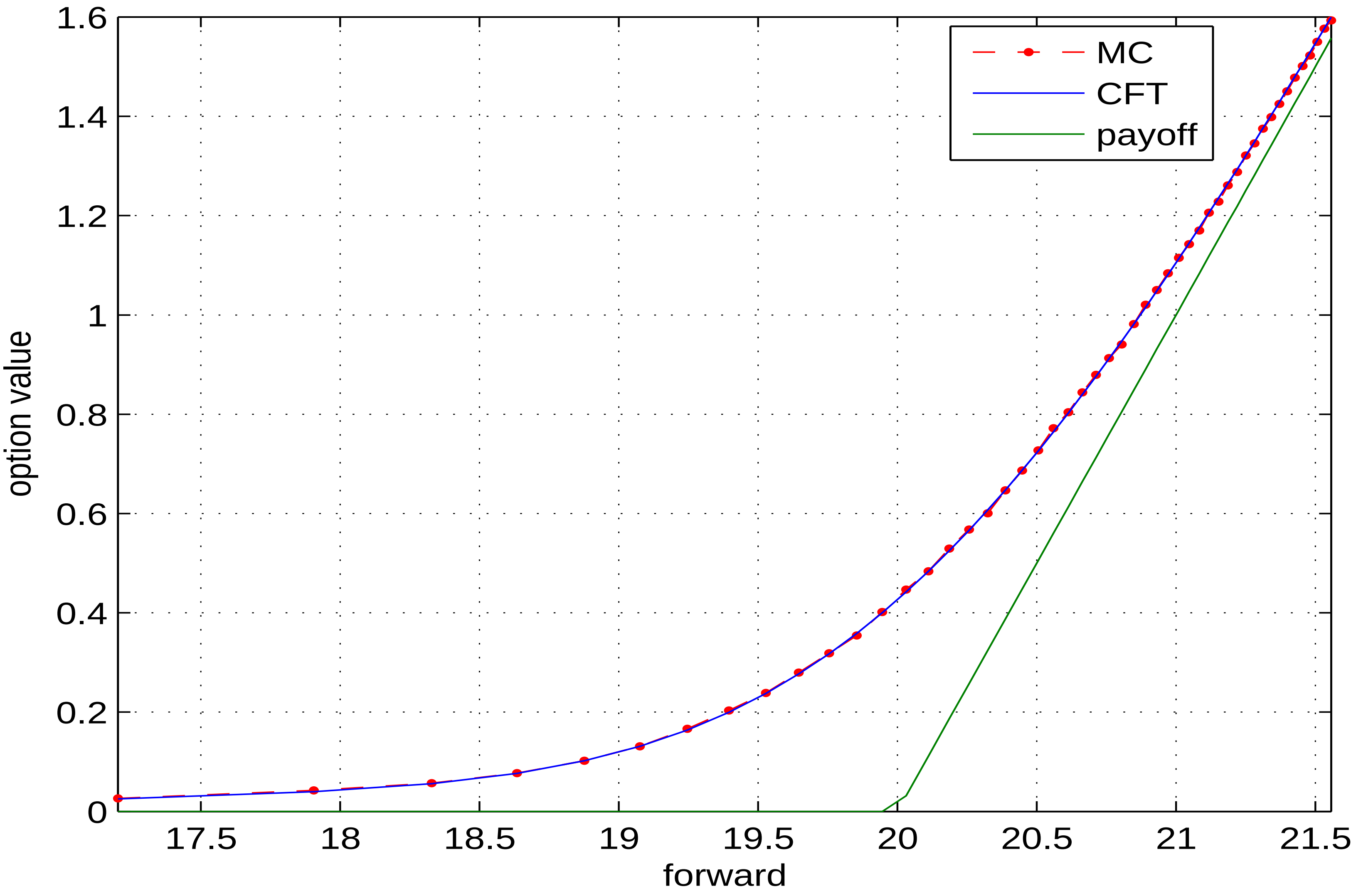
Option Pricing -European Call Option (I)

Call on a Forward; $T' = 60$, $T = 30$, $K = 25$, $\alpha\downarrow = 10$



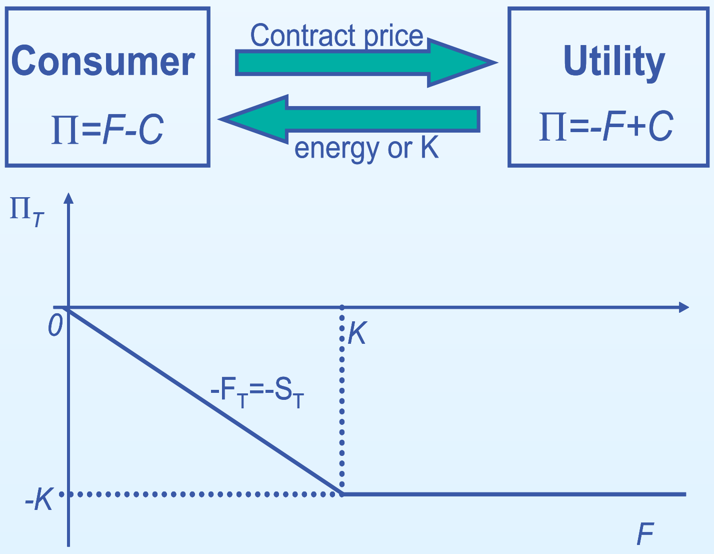
Option Pricing -European Call Option (II)

Call on a Forward; $T' = 10$, $T = 7$, $K = 20$, $\alpha \uparrow = 104$



Interruptible Contracts –Description (I)

- Callable forwards, introduced as early as 1994 by Gedra (1994), replicate an interruption strategy on supply of electricity.
- The portfolio held by the supplier/utility is given by $\Pi_t = -F_t^{T'} + C(S_t; K, T')$.
- The user, who owns the opposite portfolio, earns a discount on a the forward bought.
- The supplier benefits by earning the possibility of calling off supply at expiry.



Interruptible Contracts –Description (II)

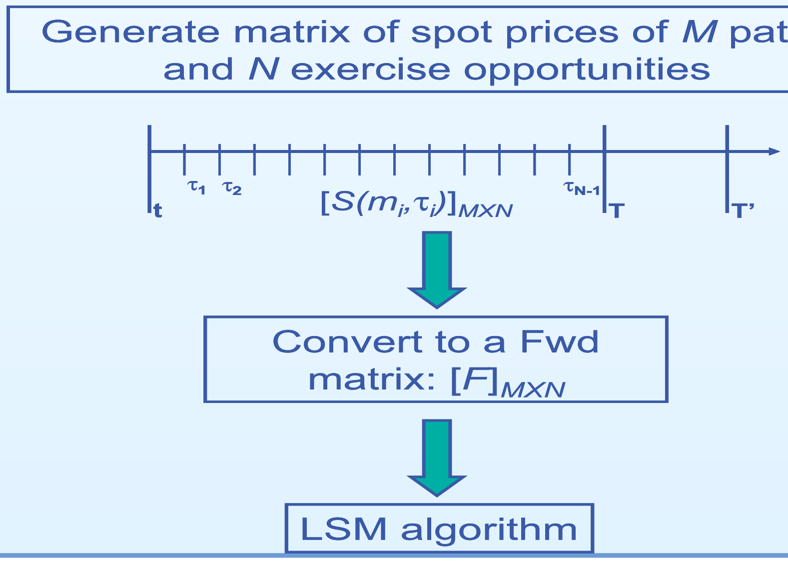
- A consumer entering such a contract must trade off the probability of interruption against its shortage cost ($\sim K$).
- The probability of interruption decreases as the strike price increases, hence the discount on the forward is lower for the consumer.
- Those consumers with lower shortage costs will be more likely to be interrupted and will receive higher discounts.
- A clear drawback is that there will be consumers whose short-notice interruption costs are too high, thus not providing a viable strike price.
- Kamat & Oren (2002) introduce an earlier notification date, and price in closed form with the use of compound options.
- However, only assuming one early possible exercise point is still unrealistic.

Pricing a Bermudan Interruptible on a Forward (I)

- We extend to interruptible contracts on forwards, by which a utility holds $\Pi_t = -F_t^{T'} + C(F_t^{T'}; K, T)$.
- It allows for the canceling of the forward obligation when exercised at any early stopping time τ_i.
- If held until expiry and if $T = T'$ it reduces to the previous interruptible contract.
- Possible incentives for a utility to enter such a contract:
  - ◇ future unpredicted capacity constraints;
  - ◇ high volatility in the spot due to extreme variations in weather, demand or others.
- The consumer benefits in a discount on the forward price.

Pricing a Bermudan Interruptible on a Forward (II)

- At intermediate exercise dates τ_i we compare the immediate exercise value with the expected cash flows from continuing, exercising if immediate exercise is more valuable.
- The key is defining the conditional expected value of continuation.
- LSM: regresses subsequent realized cash flows from continuation on a set of basis functions of the values of the relevant state variables.
- Longstaff & Schwartz show that results are robust for different choices of basis functions.



Preliminary results

	$F=10$		$F=25$	
	CFT	MC	CFT	MC
Δ	7.66×10^{-12}	5.10×10^{-3}	3.54×10^{-11}	2.86×10^{-2}
τ	0.13 sec.	17.06 sec.	0.14 sec.	17.00 sec.

Parameters: $K=25$; $t=0$; $T=1$; $T'=1.5$; $\alpha=1.18$; $\sigma=1.77$; $r=0.15$; $n=100$; $m=100,000$; $C(10)=0.23$ and $C(25)=5.14$. $C(\cdot)$ denotes the analytical values of the call option on the forward for Schwartz' model.

	European	Bermudan	Premium
GBM –Put	3.8443	4.4702 (0.0092)	0.6259
MRJD –Call	1.3931 (0.0079)	1.6804 (0.0070)	0.2873

Parameters-GBM-Put: $K=40$; $r=0.06$; $S_t=36$; $\sigma=0.20$; $t=0$; $T=1$; $N=50$.
Parameters-MRJD-Call: $r=0.15$; $K=23.5$; $S_t=18$; $\sigma=1.60$; l=8.58; n=365; $\lambda^*=-0.23$; $\alpha=104$; $\eta_1=2.80$; $\eta_2=3.85$; $p=0.54$; $q=0.46$; $t=15/09/05$; $T'=31d+t$; $T=30d+t$; $N=30$; $F_t^{T'}=24.2788$.

Conclusions and future research

- The appeal of spot-based models is that they provide realistic simulations of spot-price paths.
- In general, it is always possible to obtain closed form expressions for the forwards, as shown in CF (2005).
- In particular, when assuming exponential jumps the model becomes very tractable, and closed form solutions using CFT are obtained; these are very accurate and fast.
- Interruptible contracts are an important tool in risk management and there is a realistic market interest in such contracts.
- However, it is paramount to be able to solve some critical aspects of these models which might affect their range of applicability, such as
  - ◇ speed of mean reversion –should it be constant?
  - ◇ arrival of jumps;
  - ◇ calibration (with scarcity of data) under higher-factor models.