

Maturity Effect on Risk Measure in a Ratings-Based Default-Mode Model

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1. Introduction: Maturity Effects in CRM
2. Key Issues for a Default Mode Model
3. Maturity Effects in the Merton/Vasicek-Model
 - 3.1 The “Capital to Maturity” Approach
 - 3.2 The “Capital for one Period” Approach
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5. Conclusion

Introduction: Maturity Effects in CRM [1]

- Today, in order to measure the “risk” arising from credit portfolios, merely two state discrete time models are common both in academic research as well as in practise.
- Well known models are CreditPortfolioViewTM, CreditRisk+TM, and CreditMetricsTM, that determine the full PDF of the portfolio on a one-year time horizon (risk horizon) and analyse it with respect to mean, standard deviation, and Value at Risk or Unexpected Loss.
- Commonly, the Value at Risk is used as a measure for the economic capital, that the bank should hold against future losses.
- Especially investment loans often have a time to maturity longer than the (one-year) time horizon of the model, but credit portfolio models do not account for this misspecification and the potential risk arising from this fact.
- Only few papers deal with the effect of (longer) time to maturity on risk measure/economic capital.

Introduction: Maturity Effects in CRM [2]

- Mark-to-Market (MTM) Models (e.g. CreditMetricsTM)
- Modelling the market value of the credits (and the portfolio), that depends on interest rates, term structure of credit spreads and the rating migration matrix.
- Risk horizon is fixed (one year) and maturity influences on the risk measure, since the different times to maturity lead to different credit spreads.
- related papers:

- Grundke (2003)
The Term Structure of Credit Spreads as a Determinant of the Maturity Effect on Credit Risk Cap.
 - Analytic determination of portfolio values using different term structures of credit spreads
 - Term structure of credit spreads via Merton(1974)-Model
 - Term structure is one of the most important determinants for the maturity effect.
 - The Basel II-maturity-adjustment is explainable.
- Kalkbrener/Overbeck (2001/2002)
Maturity as a factor for credit risk capital/The maturity effect on credit risk capital
 - Simulation based analysis using market credit spreads (US industrial bonds 1997/2001)
 - The Basel II-maturity-adjustment is very conservative.
- Barco (2004)
Bringing Credit Portfolio Modelling to Maturity
 - Analytic determination using saddle point technique.
 - The Basel II-maturity-adjustment is very conservative.

- All these models directly link their result to the Basel II adjustment formula, but suffer from the problem that they use a MTM approach.
- In a Default Mode (DM) model (e.g. CreditRisk+™, Basel II-IRB-Model) changes in the market value of credits are not of interest.
- Literature on incorporating possible risk, that arises from longer time to maturities in a DM framework are scarce.
- Li/Song/Ong (1999)
Maturity Mismatch
 - Models for credits, that mature before the risk horizon.
- Gordy/Heitfield (2001)
Maturity effects in a class of multi-period default mode models
 - unpublished

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Key Issues for a Default Mode Model [1]

- Some assumptions due to the key issues on default mode models from BIS (1999):
Credit Risk Modelling: Current Practise and Applications, Bank for International Settlements
- The exposure is intended to be held to maturity (buy and hold). IAS 39 (loans and receivables shall be “measured at amortised cost”)
 - The Fair value is not observable (IAS 39.46/47)
 - Losses only occur, if there is “objective evidence that an impairment loss has been incurred” (IAS 39.63)
 - This especially is valid for so called “held-to-maturity” investment (IAS 39.9)
- Due to limited markets, the credit could not be traded before maturity.

Key Issues for a Default Mode Model [2]

- “Liquidation Period” approach: each facility is associated with a unique interval coinciding with instrument’s maturity.
- Our approach: “Capital to Maturity”
For the model a time horizon equal to the maturity of the credits is considered.
- “Constant Time Horizon for all Asset Classes” approach: a one year time horizon for all facilities is adopted.
- Additionally:
 - New economic capital could not be raised for the following period.
 - The bank could not hedge perfectly future potential losses.
- Our approach: “Capital for one Period”
Risk of credits with long maturity arises from a increasing probability of default (-> increasing economic capital) of non-defaulted loans.

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- Assets of the borrower are log normally distributed and follow a geometric Brownian motion. The liabilities grow with a constant rate.

$$\ln \tilde{A}_i^{(T)} = \ln \tilde{A}_i^{(0)} + \mu_i^{(T)} + \sigma_i^{(T)} \cdot \tilde{a}_i^{(T)} \quad B_i^{(T)} = B_i^{(0)} \cdot \exp(r \cdot T)$$

- Default occurs, if the assets at $t=T$ falls short of the outside liabilities.

$$PD_i^{(T)} = P(\tilde{A}_i^{(T)} < B_i^{(T)}) = N(b_i^{(T)}) \quad b_i^{(T)} = [\ln(B_i^{(0)} / A_i^{(0)}) - \mu_{i,\text{eff}}^{(T)}] / \sigma_{A,i}^{(T)}$$

- The credit portfolio is “infinitely homogeneous” and assets follow a one-factor approach.

$$\tilde{a}_i^{(T)} = \sqrt{\rho_i} \cdot \tilde{x}^{(T)} + \sqrt{1-\rho_i} \cdot \tilde{\varepsilon}_i^{(T)} \quad \tilde{x}^{(T)}, \tilde{\varepsilon}_i^{(T)} \sim N(0,1)$$

- The (portfolio invariant) credit risk contribution is quantified by the difference (Unexpected Loss) between Expected Loss and VaR of the potential gross loss rate

$$UL(\tilde{\ell}_i^{(T)}) := VaR_z(\tilde{\ell}_i^{(T)}) - E(\tilde{\ell}_i^{(T)})$$

$$E(\tilde{\ell}_i^{(T)}) = PD_i^{(T)} \quad VaR_z(\tilde{\ell}_i^{(T)}) = N\left(\left[N^{-1}(PD_i^{(T)}) - \sqrt{\rho_i} \cdot x_{q_{1-z}}^{(T)}\right] / \sqrt{1-\rho_i}\right)$$

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- In the “Capital to Maturity” approach the time horizon $t = T$ is set to the maturity of the loan.
- The effect of increasing maturity ($t = m \cdot T$) on the probability of default due to Merton is (here $m = 2$)

$$PD_i^{(2 \cdot T)} = N(b_i^{(2 \cdot T)}) \quad b_i^{(2 \cdot T)} = \frac{1}{\sqrt{2}} \left(b_i^{(T)} - \frac{\mu_{i, \text{eff}}^{(T)}}{\sigma_{A,i}^{(T)}} \right)$$

- The probability of default rises, if

$$\mu_{i, \text{eff}}^{(T)} / \sigma_{A,i}^{(T)} < -(\sqrt{2} - 1) \cdot b_i^{(T)}$$

- Since this bound increases with lower probabilities of default (for $PD < 0,5$), so it seems to be more likely, that the default probability rises with shifting to higher maturity, when probability initially is low.

- In order to calculate the unexpected loss contribution we use the probability of default at maturity $t = m \cdot T$.

$$UL(\tilde{\ell}_i^{(m \cdot T)}) := VaR_z(\tilde{\ell}_i^{(m \cdot T)}) - E(\tilde{\ell}_i^{(m \cdot T)})$$

$$E(\tilde{\ell}_i^{(m \cdot T)}) = PD_i^{(m \cdot T)} \quad VaR_z(\tilde{\ell}_i^{(m \cdot T)}) = N\left(\left[N^{-1}(PD_i^{(m \cdot T)}) - \sqrt{\rho_i} \cdot x_{q_{l-z}}^{(m \cdot T)}\right] / \sqrt{1 - \rho_i}\right)$$

- The probability of default at $t = m \cdot T$ is a function of the probability of default at $t = T$

$$PD_i^{(m \cdot T)} = N(b_i^{(m \cdot T)}) = f\left(N^{-1}(PD_i^{(T)}), \mu_{i,eff}^{(T)} / \sigma_{A,i}^{(T)}, m\right)$$

- The maturity adjustment specifies the function, that links the maturity adjustment at maturity $t = T$ to maturity $t = m \cdot T$

$$UL(\tilde{\ell}_i^{(m \cdot T)}) = UL(\tilde{\ell}_i^{(T)}) \cdot g_{CtM}(PD_i^{(T)}, \mu_{i,eff}^{(T)} / \sigma_{A,i}^{(T)}, m, \rho_i)$$

- Since in a rating based model most parameters are not observable, the maturity adjustment function should be estimated empirically.

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- In the “Capital to Maturity” approach the time horizon $t = T$ is constant, but it has to be taken into account, that the probability of default of a loan possibly rises over time.
- The (expected) probability of default of a loan with maturity $m > 1$ for the period $(T, m \cdot T)$ is (here $m = 2$)

$$PD_i^{(T,2T)} = PD_i^{(2T)} | (\tilde{A}_i^{(T)} > B_i^{(T)}) = \frac{\Delta PD_i^{(T,2T)}}{1 - PD_i^{(T)}}$$

- The probability of default with respect to the first period rises, if

$$PD_i^{(T)} > 0,5 - \sqrt{0,25 - \Delta PD_i^{(T,2T)}} \quad \Delta PD_i^{(T,2T)} < 0,25$$

- The marginal probability of default is

$$\Delta PD_i^{(T,2T)} = N\left(\frac{b_i^{(T)}}{\sqrt{2}} - \frac{\mu_{i,\text{eff}}^{(T)}}{\sqrt{2} \cdot \sigma_{A,i}^{(T)}}\right) - N^2\left(b_i^{(T)}, \frac{b_i^{(T)}}{\sqrt{2}} - \frac{\mu_{i,\text{eff}}^{(T)}}{\sqrt{2} \cdot \sigma_{A,i}^{(T)}}, \sqrt{\frac{1}{2}}\right)$$

- Therefore, a high probability of default in the first period leads to low marginal probabilities in the second period.

- In order to calculate the unexpected loss contribution only for one period

$$UL(\tilde{\ell}_i^{(m \cdot T)}) = VaR_z(\tilde{\ell}_i^{(m \cdot T)}) - E(\tilde{\ell}_i^{(m \cdot T)})$$

$$E(\tilde{\ell}_i^{(m \cdot T)}) = \widehat{PD}_i^{(m \cdot T)} \quad VaR_z(\tilde{\ell}_i^{(m \cdot T)}) = N \left(\left[N^{-1}(\widehat{PD}_i^{(m \cdot T)}) - \sqrt{\rho_i} \cdot x_{q_{1-z}}^{(m \cdot T)} \right] / \sqrt{1 - \rho_i} \right)$$

- We use the highest expected one-period probability of default until maturity $t = m \cdot T$

$$\widehat{PD}_i^{(m \cdot T)} = \max \left(PD_i^{(T)}, PD_i^{(T,2T)}, PD_i^{(2T,3T)}, \dots, PD_i^{((m-1) \cdot T, m \cdot T)} \right)$$

- The maturity adjustment specifies the function, that links the maturity adjustment at maturity $t = T$ to maturity $t = m \cdot T$

$$UL(\tilde{\ell}_i^{(m \cdot T)}) = UL(\tilde{\ell}_i^{(T)}) \cdot g_{CoP}(PD_i^{(T)}, \mu_{i,eff}^{(T)} / \sigma_{A,i}^{(T)}, m, \rho_i)$$

- The function is fitted empirically.

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- We analysed average cumulative default rates of rating data on a yearly basis from
 - Standard & Poors (cumulative default rates/average transition rates, up to 15 years)
 - Moody's (cumulative default rates/average migration rates, up to 20 years)
here: Mood's cumulative default rates, 7 classes
 - Fitch (cumulative default rates, 5 years)
 - Creditreform Rating AG (average migration rates, 5 years)
- In the "Capital to Maturity"-Approach we used the cumulative default rates as a estimator for the probability of default

$$\widehat{PD}_i^{(m \cdot T)} = \overline{DR}_i^{(m \cdot T)}$$

- In the "Capital for One Period"-Approach we used the conditional default rates as an estimator for the conditinal (periodical) probability of default.

$$\widehat{PD}_i^{(T, T+1)} = \left(\overline{DR}_i^{(T+1)} - \overline{DR}_i^{(T)} \right) / \left(1 - \overline{DR}_i^{(T)} \right) =: \overline{DR}_i^{(T, T+1)}$$

- In order to derive the unexpected loss contribution we used the asset correlation due to the calibration formula in Basel II

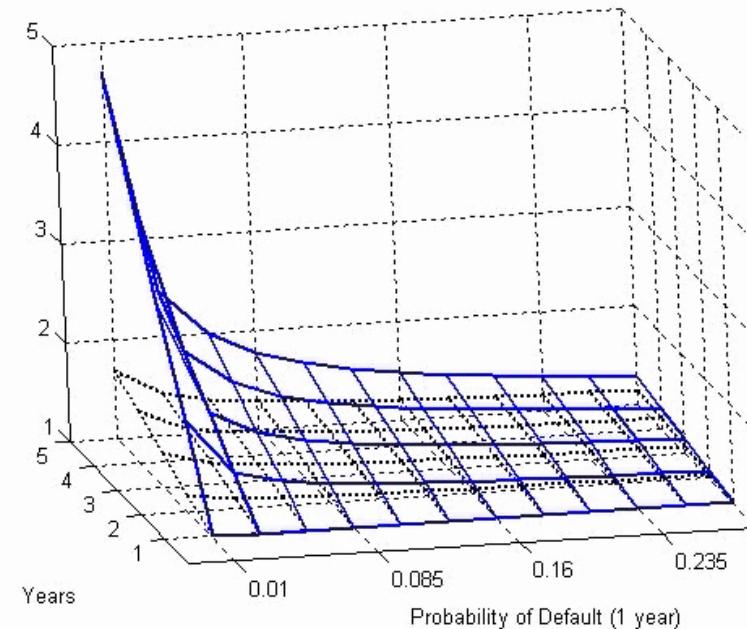
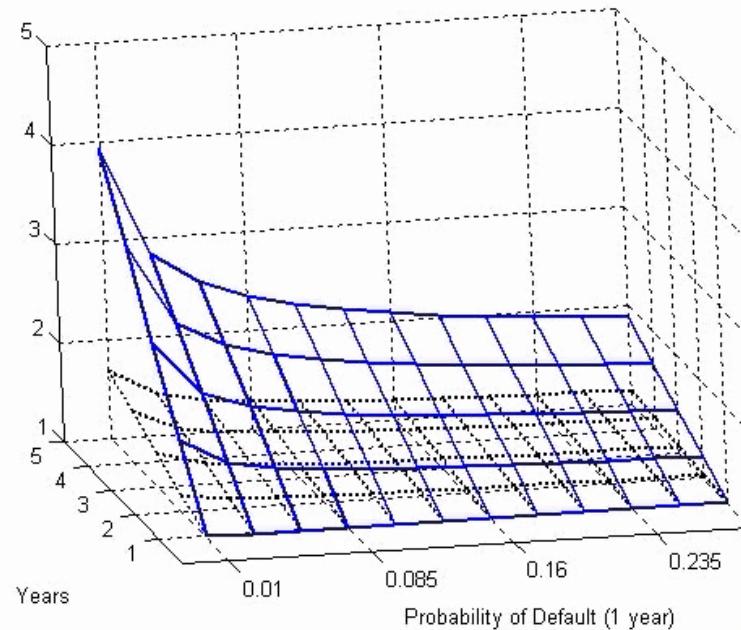
$$\hat{\rho}_i = f(\widehat{PD}_i^{(1 \text{ year})}) = 0.12 \cdot \frac{1 - \exp(-50 \cdot \widehat{PD}_i^{(1 \text{ year})})}{1 - \exp(-50)} + 0.24 \cdot \left(1 - \frac{1 - \exp(-50 \cdot \widehat{PD}_i^{(1 \text{ year})})}{1 - \exp(-50)} \right)$$

- As a function for the maturity adjustment we used the function of the form like in Basel II

$$g_*(\widehat{PD}_i^{(1 \text{ year})}, m) = \frac{1 + (m - 2.5) \cdot \left(a - b \cdot \ln(\widehat{PD}_i^{(1 \text{ year})}) \right)^2}{1 - 1.5 \cdot \left(a - b \cdot \ln(\widehat{PD}_i^{(1 \text{ year})}) \right)^2}$$

- The function was fitted to the empirical UL contribution using leasted squares.

- The maturity adjustment function seems to be a good choice since only two parameters has to be analysed where

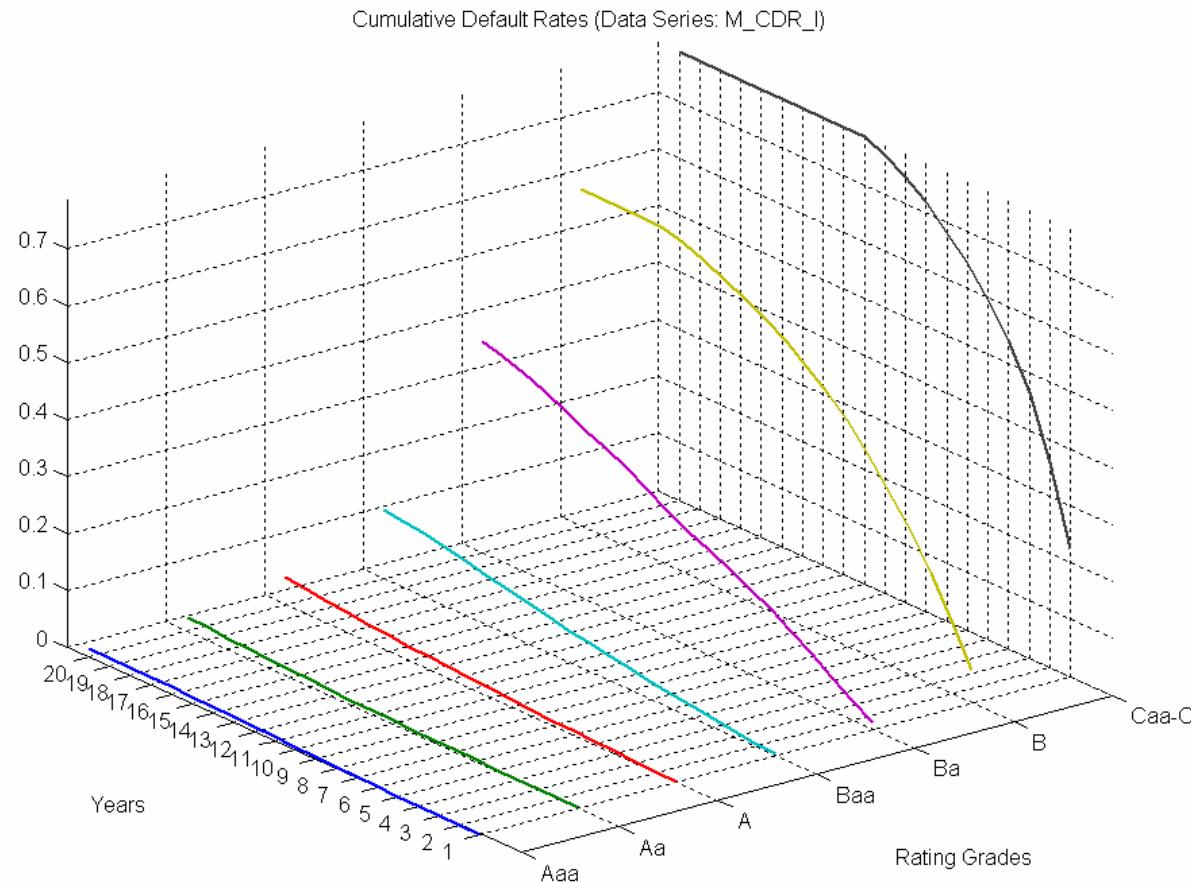


- the parameter a especially controls the slope with respect to m ,
- The parameter b especially controls the slope with respect to PD

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The “Capital to Maturity” Approach [1]

- Analysis of the cumulative default rate

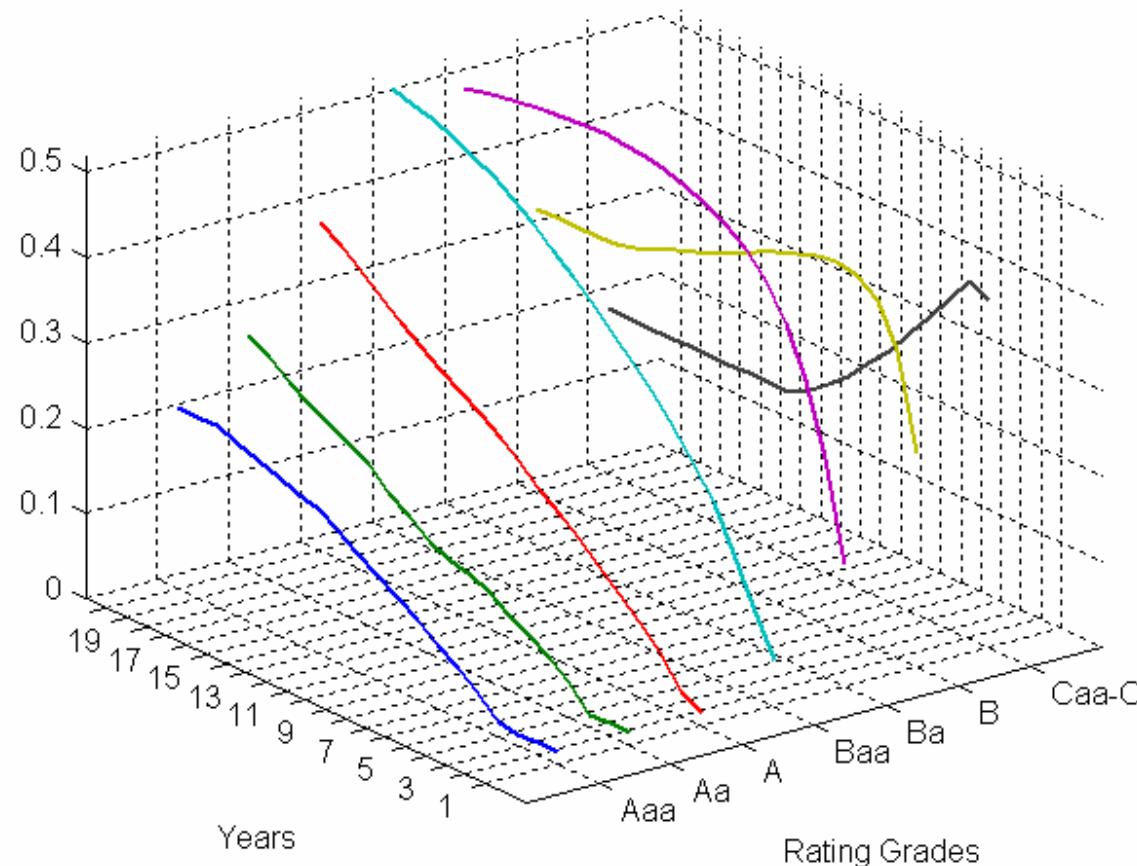


- The cumulative default rate has a concave characteristic for speculative grades and a convex/linear characteristic for investment grades.

The “Capital to Maturity” Approach [2]

- Analysis of the UL contribution

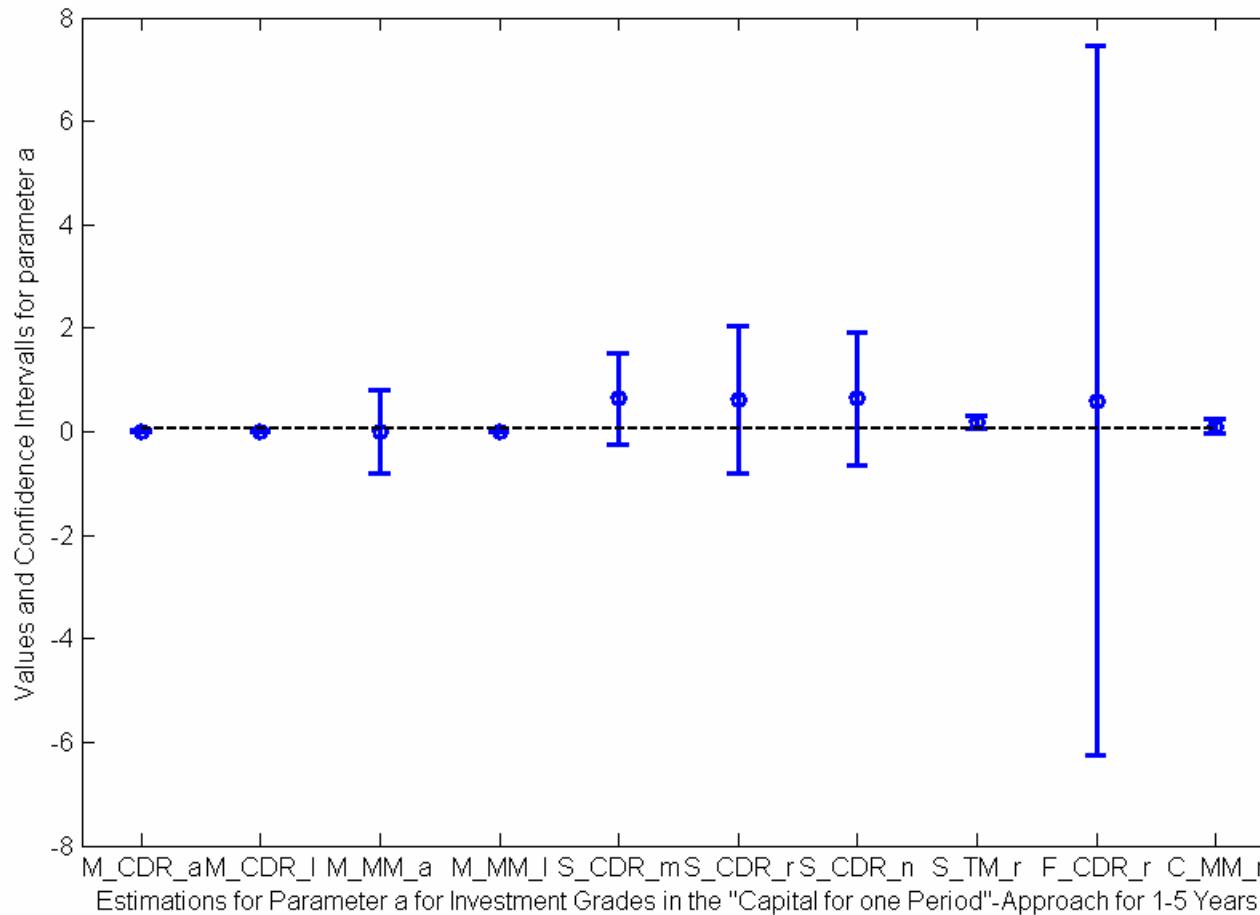
Unexpected Loss (CtM) with $\rho = f(PD^{1 \text{ year}})$ (Data Series: M_CDR_I)



- The UL contribution rises rapidly for investment grades and with decreasing slope / is at a stretch declining for speculative grades.

The “Capital to Maturity” Approach [3]

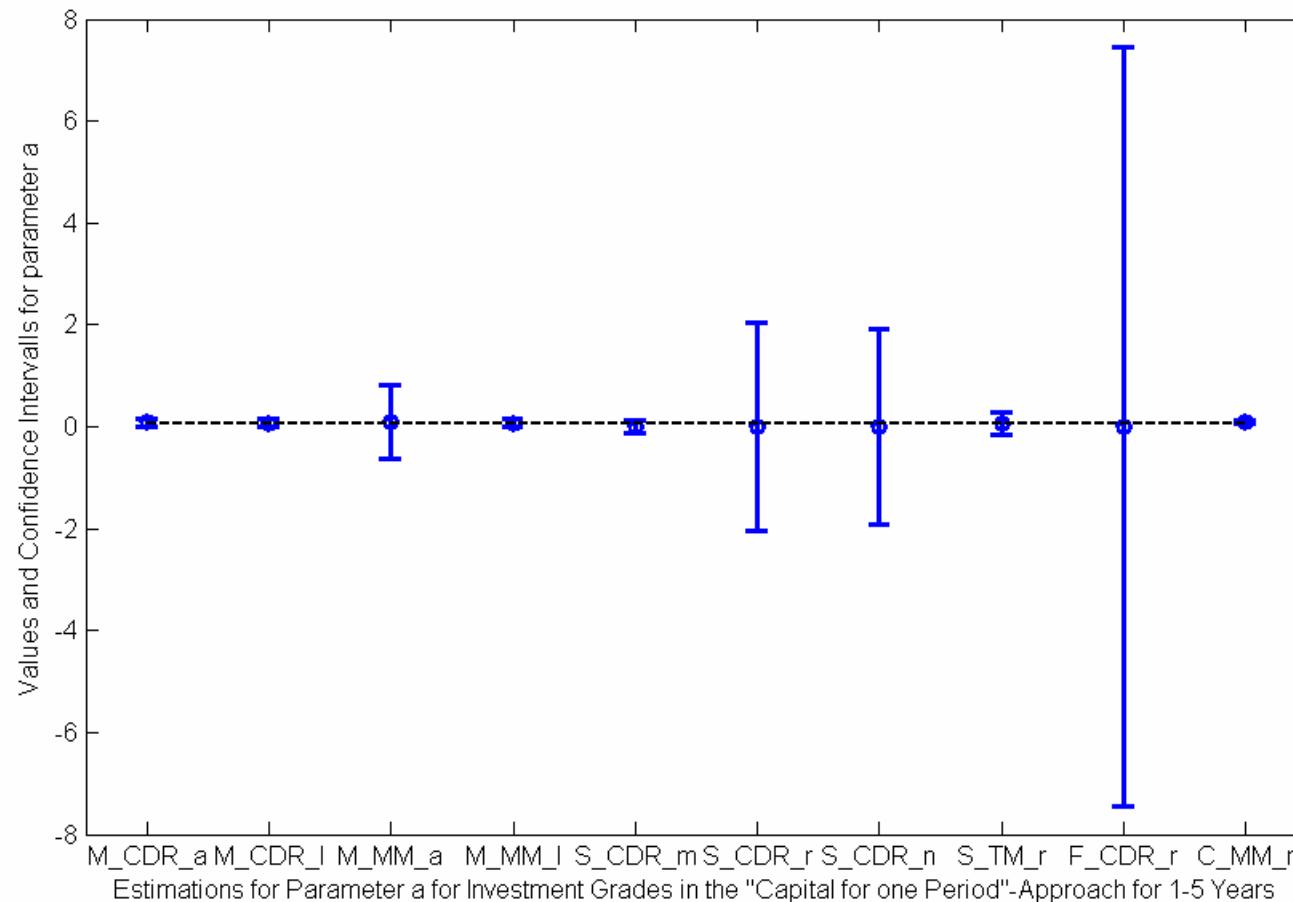
- Estimations for parameter a for a maturity of up to 5 years



- Our estimates range from 0.06 to 0.22 in comparison to 0.12 in Basel II.

The “Capital to Maturity” Approach [3]

- Estimations for parameter b for a maturity of up to 5 years

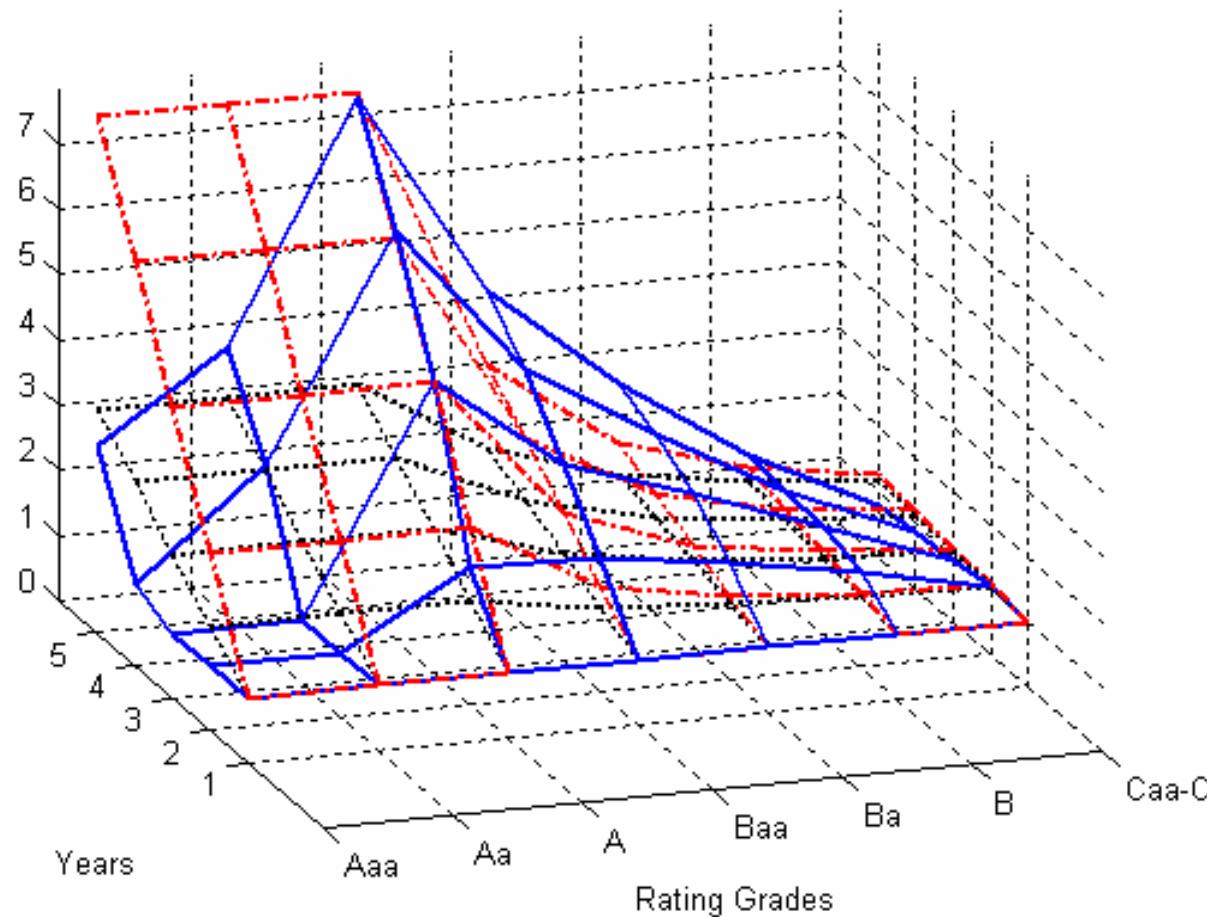


- Our estimates range from 0.06 to 0.1 in comparison to 0.05 in Basel II.

The “Capital to Maturity” Approach [4]

- Comparison of the maturity adjustments:

Comparision of the Maturity Adjustment (CtM) (Data Series: M_CDR_I)

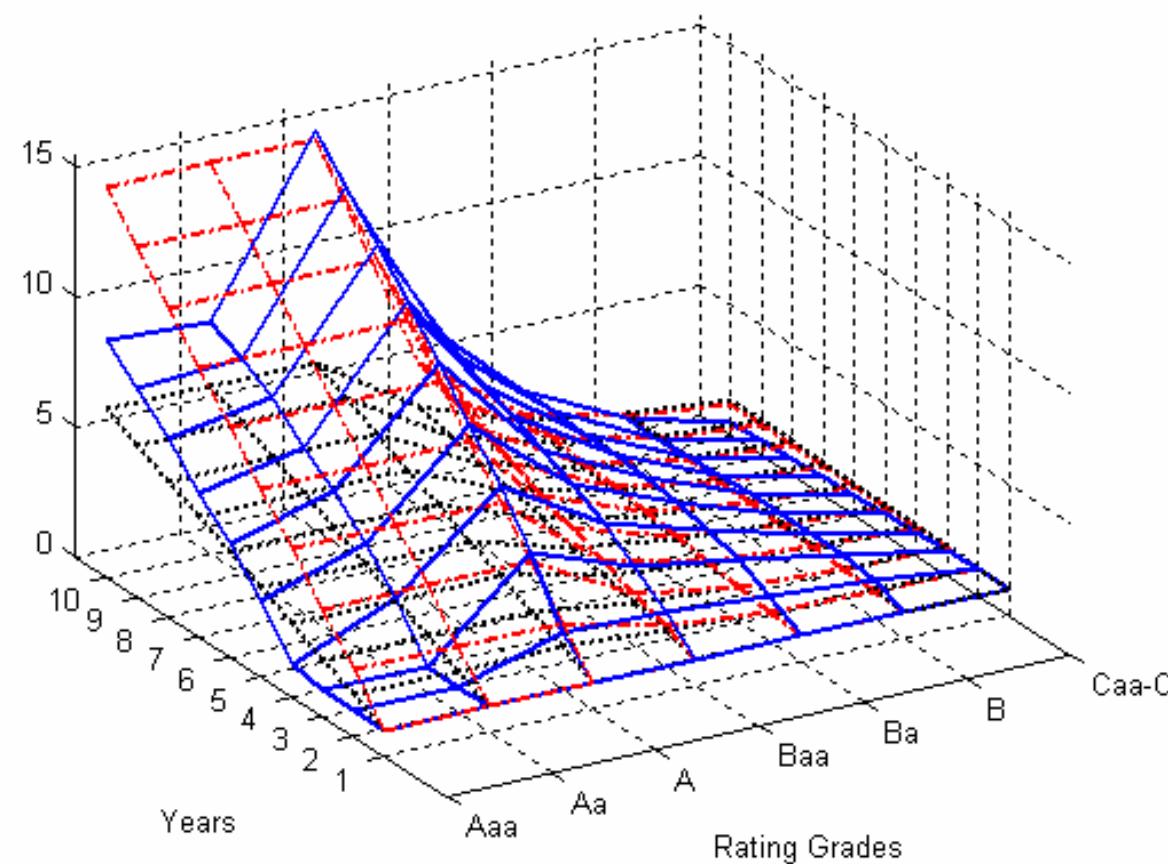


- Our adjustment especially fits with the empirical values at speculative grade and overestimates the effect if compared to Basel II.

The “Capital to Maturity” Approach [4]

- Comparison of the maturity adjustments using maturities up to 10 years:

Comparision of the Maturity Adjustment (CtM) (Data Series: M_CDR_I)

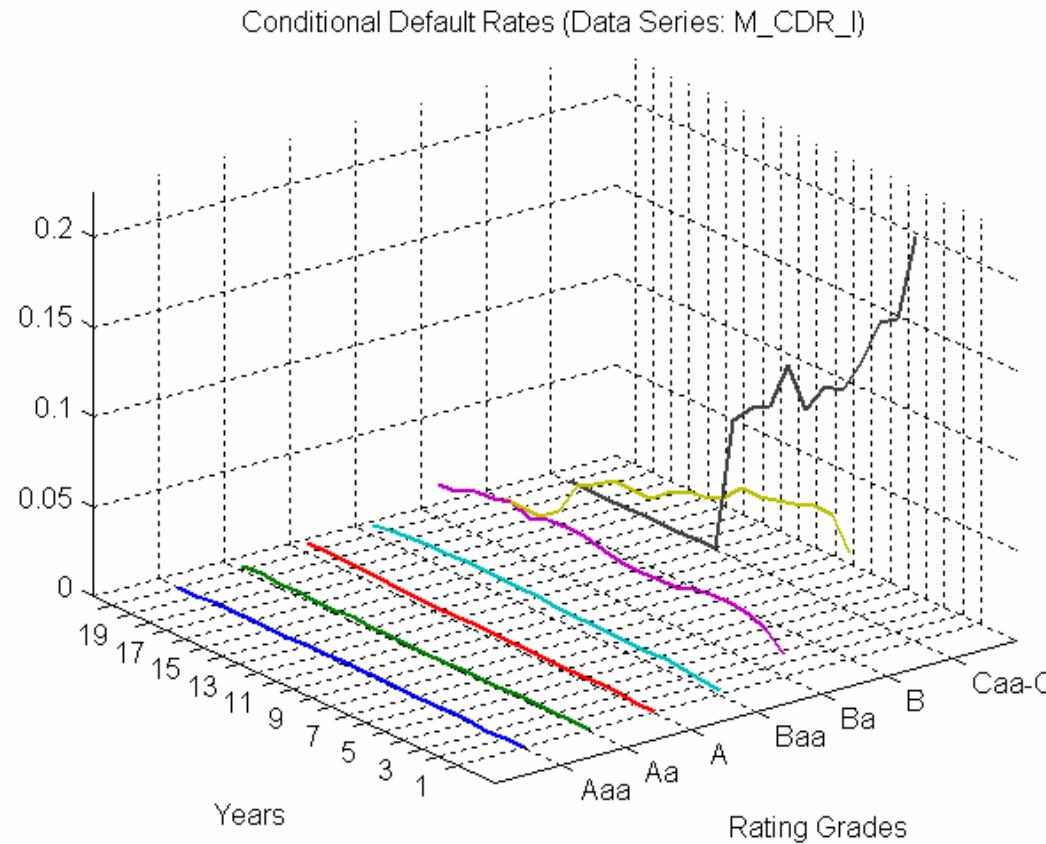


- The result does not change: the BaselII formula underestimates the effect in the “Capital to Maturity” approach.

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The “Capital for one period” Approach [1]

- Analysis of the conditional one-year default rate

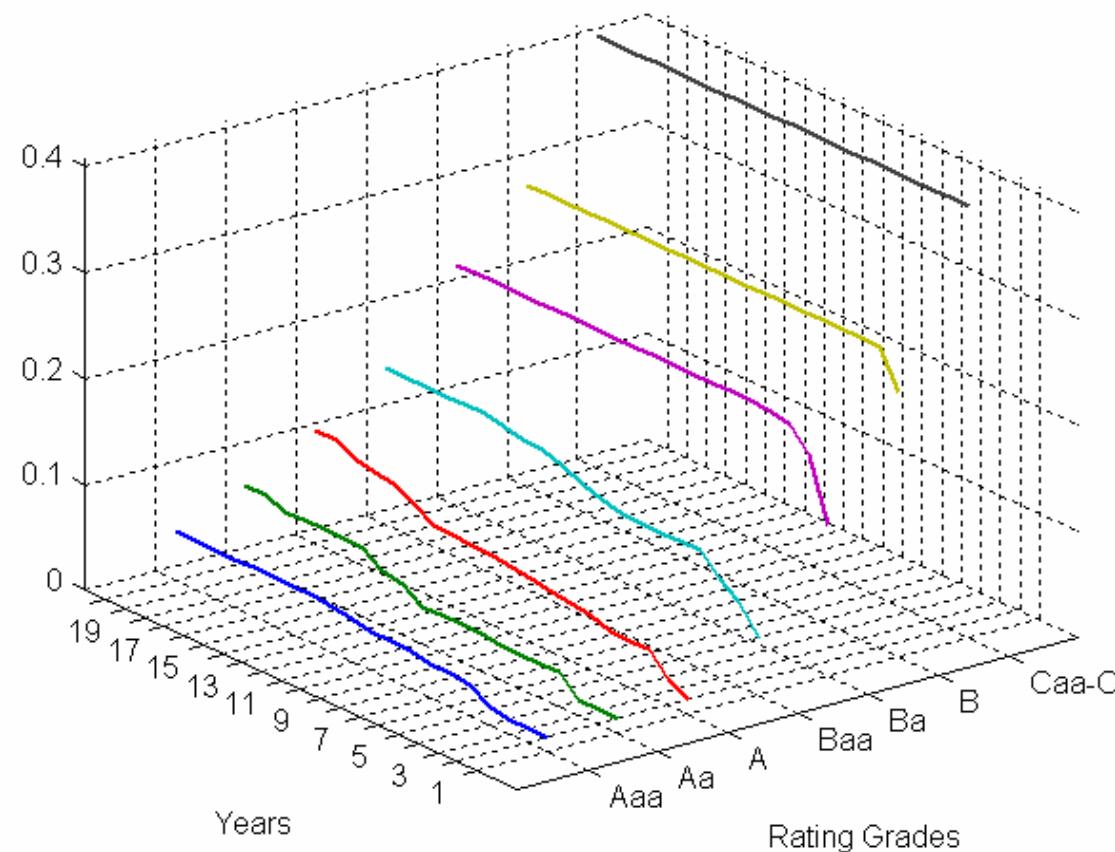


- The conditional default declines for the lower speculative grades and rises / stays equal for the investment grades.

The “Capital for one period” Approach [2]

- Analysis of the UL contribution

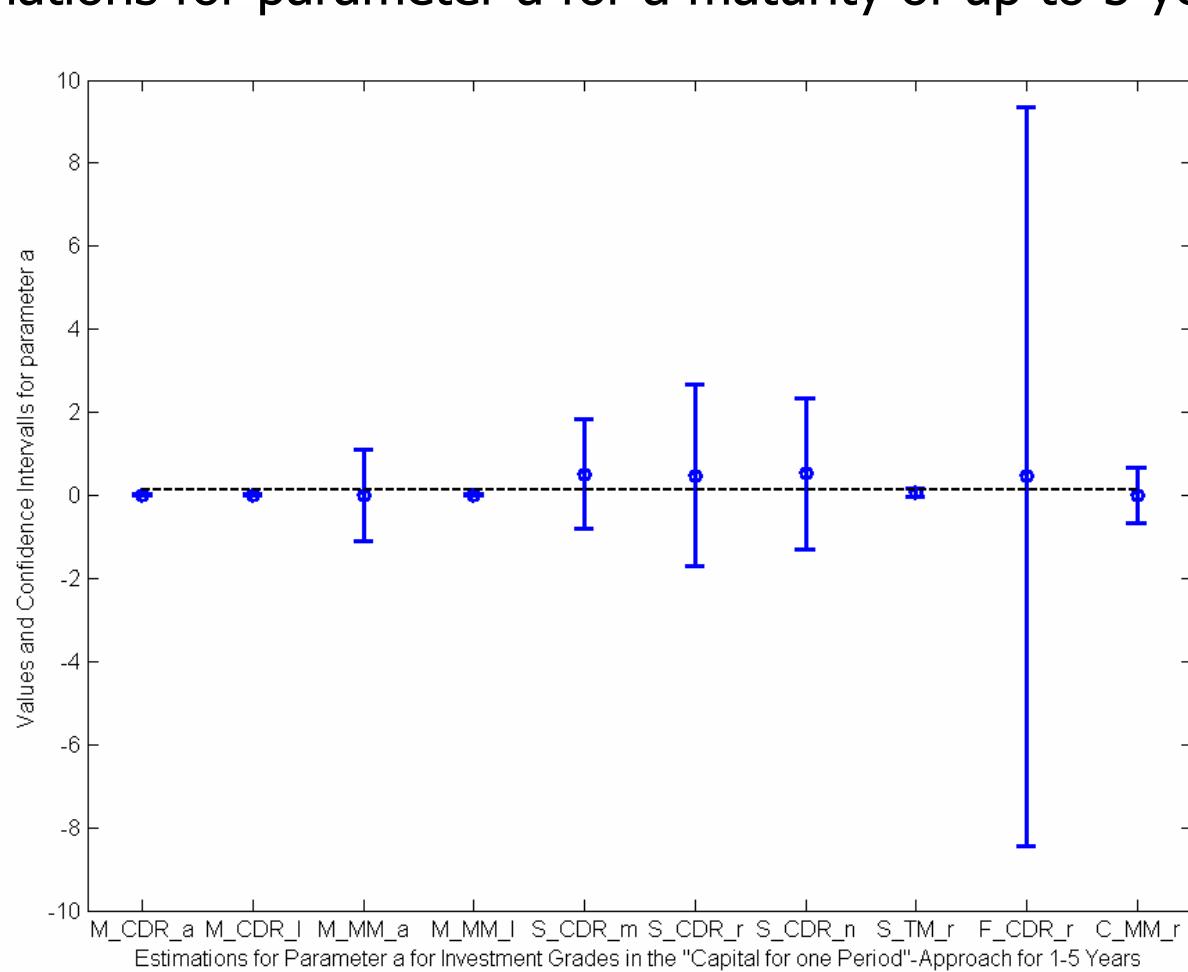
Unexpected Loss (CoP) with $\rho=f(\text{PD}^{\frac{1}{\text{year}}})$ (Data Series: M_CDR_I)



- The UL contribution rises linear for investment grades and stays at an equal level for speculative grades.

The “Capital for one period” Approach [3]

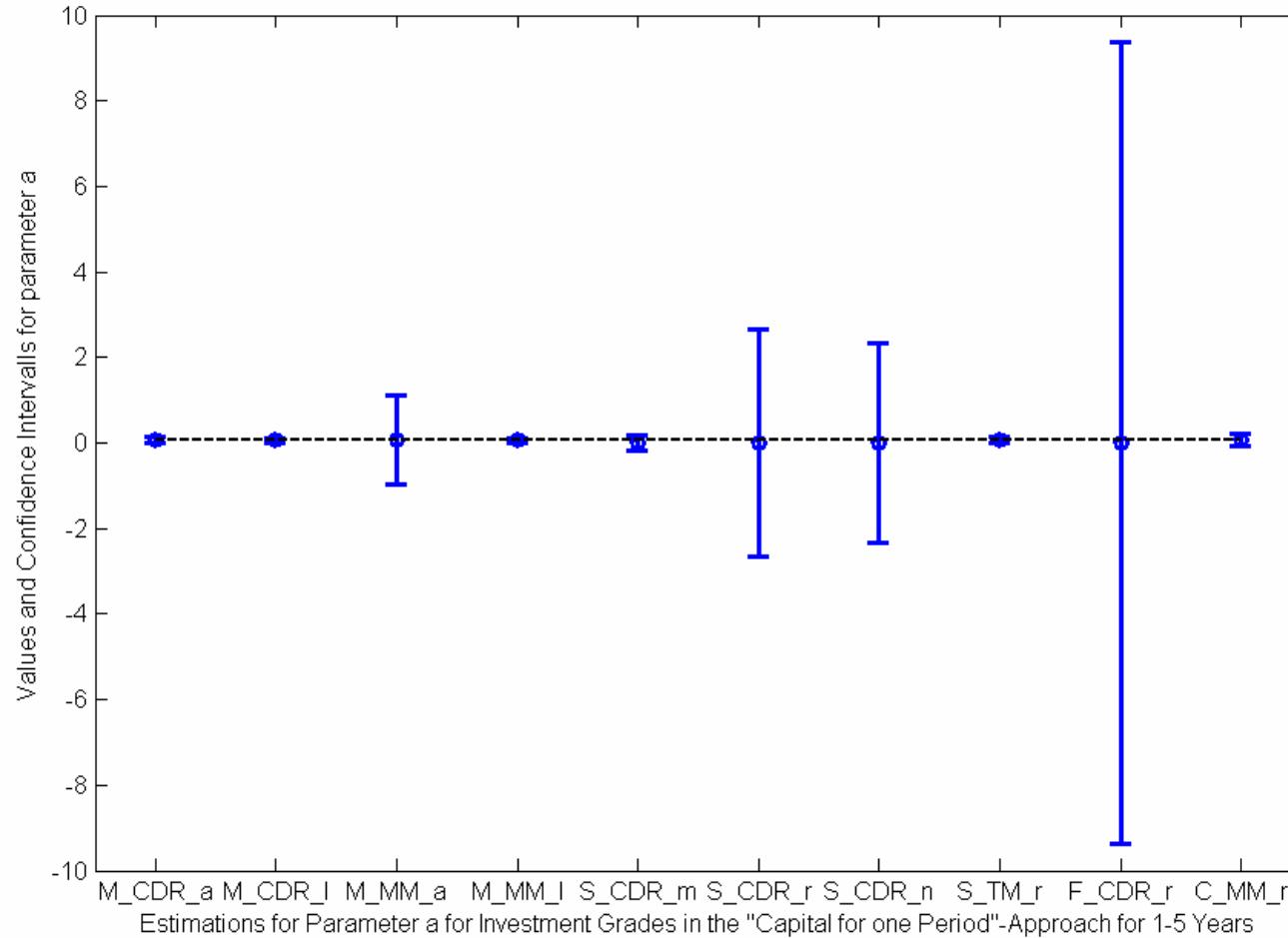
- Estimations for parameter a for a maturity of up to 5 years



- Our estimates range from 0.00 to 0.07 in comparison to 0.12 in Basel II.

The “Capital for one period” Approach [3]

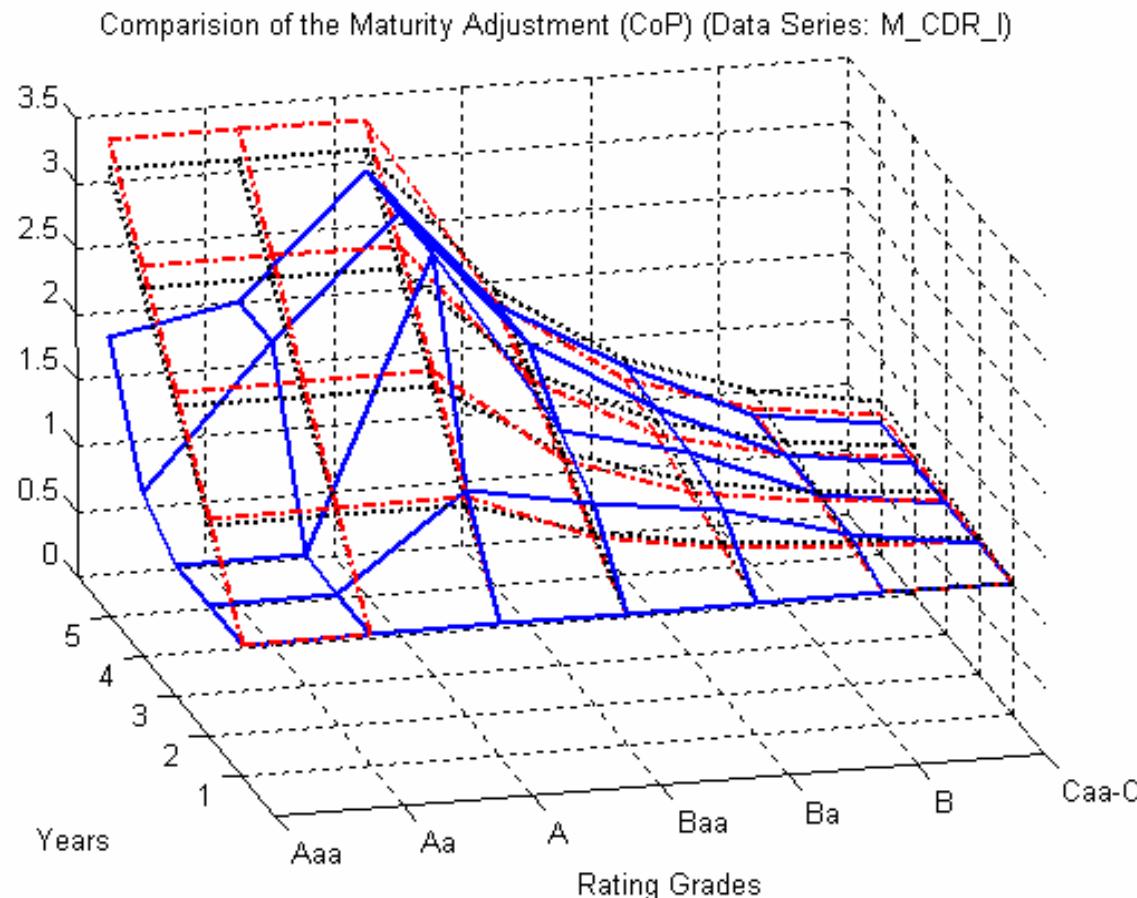
- Estimations for parameter b for a maturity of up to 5 years



- Our estimates range from 0.06 to 0.08 in comparison to 0.05 in Basel II.

The “Capital for one period” Approach [3]

- Comparison of the maturity adjustments:

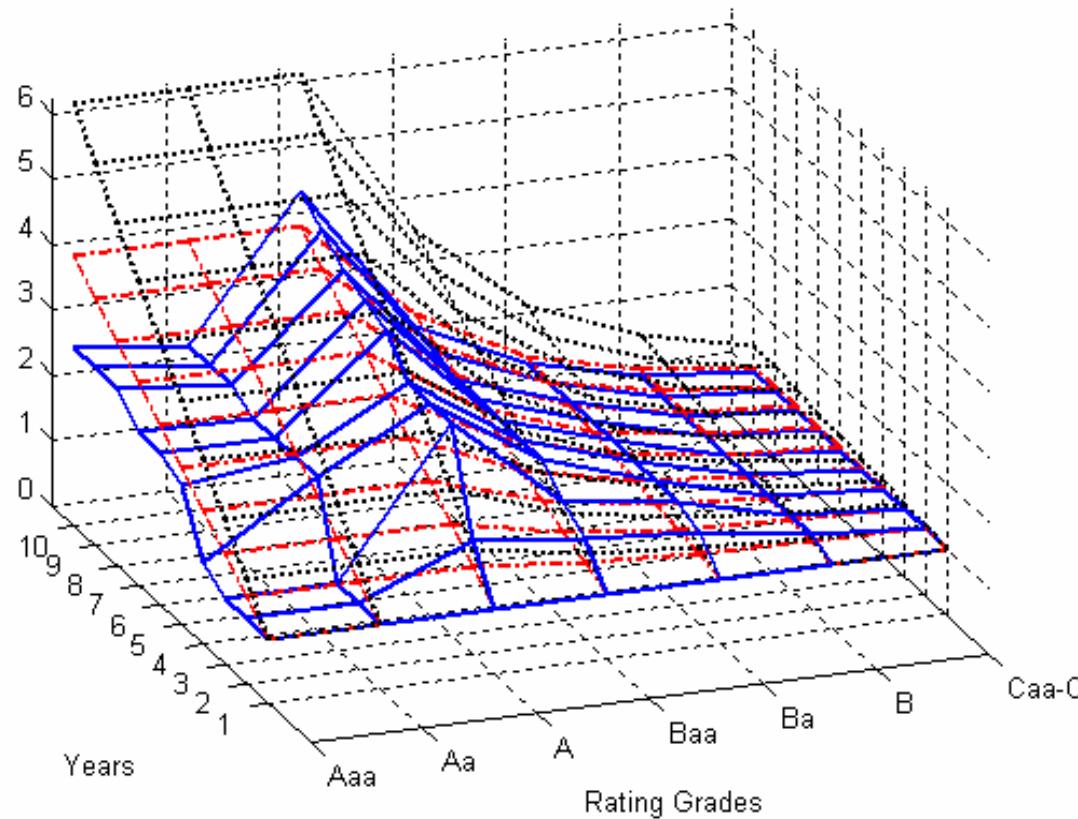


- Our adjustment especially fits with the empirical values at speculative grade and is very close to the results from Basel II.

The “Capital for one period” Approach [3]

- Comparison of the maturity adjustments using maturities up to 10 years:

Comparision of the Maturity Adjustment (CoP) (Data Series: M_CDR_I)



- For longer maturities the adjustment declines and therefore the Basel II formula overestimates the effect in the “Capital for one Period” approach.

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- We have investigated the effect of the time to maturity on economic capital in the CreditMetrics™-one-factor default mode model.
- Therefore, we motivated two approaches based on key issues in credit risk modelling:
 - the “Capital to Maturity” Approach
 - the “Capital for one period” Approach.
- The qualitative impact of both approaches was examined using the Merton/Vasicek-model for credit risk.
- Additionally, our Approach were implemented on empirical data.
- The empirical data shows characteristics as expected from the model. Our results for the maturity adjustment (especially under the “Capital for one period” approach) is close to the result in Basel II.