Pricing of electricity futures with modified Schwartz and Smith two factor model

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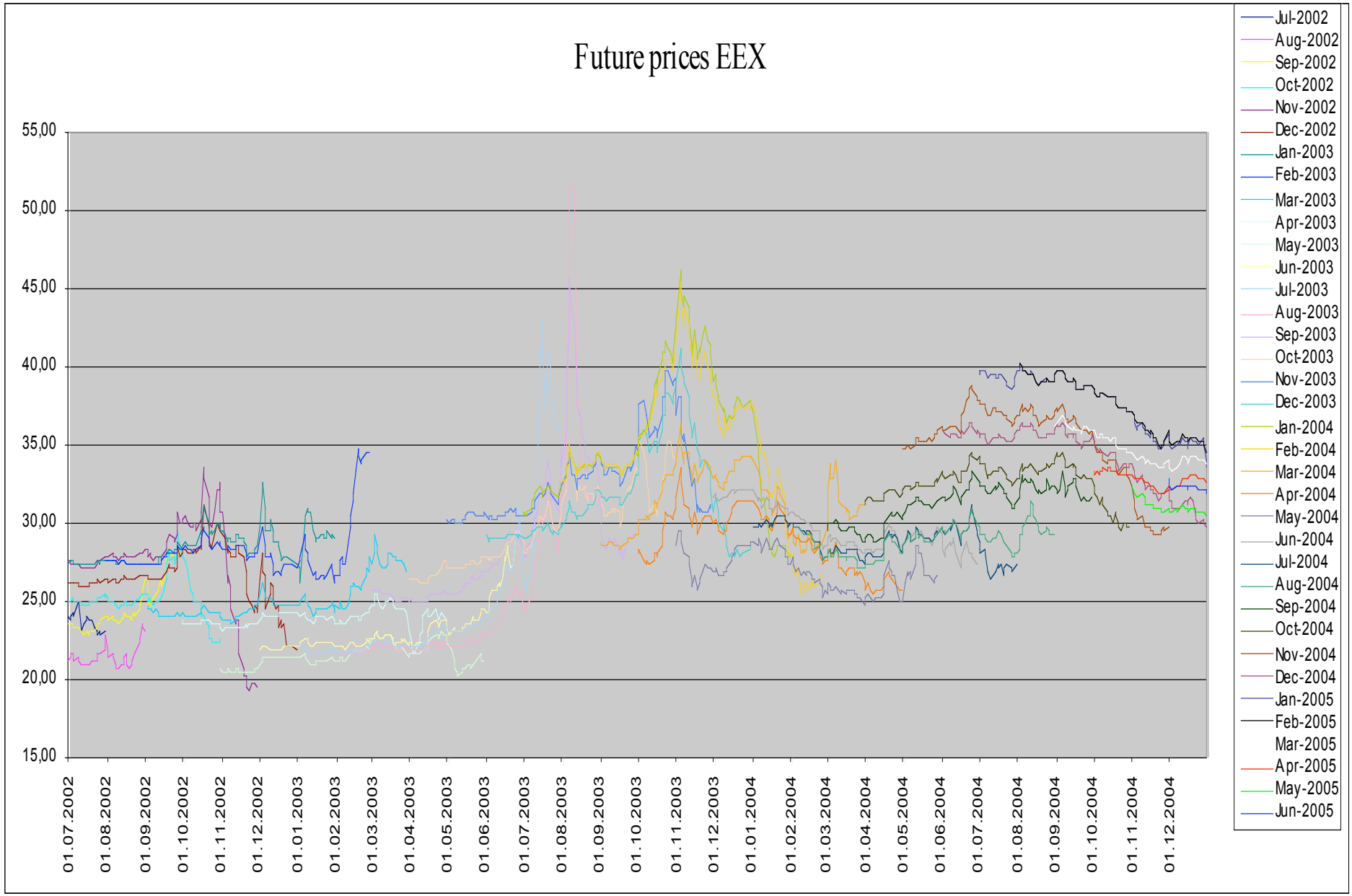
Plan of presentation

- Motivation
- Schwartz and Smith model for commodity futures pricing and modifications of the model for electricity
- Calibration issues
- Results
- Further research
- Questions and discussions


Future prices EEX
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Oct-2004
Nov-2004
Dec-2004
Jan-2005
Feb-2005
Mar-2005
Apr-2005
May-2005
Jun-2005

Possible approaches to futures modeling

- Derive future prices from the spot

  example: Schwartz and Smith two-factor model

- Model future dynamics directly

  example: Heath-Jarrow-Morton (HJM) approach

Electricity futures

- Delivery over month/quarter and over a year
- Overlapping delivery periods
- Average spot price is used for reference price of the future contract
- Two possibilities
  1. Model fixed-delivery futures F(t,T) and derive electricity futures F(t,T0,T)
  2. Model directly electricity futures F(t,T0,T)

Schwartz and Smith model for commodity futures

Schwartz E. and Smith J.E. (2000), Short-term variations and long-term dynamics in commodity prices, *Management Science*, vol. 46, No. 7, pp. 893-911.

S_t (or $S(t)$) the spot price of electricity at time t.

$$\ln(S_t) = \chi_t + \xi_t, \tag{1}$$

χ_t (or $\chi(t)$) - the short-term deviation in price
ξ_t the equilibrium price level

$$d\chi(t) = -\kappa\chi(t)\,dt + \sigma_\chi\,dW_\chi(t), \tag{2}$$

$$d\xi(t) = \mu_\xi\,dt + \sigma_\xi\,dW_\xi(t).$$

Here

$dW_\chi(t)dW_\xi(t) = \rho_{\chi\xi}dt$

under an real-world measure $\mathbb{P}$.

Consider risk-neutral measure $\mathbb{Q}^*$.

$$d\chi(t) = \left(-\kappa\chi(t) - \lambda_\chi \sigma_\chi\right) dt + \sigma_\chi \, dW_\chi^\star(t) \tag{6a}$$

$$d\xi(t) = \left(\mu_\xi - \lambda_\xi \sigma_\xi\right) dt + \sigma_\xi dW_\xi^*(t), \tag{6b}$$

Here

$dW_\chi^*(t) dW_\xi^*(t) = \rho_{\chi\xi} dt$

under the risk-neutral measure $\mathbb{Q}^*$.

(χ_t) is the process reverting to $-\lambda_\chi \sigma_\chi / \kappa$

(ξ_t) is again a geometric Brownian motion, but now with a drift $\mu_\xi^* = \mu_\xi - \lambda_\xi \sigma_\xi$.

t - current time

T_0 - one day before starting date of delivery

T - an ending date of delivery.

For $i = 1, \ldots, n$ consider n averaging points

$t_i = T_0 + i \cdot \Delta t$

$t_n = T$

$\Delta t = (T - T_0)/n.$

G_n - geometric average of n prices at times t_1, $t_2, \ldots, t_n$

$$G_n = \left[\prod_{i=1}^{n} S(t_i) \right]^{1/n} . \tag{5}$$

Now lets denote the future price of electricity at time t with delivery period $(T_0, T]$ as $F(t, T_0, T)$. Under risk-neutral measure

$$\begin{aligned} F(t, T_0, T) &= \mathbb{E}_t^\star \left[G_n \right] \\ &= \exp \left\{ m_A(t, T_0, T, n) + \frac{1}{2} \sigma_A^2(t, T_0, T, n) \right\} \\ &= \chi_t e^{-\kappa(T-t)} \varphi(T_0, T, n) + \xi_t + B(T_0, T, n), \end{aligned} \tag{13}$$

Now we can calculate term premium coefficient R:

$$R = \log\left(\frac{\mathbb{E}_t\left[G_n\right]}{F(t, T_0, T)}\right) = \log\left(\frac{\mathbb{E}_t\left[G_n\right]}{\mathbb{E}_t^*\left[G_n\right]}\right).$$

In the case where $t < t_1$ term premium coefficient can be expressed as

$$\begin{aligned} R \;=\; & \frac{\lambda_\chi \sigma_\chi}{\kappa}\left(1 - e^{-\kappa(T-t)}\varphi(T_0, T, n)\right) \\ & + \lambda_\xi \sigma_\xi \left(T - t - \frac{(n-1)}{2}\Delta t\right) \end{aligned}$$

For the case where $t_1 \leq t \leq T$

$$R = \frac{\lambda_\chi \sigma_\chi}{\kappa}\left(\frac{n-i^*+1}{n} - \frac{1}{n}e^{-\kappa(t_{i^\star}-t)}\frac{1-e^{-(n-i^\star+1)\kappa\Delta t}}{1-e^{-\kappa\Delta t}}\right)$$
$$+\lambda_\xi \sigma_\xi \frac{n-i^*+1}{n}\left[(T_0-t)+\frac{(i^\star+n)}{2}\Delta t\right] \qquad (20)$$

where i^* is such that $t_{i^*-1} < t \leq t_{i^*}$.

If $n = 1$

$$R = \lambda_\chi \sigma_\chi \left(\frac{1-e^{-\kappa(T-t)}}{\kappa}\right) + \lambda_\xi \sigma_\xi \left(T-t\right).$$

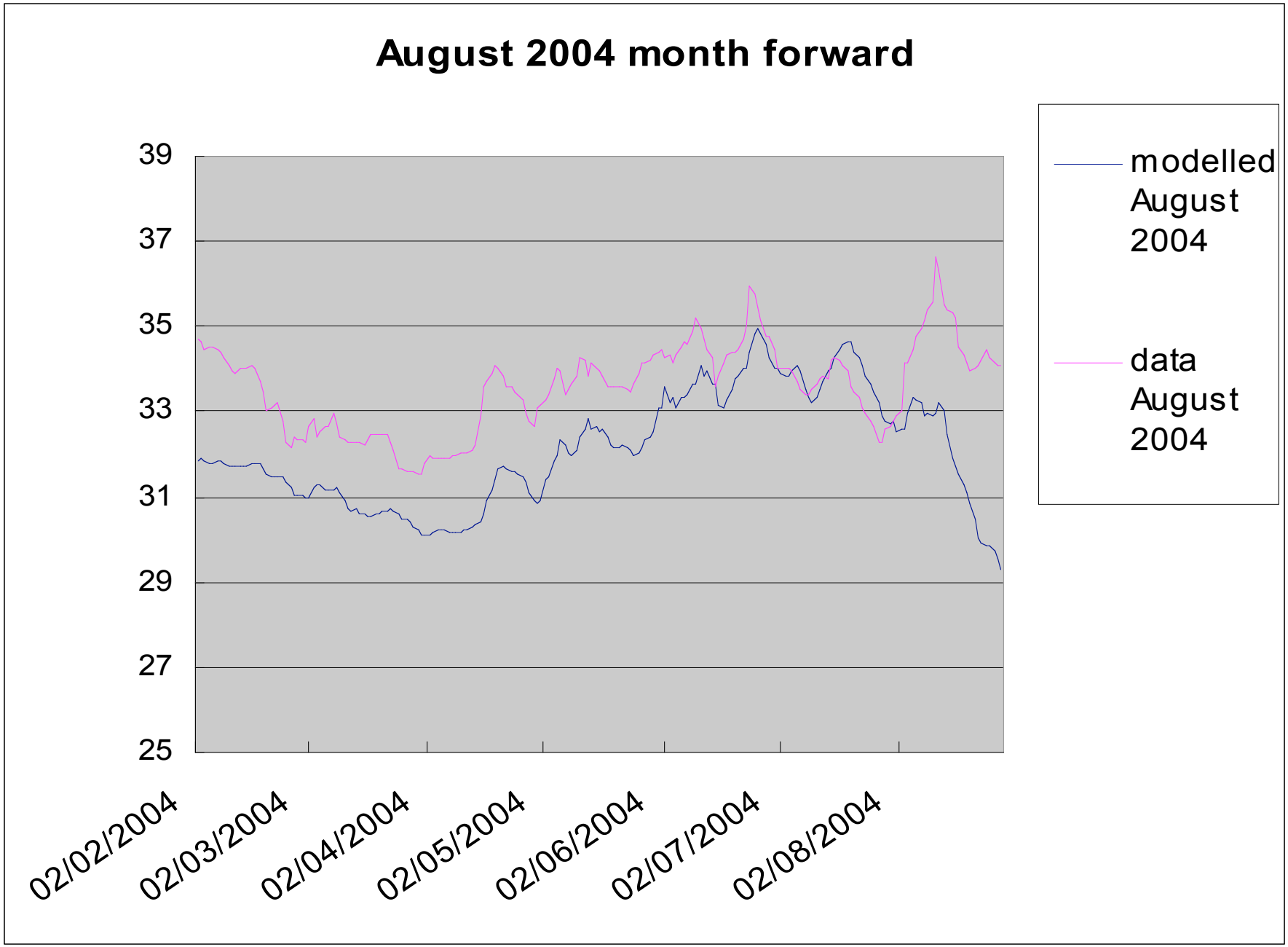
- Estimation of the model parameters

$(\kappa, \mu_\xi, \sigma_\chi, \sigma_\xi, \rho_{\chi\xi}, \lambda_\chi, \lambda_\xi)$
for month futures using Kalman filter and maximum likelihood

Calibration

- Simple estimation of the parameters leads to unrealistic sets of parameters which are also depended on starting values and boundaries for parameters estimation.
- Example:

κ	λ_χ	λ_ξ	σ_χ	σ_ξ	μ_ξ	ρ
3.99	-0.56	0.08	0.29	0.01	0.02	0.67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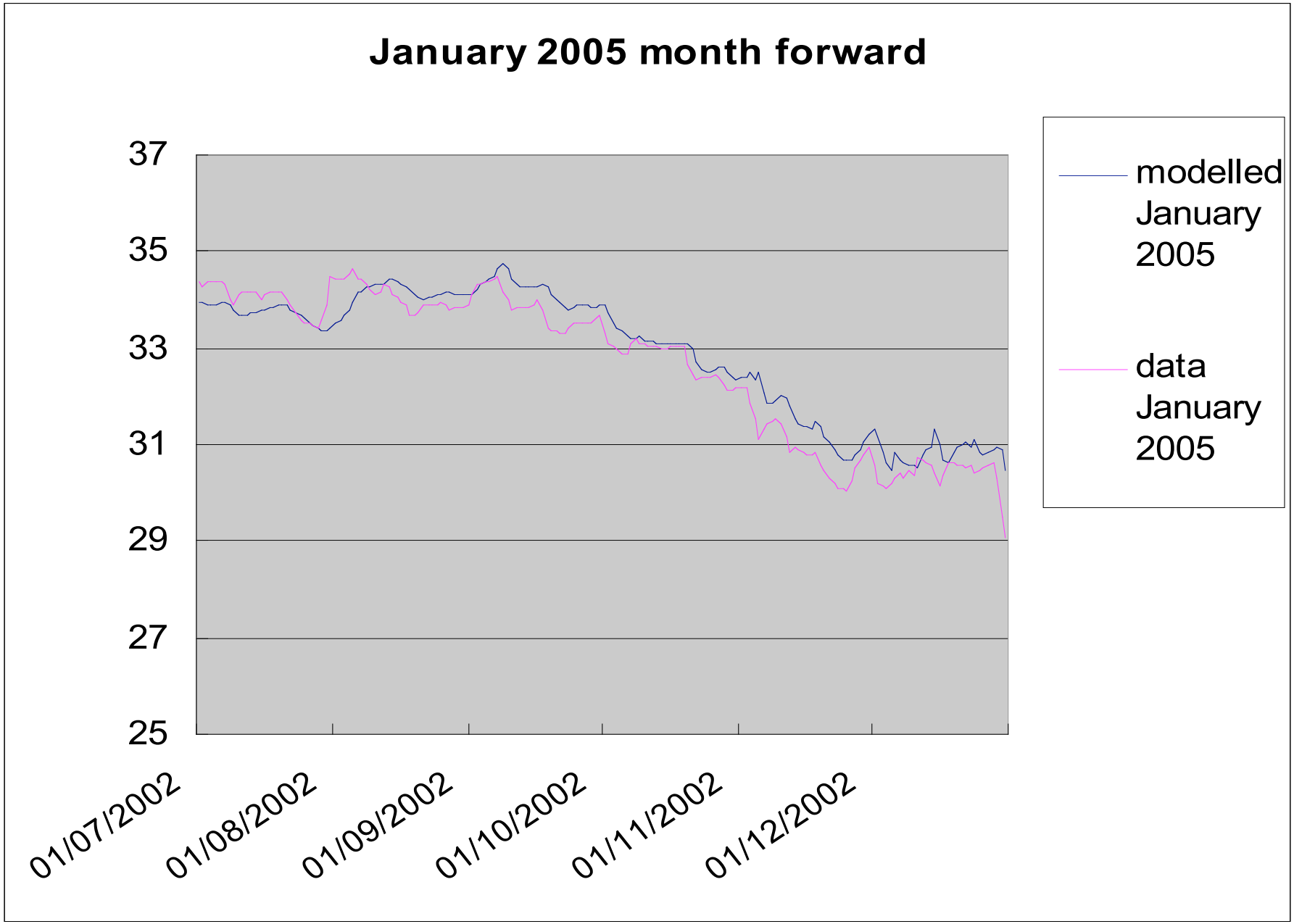
37
35
33
31
29
27
25
02/02/2004
02/03/2004
02/04/2004
02/05/2004
02/06/2004
02/07/2004
02/08/2004
modelled August 2004
data August 2004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35
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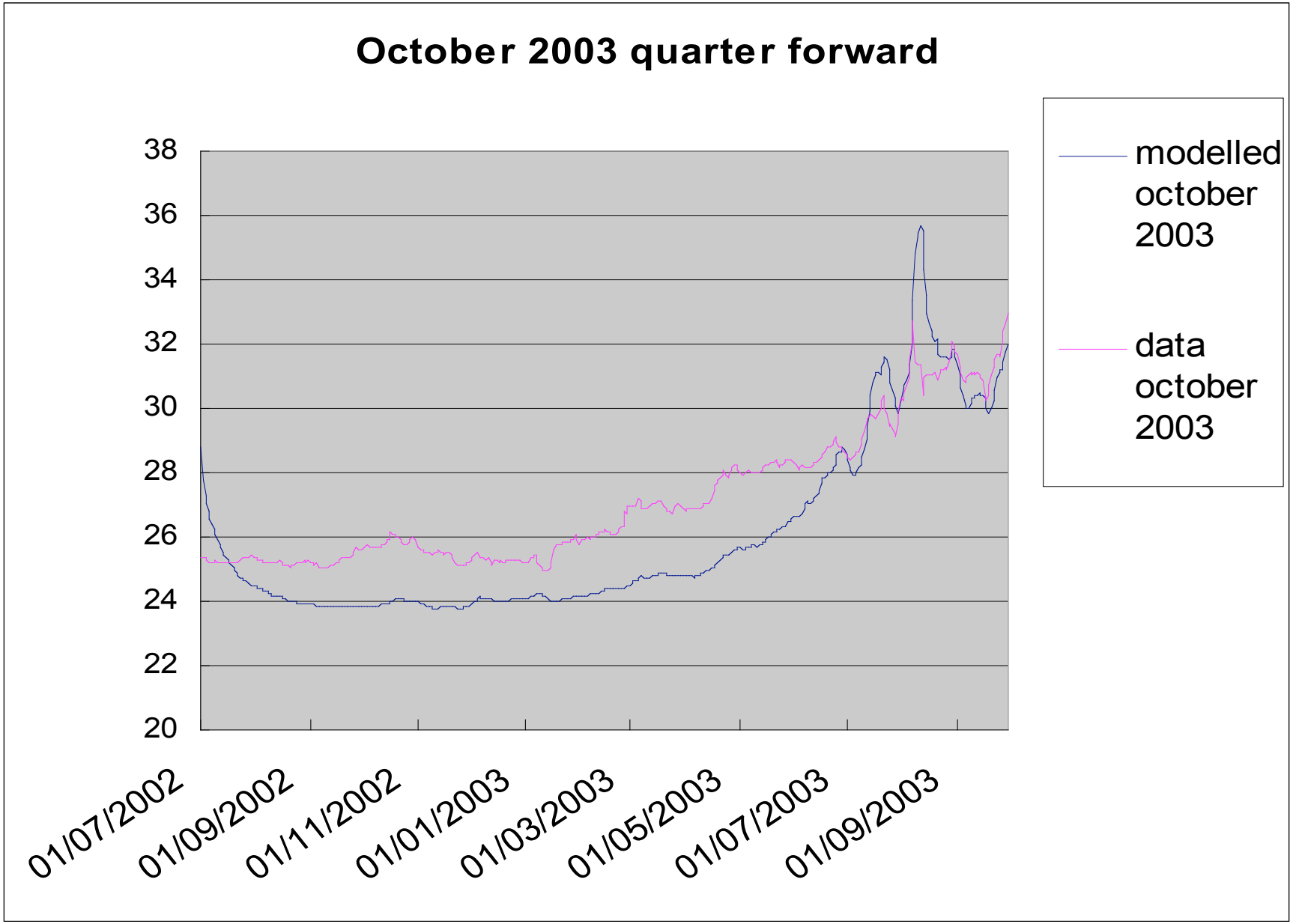
01/07/2002
01/08/2002
01/09/2002
01/10/2002
01/11/2002
01/12/2002
modelled January 2005
data January 2005


October 2003 quarter forward
38
36
34
32
30
28
26
24
22
20
01/07/2002
01/09/2002
01/11/2002
01/01/2003
01/03/2003
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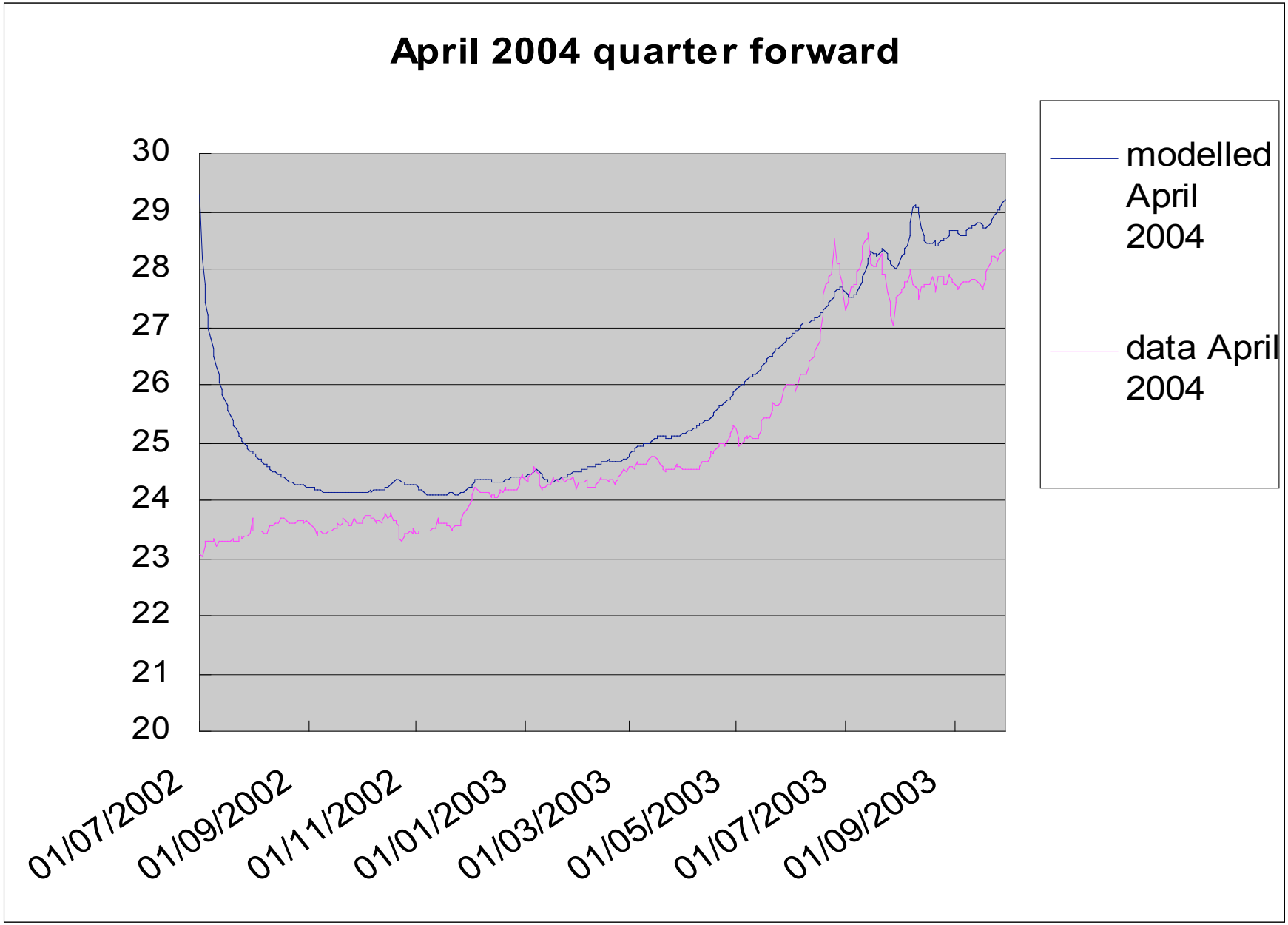
01/07/2003
01/09/2003
modelled october 2003
data october 2003


April 2004 quarter forward
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23
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21
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01/07/2002
01/09/2002
01/11/2002
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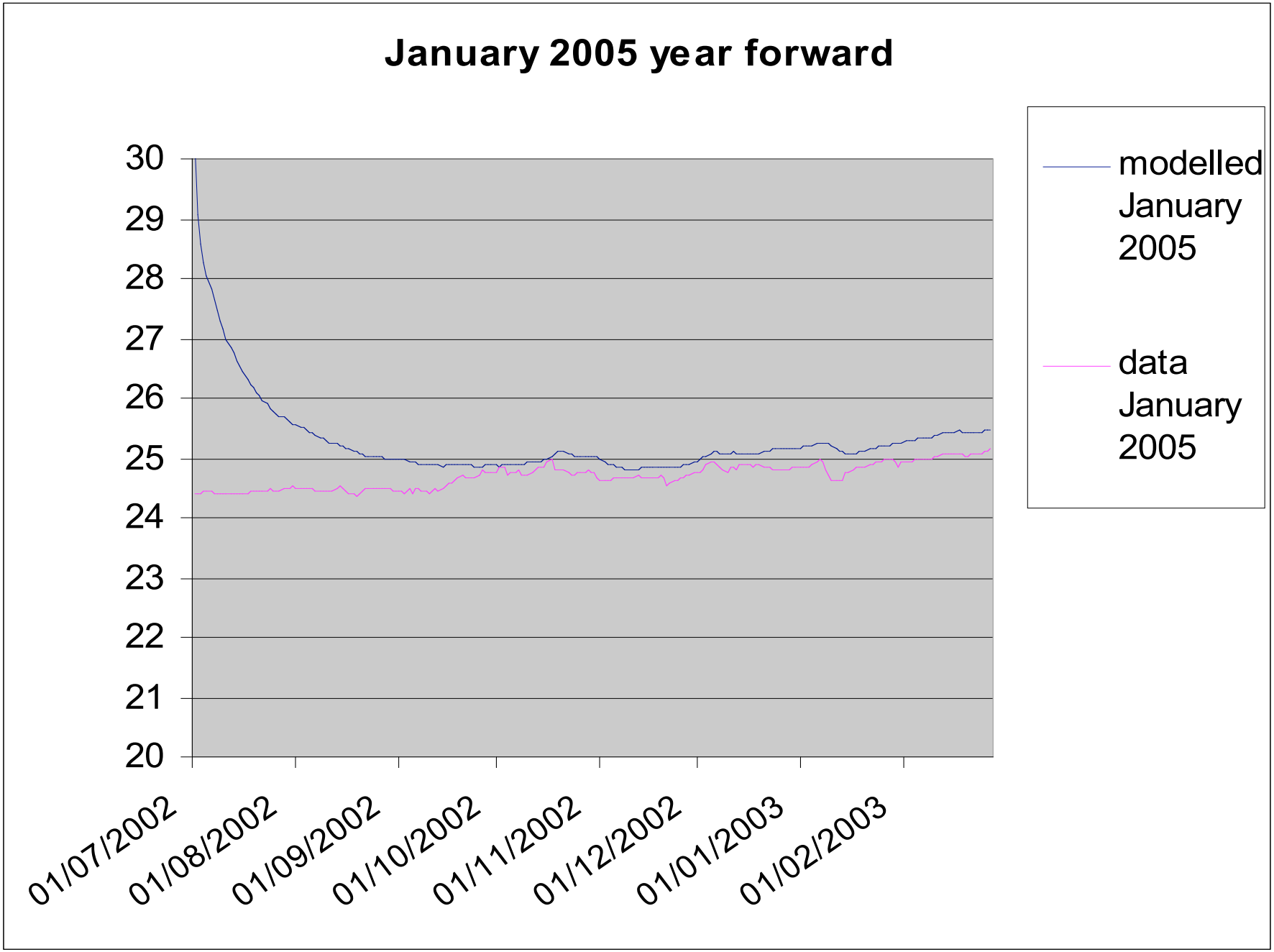
01/03/2003
01/05/2003
01/07/2003
01/09/2003
modelled April 2004
data April 2004


January 2005 year forward
30
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01/07/2002
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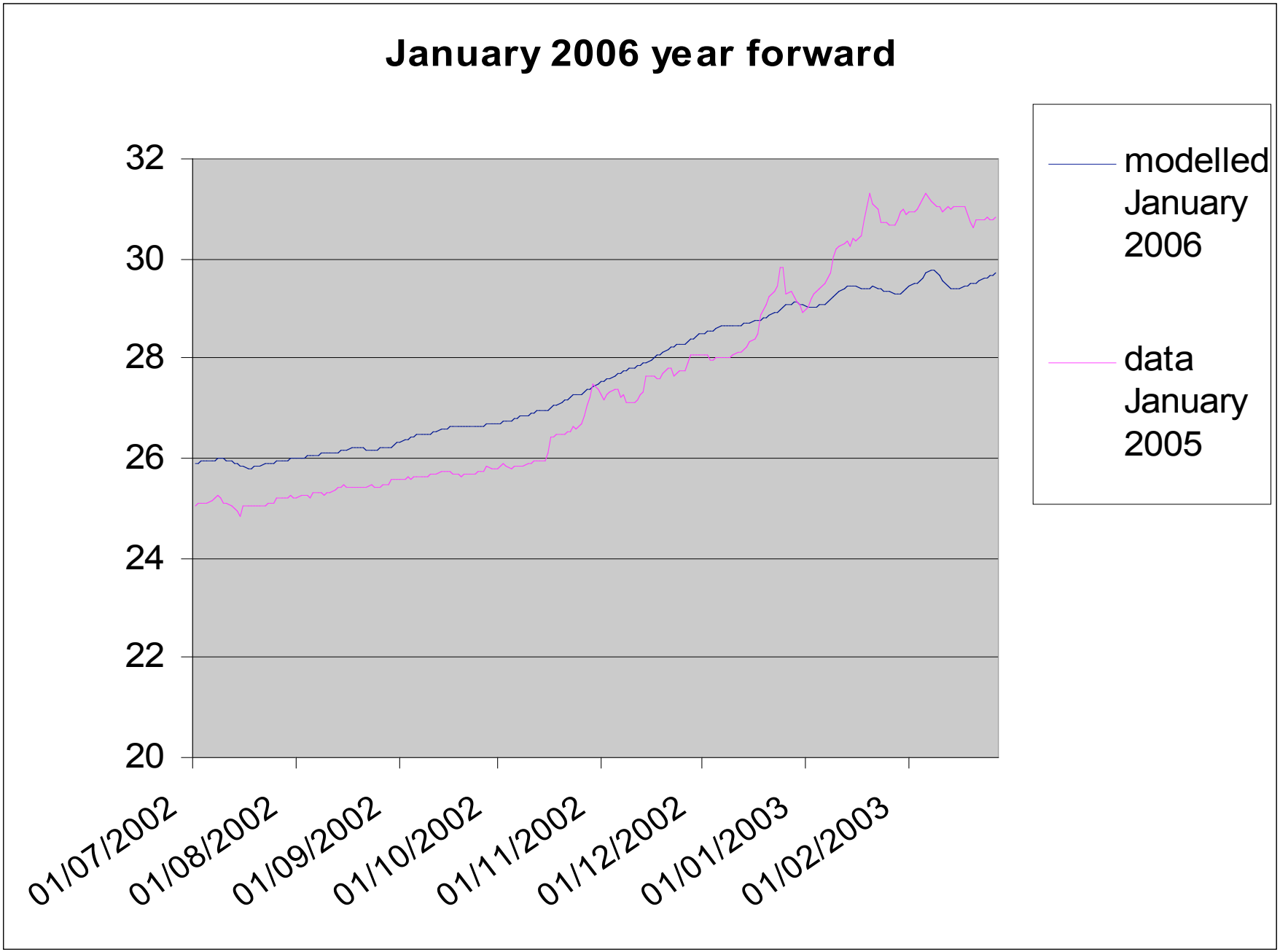
01/02/2003
modelled January 2005
data January 2005


January 2006 year forward
32
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01/08/2002
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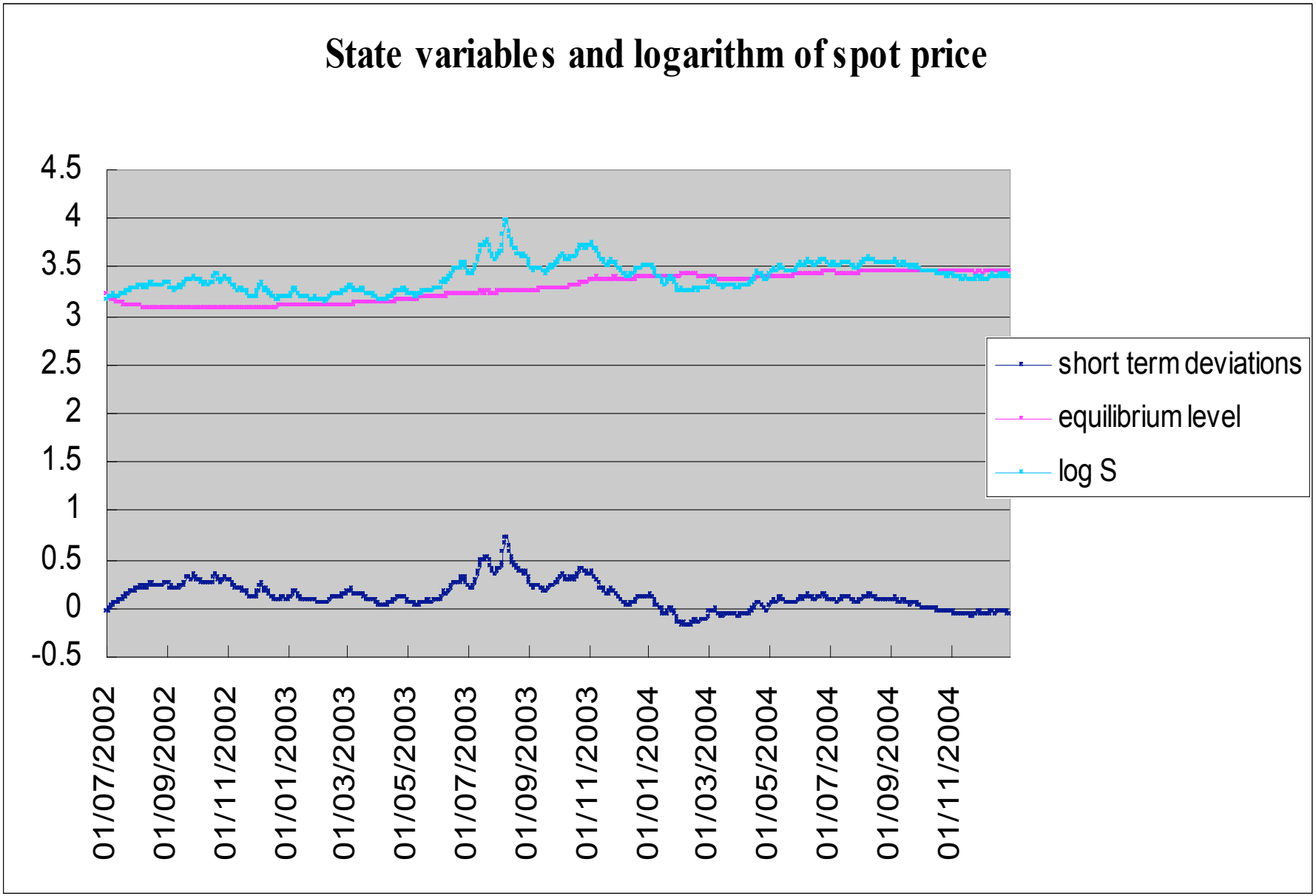
modelled January 2006
data January 2005


State variables and logarithm of spot price
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4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
01/07/2002
01/09/2002
01/11/2002
01/01/2003
01/03/2003
01/05/2003
01/07/2003
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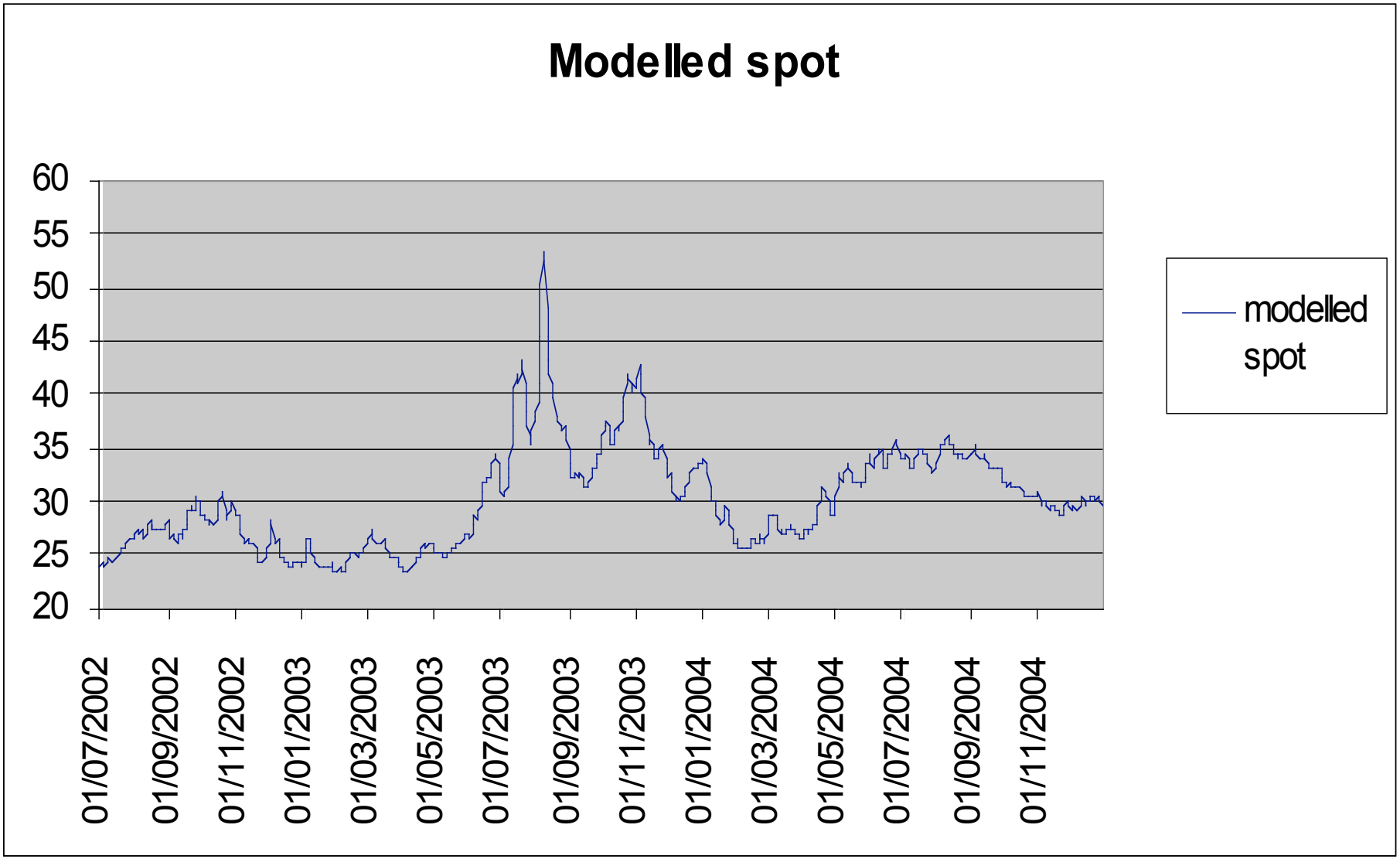
short term deviations
equilibrium level
log S


Modelled spot
60
55
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01/07/2002
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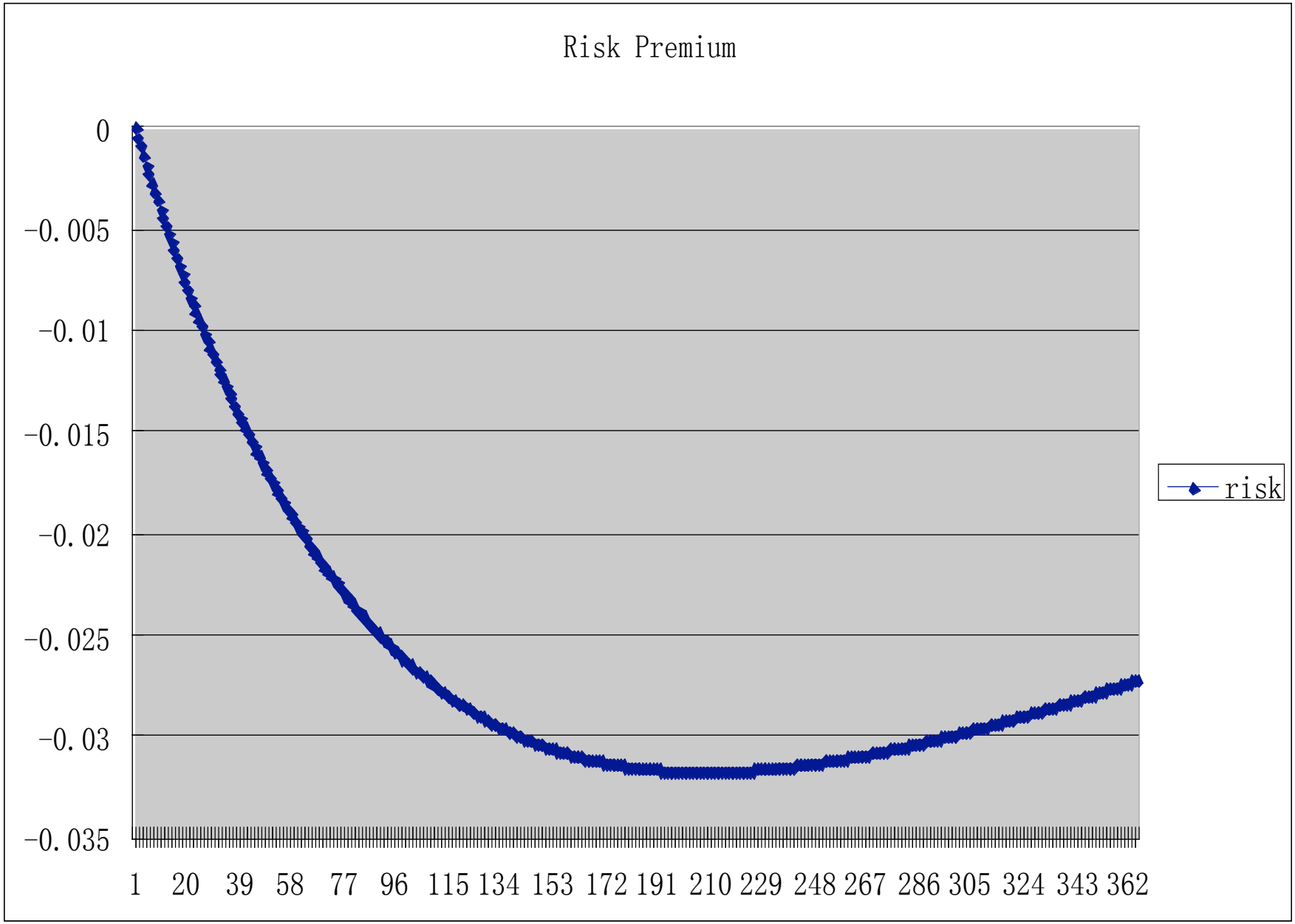
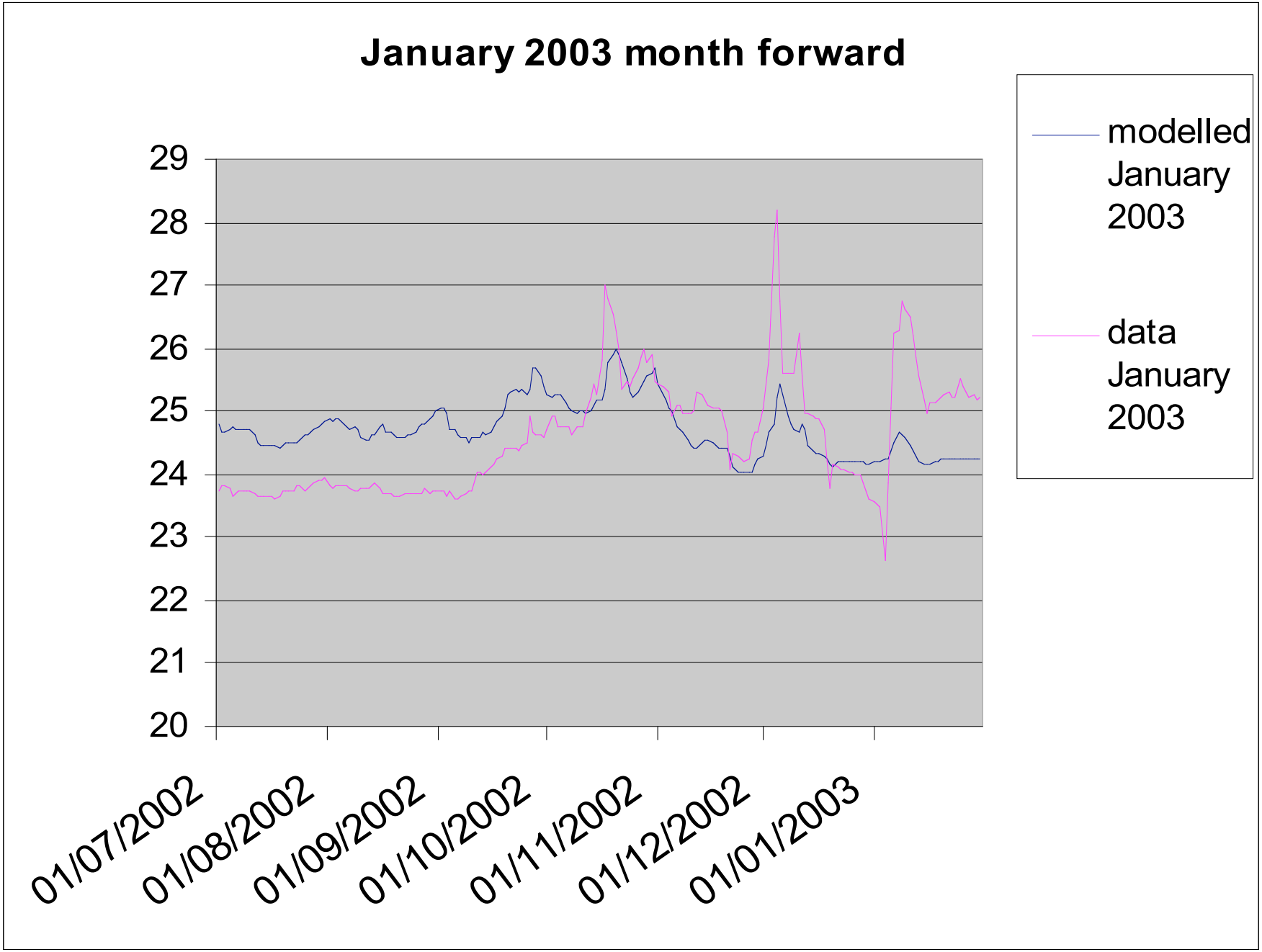
01/11/2004
modelled spot


Risk Premium
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
-0.035
1 20 39 58 77 96 115 134 153 172 191 210 229 248 267 286 305 324 343 362
risk

Calibration

- Now we first estimate κ from spot data (=0.427)
- Example:

κ	λ_χ	λ_ξ	σ_χ	σ_ξ	μ_ξ	ρ
0.43	0.60	0.58	0.34	0.04	0.06	0.70


January 2003 month forward
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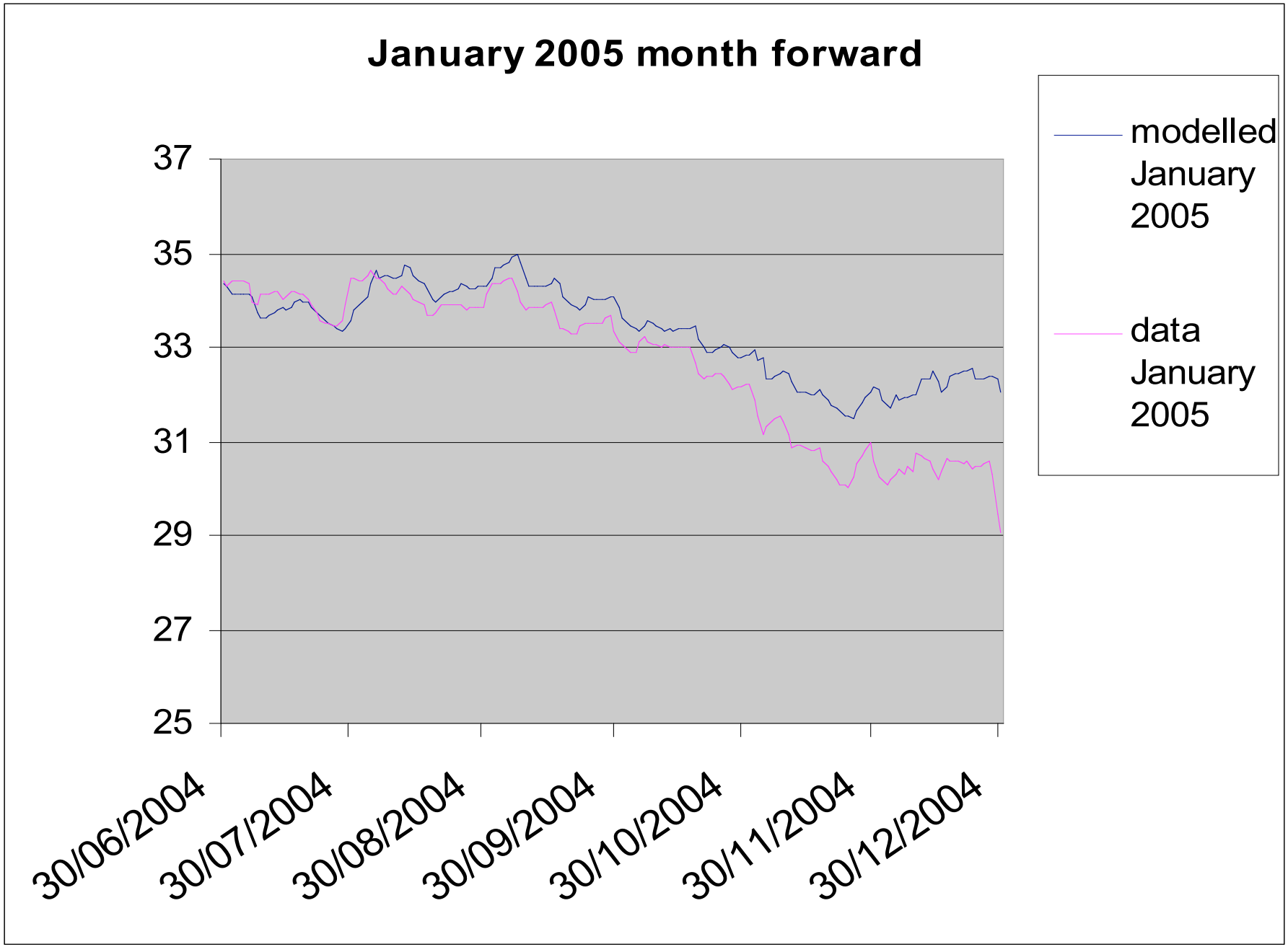
modelled January 2003
data January 2003


January 2005 month forward
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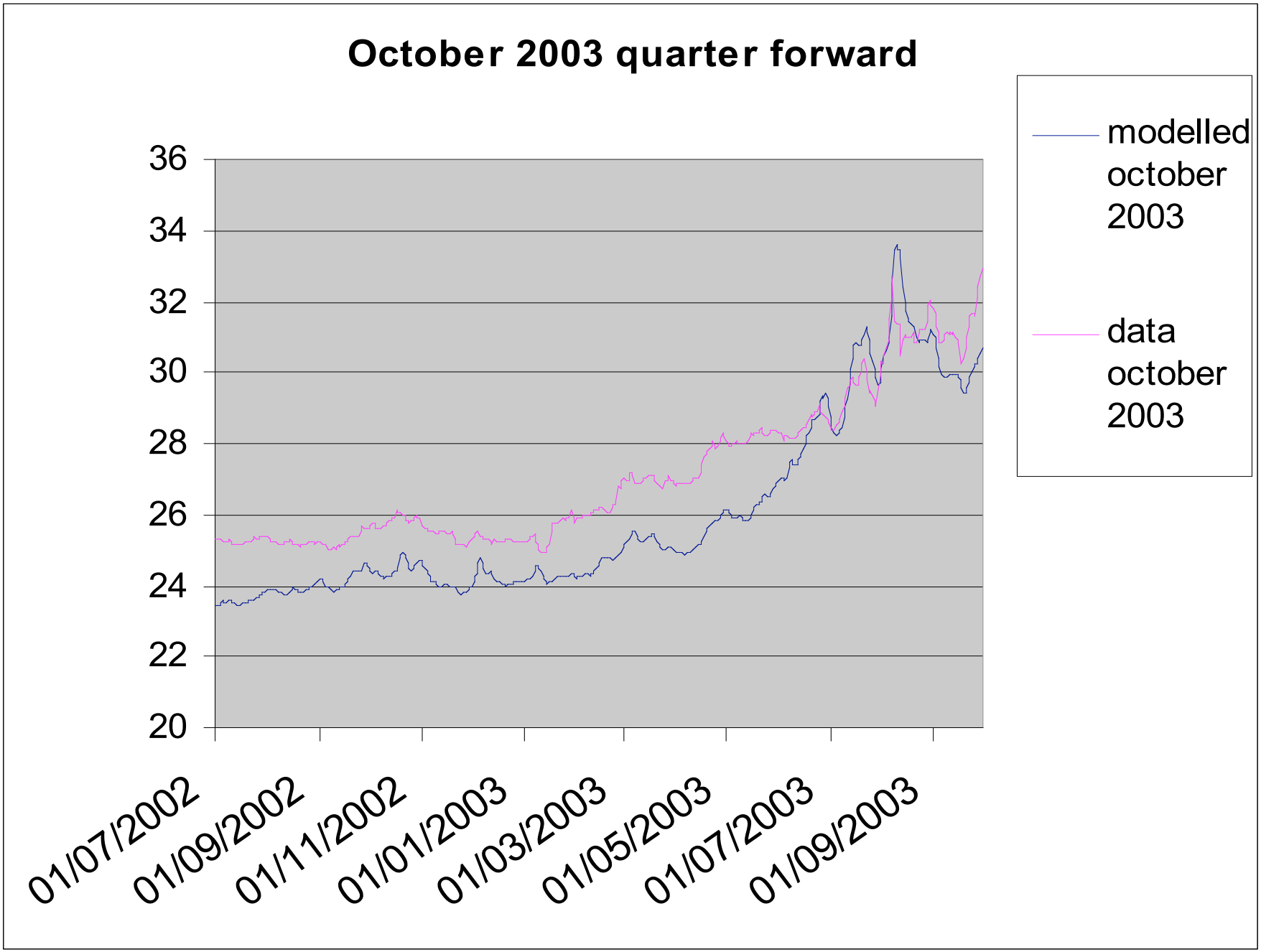
30/12/2004
modelled January 2005
data January 2005


October 2003 quarter forward
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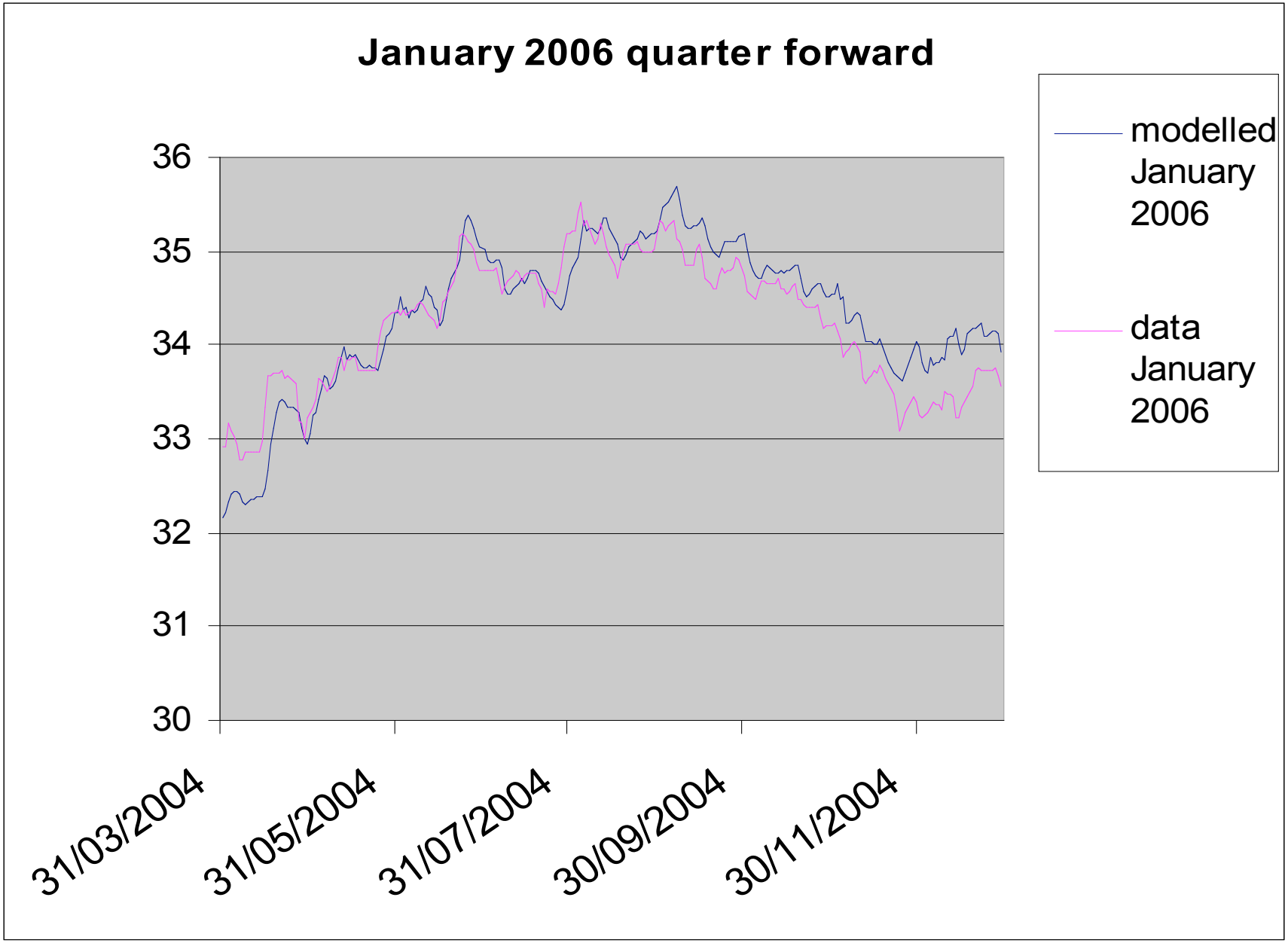
01/07/2003
01/09/2003
modelled october 2003
data october 2003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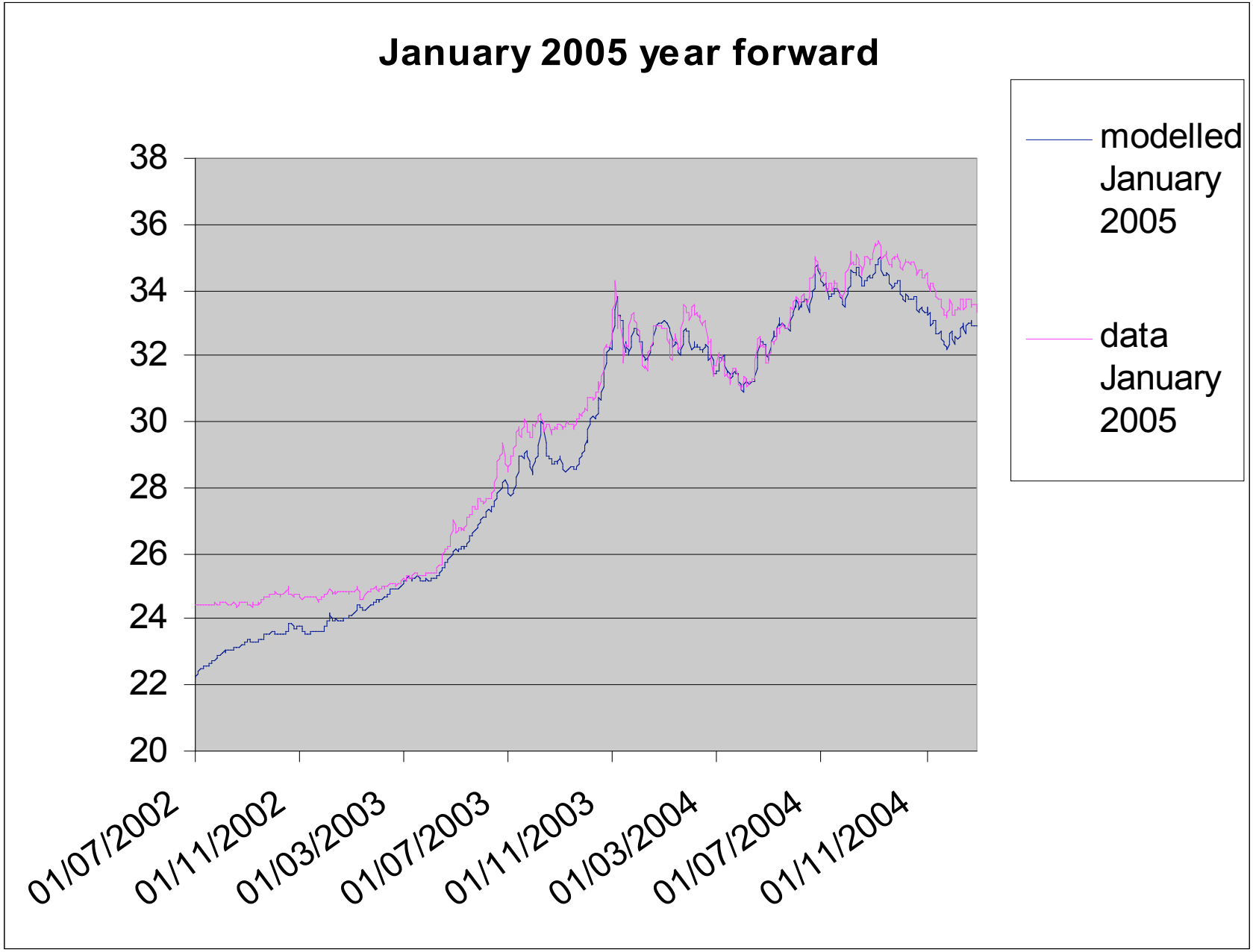
data January 2006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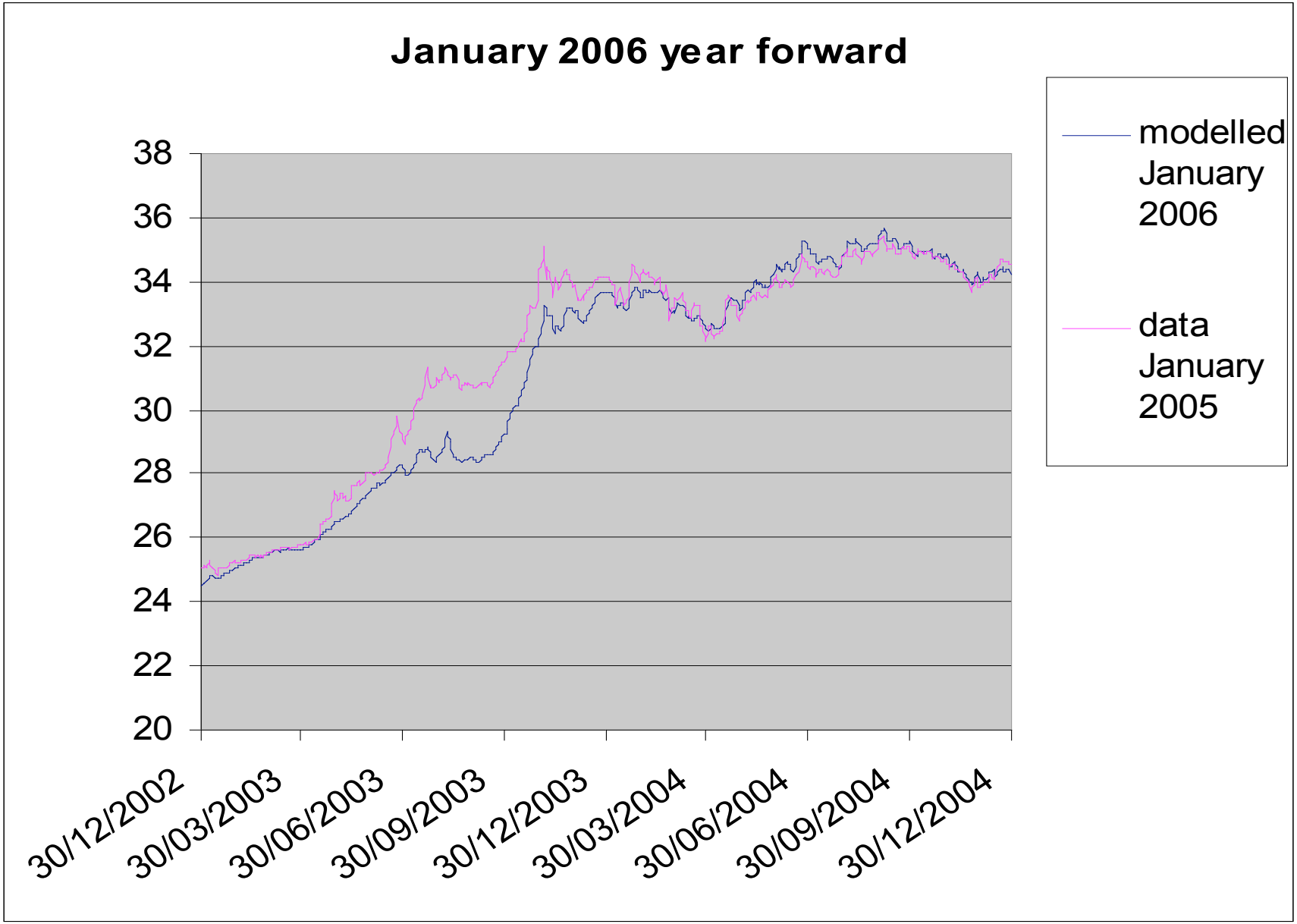
modelled January 2005
data January 2005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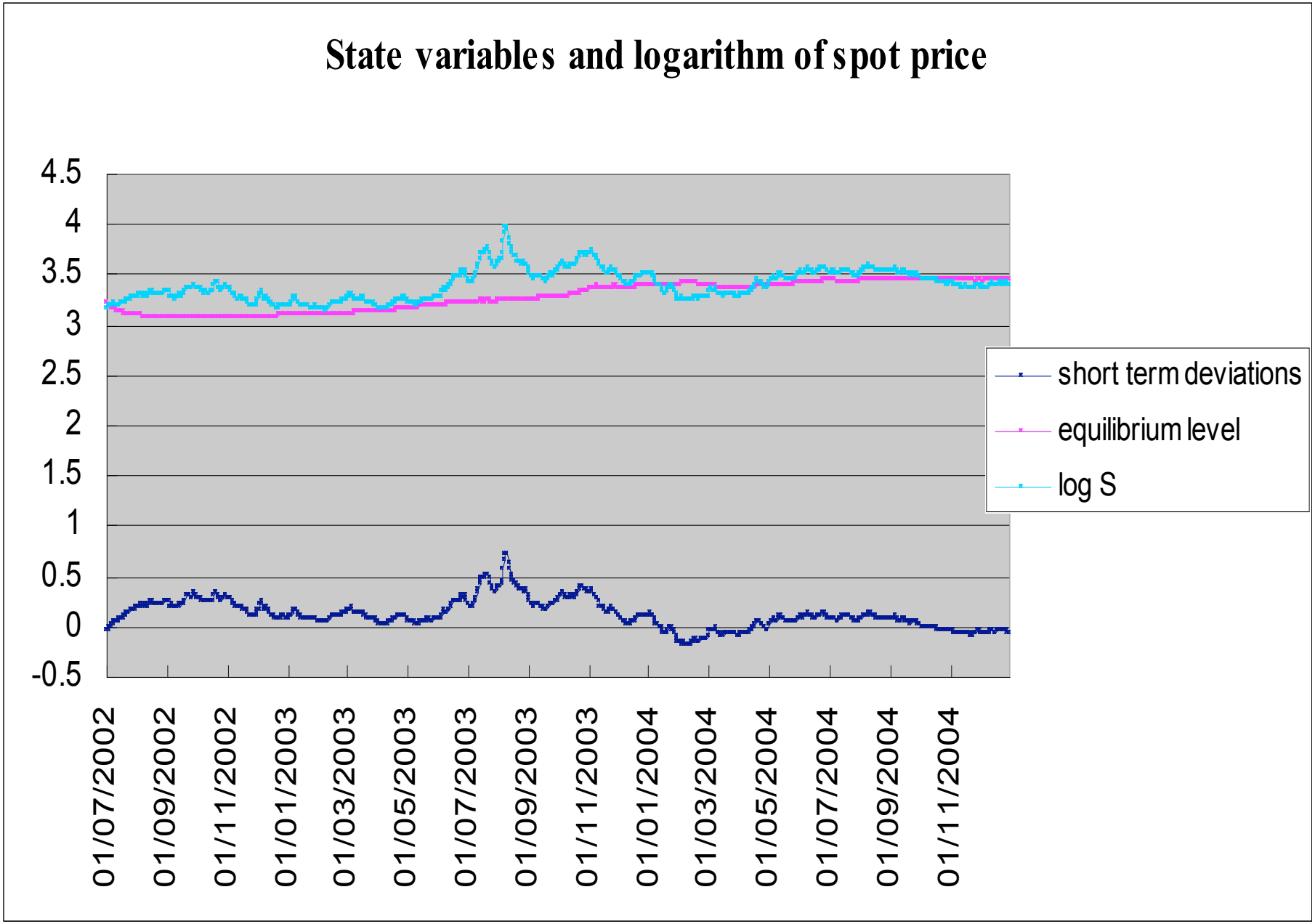
30/12/2004
modelled January 2006
data January 2005


State variables and logarithm of spot price
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01/07/2002
01/09/2002
01/11/2002
01/01/2003
01/03/2003
01/05/2003
01/07/2003
01/09/2003
01/11/2003
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01/03/2004
01/05/2004
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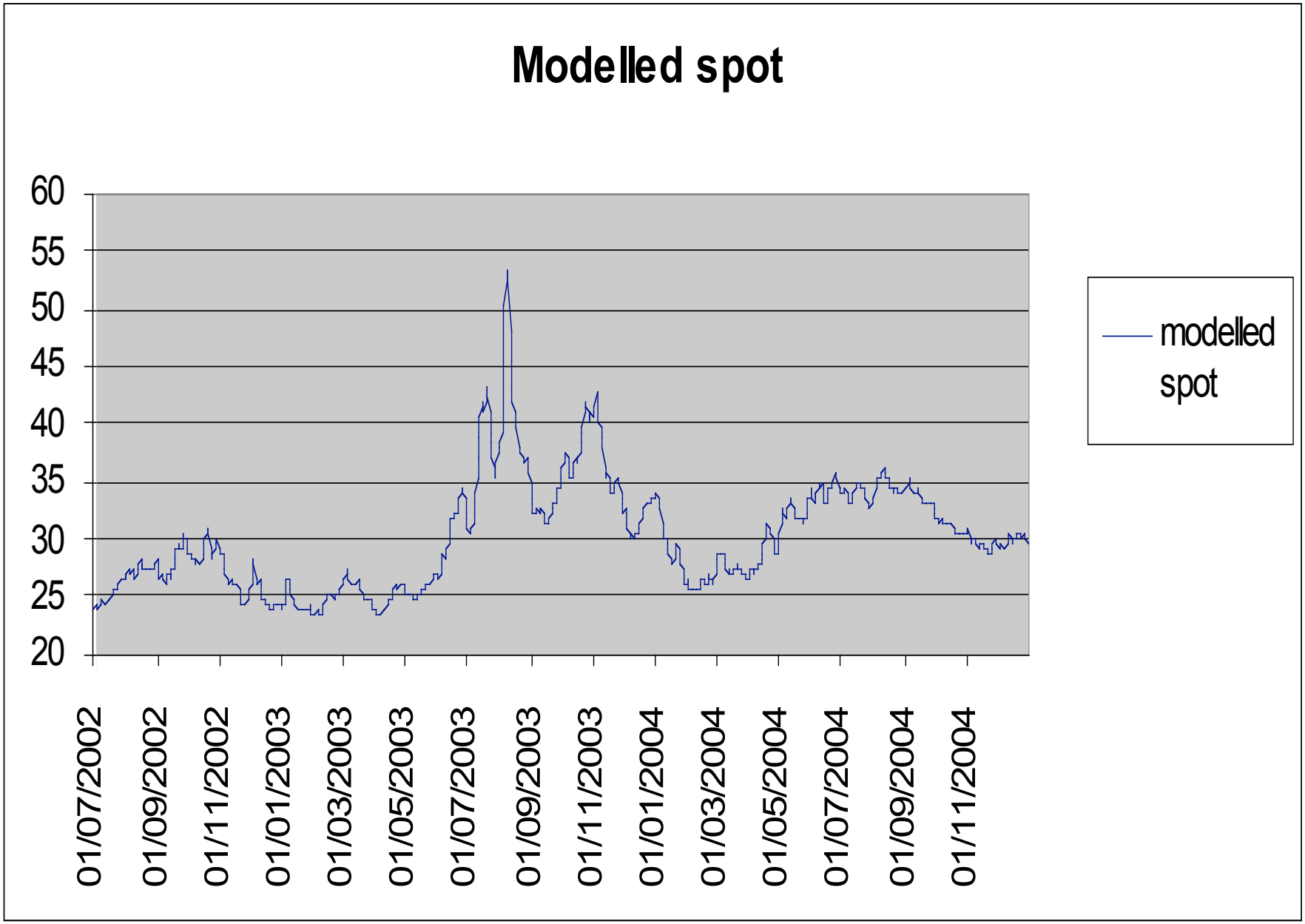
01/09/2004
01/11/2004
short term deviations
equilibrium level
log S


Modelled spot
60
55
50
45
40
35
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25
20
01/07/2002
01/09/2002
01/11/2002
01/01/2003
01/03/2003
01/05/2003
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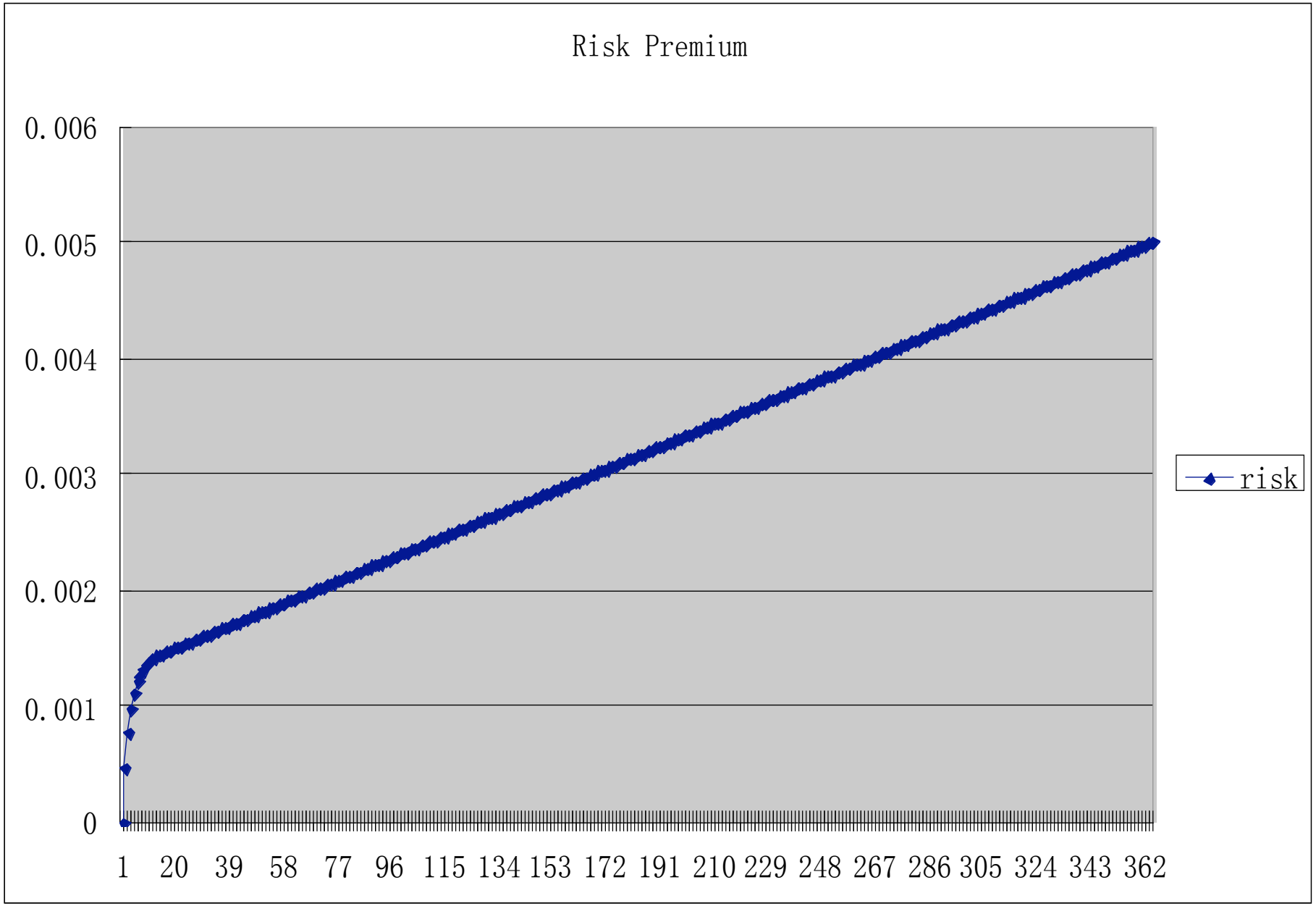
modelled spot


Risk Premium
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0.004
0.003
0.002
0.001
0
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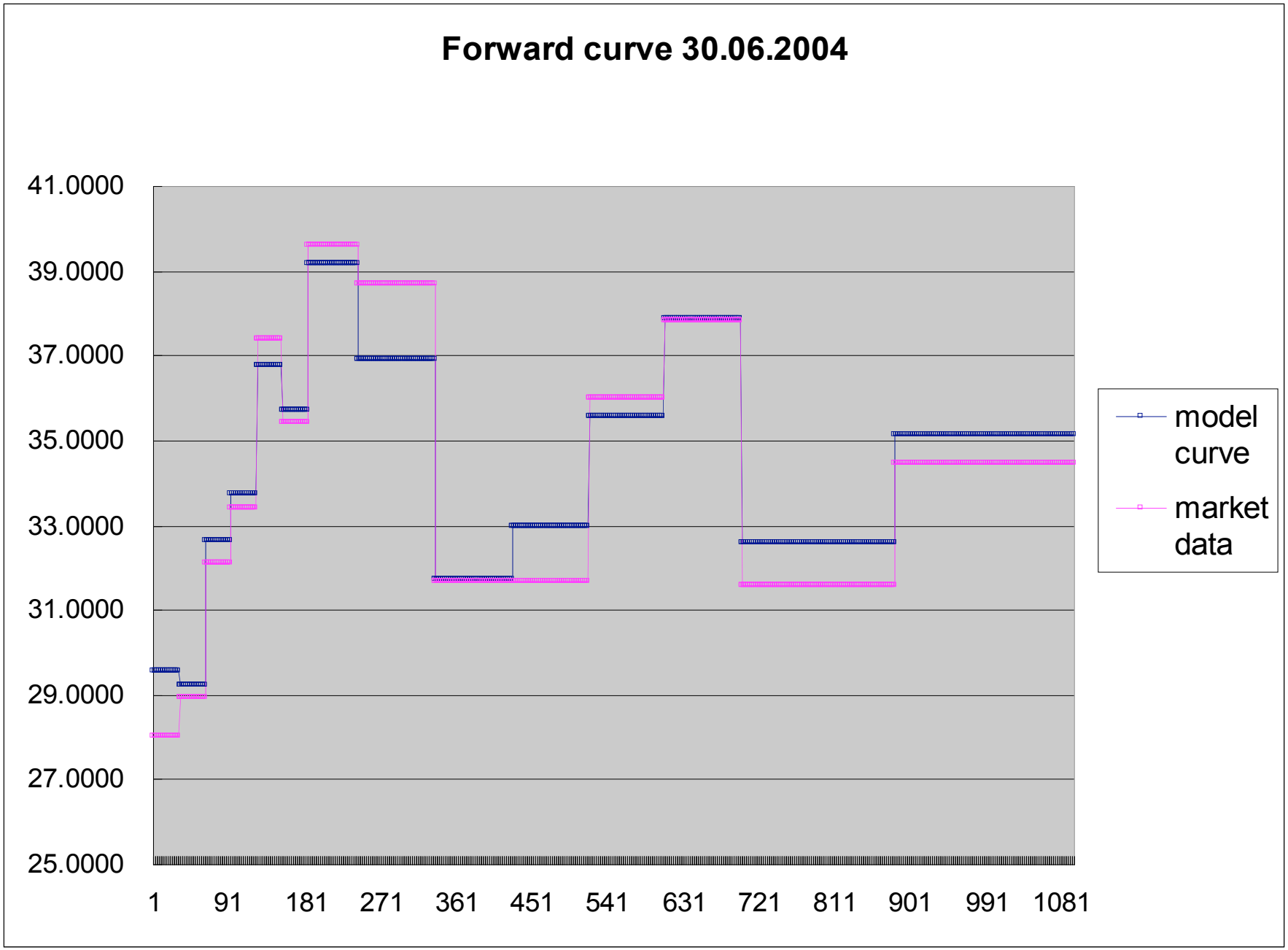
risk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37.0000
35.0000
33.0000
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27.0000
25.0000
1
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181
271
361
451
541
631
721
811
901
991
1081
model curve
market data

Further research

- Optimize estimation of model parameters: some parameters could be estimated a priory.
- Estimation of model parameters using different approach: for example simple calibration with minimizing squared errors (fitting to monthly futures)
- Forecasting
- Option pricing

Questions?

Suggestions?