Modeling the Spot Price of Electricity in Deregulated Energy Markets

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Outline

- Empirical Analysis of Electricity Spot Prices
  - Physical *vs.* financial assets
  - Trajectorial *vs.* statistical properties of market prices
  - Endogenous factors
- A Jump-Diffusion Model
  - Desirable features
  - Modules: trend, noise, and spikes
  - The model
- Model Calibration
  - Step 1: Fitting structural elements
  - Step 2: Parameter estimation
  - Empirical results

Empirical Analysis of Electricity Spot Prices

I. Introduction

Market context Deregulation of energy market $\longrightarrow$ price fluctuation $\longrightarrow$ new typology of risk $\longrightarrow$ hedging needed

Issue Determine and quantify these relations

Method Empirical analysis $\longrightarrow$ modelling (qualifying risk) $\longrightarrow$ calibration (quantifying risk) $\longrightarrow$ test (performance analysis)

II. A Special Underlying

Arbitrage pricing = derivative price is the minimal capital required to set up a self-financing hedging portfolio of tradeable assets

$\rightarrow$ Hypothesis = the underlying is transferable in time (at a cost = interest rate) and space (at some transaction cost).

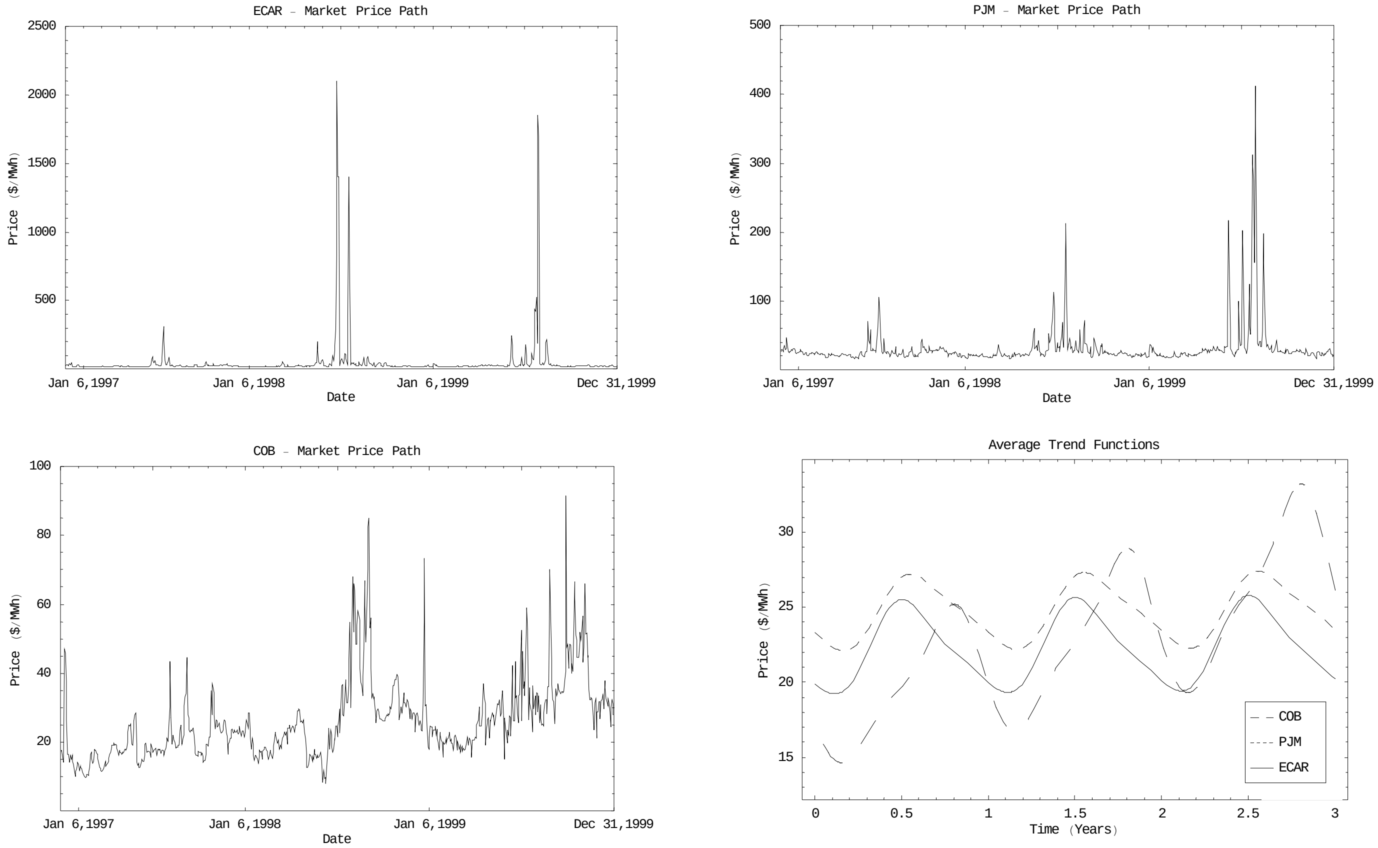
Electricity

- Limited space transferability (capacity constraints, line losses, physical market segmentation).

- Almost impossible time transferability (power cannot be easily stored).

$\Rightarrow$ singular price dynamics $\rightarrow$ new typology of nondiversifiable risk.

III. Trajectorial Properties of Market Prices



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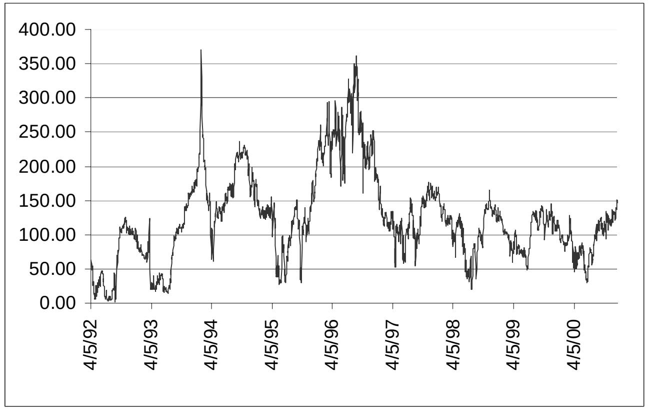
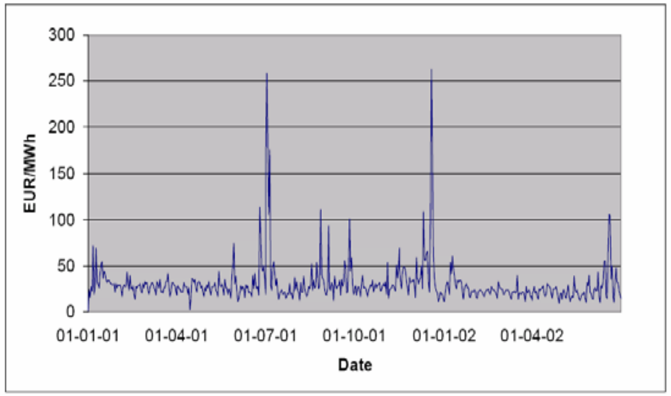




1) Drift	$=$	Periodical trend $+$ Mean reversion
2) Volatility	$=$	Local perturbations
3) Spikes	$=$	Periodical occurrence $+$ Jump reversion

$Warning$: spike $=$ sequence of upward jumps followed by downward jumps $\neq$ two jumps with opposite sign.

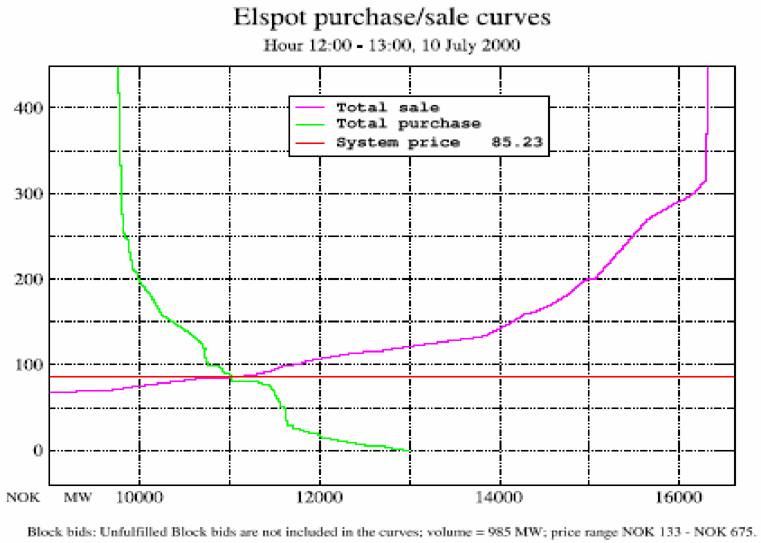
IV. Statistical Properties of Market Prices

- Daily price return of 1 MWh.
- Empirical distribution: January,6 1997 - December, 30 1999.

	ECAR	PJM	COB	NP	APX
Mean	-0.000	-0.001	0.001	0.000	0.000
Std.Dev.	0.353	0.236	0.159	0.099	0.359
Skew.	-0.557	0.395	0.159	0.461	-0.17
Kurt.Ex.	21.683	13.151	6.771	14.397	4.60

V. Driving Factors

Market: Inelastic demand function



$\longrightarrow$ Public service: Producers $\overset{\text{floating}}{\longrightarrow}$ Utilities $\overset{\text{fixed}}{\longrightarrow}$ Consumers

V. Driving Factors

Economy: Real economy growth, generation asset prices

Weather: Hot waves, cold winters

Physical: Grid balancing $\rightarrow$ marginal prices; transmission constraints $\rightarrow$ line congestion; production system:

- thermal (PJM) $\rightarrow$ non storable $\rightarrow$ capacity *shortfalls* due to demand excess

- hydro (WSCC) $\rightarrow$ stable $\rightarrow$ *shortfalls* due to outages

A Jump-Diffusion Model

I. Desirable Features: A Model Should be...

Representative and Flexible (fit trajectorial and statistical properties across different markets + embed all risk factors)

Tractable (Markovian, reduced-form) → Valuation and hedging

Easily implementable → Scenario simulation

Statistically stable → Robustness of hedging prescriptions

Simple → Acceptance by market operators

Tested with respect to these criteria

II. Trend and Local Volatility

Variable

$$E(t) := \log(\text{Spot Price } (t))$$

Trend

$$\boxed{\mu(t) = \alpha + \beta t + \gamma \cos[\varepsilon + 2\pi t] + \delta \cos[\zeta + 4\pi t]}$$

Mean reversion

$$\boxed{dE(t) = D\mu(t)\,dt + \theta_1[\mu(t) - E(t^-)]\,dt + ...}$$

Local shock

$$\boxed{... + \sigma dW(t) + ...}$$

III. Spikes

Jump component

$$\boxed{... + h\left(E\left(t^{-}\right)\right) dJ\left(t\right)}$$

- $\triangleright$ Jump sign $\rightarrow h\left(t^{-}\right) = 1$, if $E\left(t^{-}\right) < \mu\left(t\right) + \Delta$; -1 otherwise.
- $\triangleright$ Jump size $\rightarrow J_i \overset{i.i.d.}{\sim} p\left(x; \theta_3, \psi\right) \propto e^{\theta_3 f(x)}$, $0 \leq x \leq \psi$.
- $\triangleright$ Jump occurrence $\rightarrow N\left(t\right)$ counting process with freq. $\iota\left(t\right) = \theta_2 \times s\left(t\right)$.
- $\triangleright$ Jump frequency shape $\rightarrow s\left(t\right)$.
- $\triangleright$ Cumulative jump size $\rightarrow J\left(t\right) = \sum_{i=1}^{N(t)} J_i$.

IV. The Model

Dynamics

$$dE(t) = \mu'(t)\,dt + \theta_1 \left[\mu(t) - E\left(t^-\right)\right] dt + \sigma dW(t) + h\left(E\left(t^-\right)\right) dJ$$

Input parameters

Structural elements (market specific)		Parameters (time series specific)	
$\mu(t)$	Mean trend	σ	Local volatility
$s(t)$	Jump frequency shape	θ_1	Mean reversion force
Δ	Jump direction switch	θ_2	Jump frequency magnitude
$p(x;\theta_3)$	Jump size distribution	θ_3	Jump size parameter

Model Calibration

I. Fitting Structural Elements

Trend $\rightarrow \mu(t) = \alpha + \beta t + \gamma \cos[\varepsilon + 2\pi t] + \delta \cos[\zeta + 4\pi t]$

Jump frequency shape $\rightarrow s(t) = \left(\frac{2}{1+|\sin[\pi(t-\tau)/k]|} - 1\right)^2$

- $\tau =$ max. freq. date (summer peak $\rightarrow \tau = 0.5$)

- $k =$ period (annual periodicity $\rightarrow k = 1$)

Jump direction switch: $\Delta = \alpha$-quantile of detrended prices

Jump size distribution: $p(x; \theta_3) =$ truncated exponential

II. Parameter Estimation

Idea

$$
\begin{array}{lcl}
\text{Continuous time process } X & & \\
\quad\big\downarrow \text{Discretization} & & \\
\text{Discrete time approxim. } X^n & \overset{\text{Transition}}{\longrightarrow} & \text{Exact Likelihood (approx.proc.) } \mathcal{L}_{X^n}
\end{array}
$$

$$
\begin{array}{lcl}
\text{Continuous time process } X & \overset{\text{Girsanov}}{\longrightarrow} & \text{Exact Likelihood (cont.observ.) } \mathcal{L} \\
 & & \quad\big\downarrow \text{Piecewise constant obs.} \\
 & & \text{Approx. Likelihood (discr. observ.) } \mathcal{L}_X^n
\end{array}
$$

Data disentangling $\Delta E(t) \to \Delta E^c(t) + \Delta E^d(t)$

II. Parameter Estimation

Local volatility $\sigma \rightarrow$ modified local covariance estimator

$$\sigma = \sqrt{\sum_{i=0}^{n-1} \left(\Delta E^c\left(t_i\right) - \left|\theta_1 \times \left(\mu\left(t_i\right) - E\left(t_i\right)\right)\right|\right)^2}$$

Parameters $\theta_1, \theta_2, \theta_3 \rightarrow$ new estimator

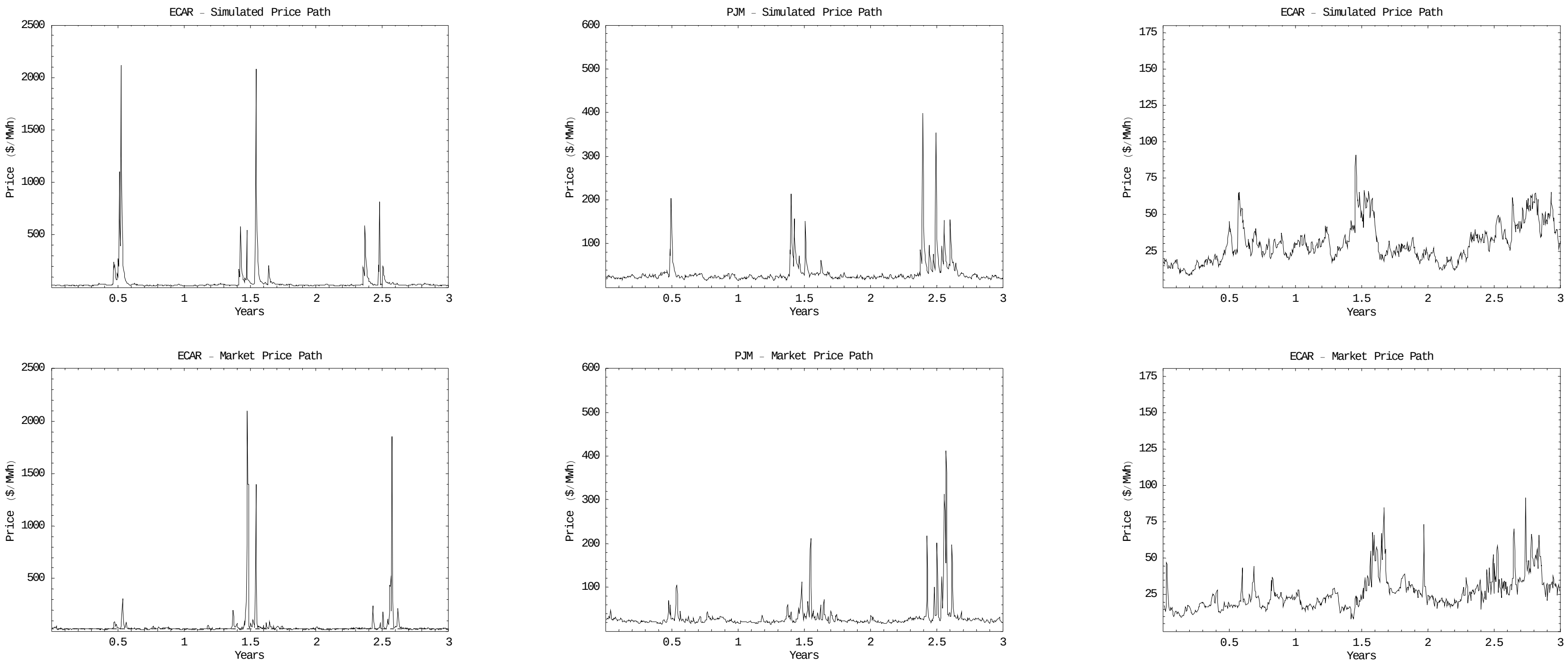
$$\mathcal{L}_{\boldsymbol{\theta}^0,\mathbf{E}}\left(\boldsymbol{\theta}\right) = \sum_{i=0}^{n-1} \frac{(\mu(t_i)-E_i)\theta_1}{\sigma^2}\left[\Delta E_i^c - \mu'\left(t_i\right)\Delta t\right] - \frac{\Delta t}{2}\sum_{i=0}^{n-1}\left(\frac{(\mu(t_i)-E_i)\theta_1}{\sigma}\right)^2$$

$$-\left(\theta_2 - 1\right)\sum_{i=0}^{n-1} s\left(t_i\right)\Delta t + \lg\theta_2 N\left(t\right)$$

$$+\sum_{i=0}^{n-1}\left[-\left(\theta_3 - 1\right)\frac{\Delta E_i^d}{h(E_i)}\right] + N\left(t\right)\lg\left(\frac{1-e^{-\theta_3\psi}}{\theta_3\left(1-e^{-\psi}\right)}\right)$$

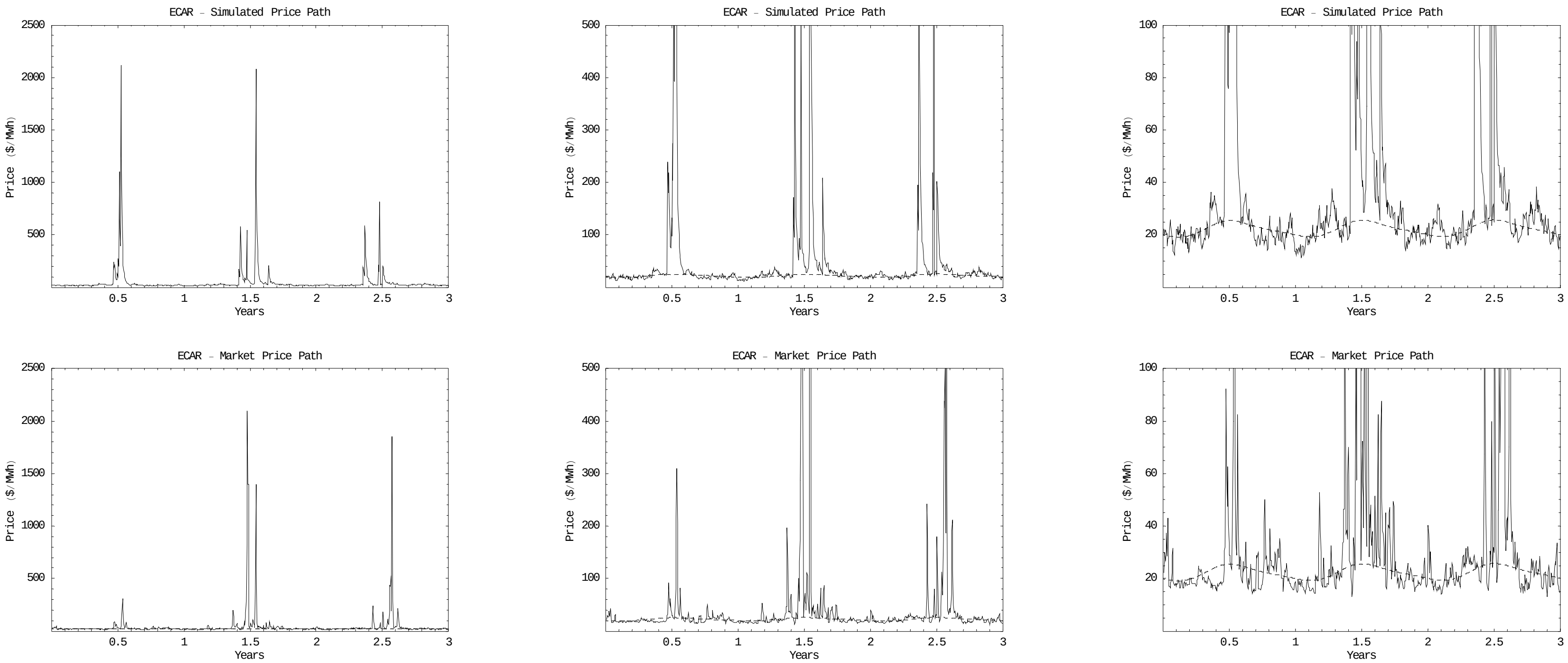
III. Empirical Results: Price Paths

Trajectorial properties (ECAR, PJM, COB)



III. Empirical Results: Price Paths

Trajectorial properties (ECAR market under varying resolution)



IV. Empirical Results: Statistics

	ECAR		PJM		COB	
	EMP	SIMUL	EMP	SIMUL	EMP	SIMUL
Average	-0.0002	-0.0001	-0.0006	0.0000	0.0009	0.0006
Std. Dev.	0.3531	0.3382	0.2364	0.2305	0.1586	0.1121
Skewness	-0.5575	2.1686	0.3949	1.6536	0.1587	0.9610
Kurtosis	21.6833	22.5825	13.1507	14.8429	6.7706	6.5402

V. Comparative Analysis of Alternative Model Specifications

Reduction: upward jumps only

Extension: price dependent jump frequency (random intensity)

	ECAR	**Up-jump** (det. freq.)	**Sgn-jump** (det. freq.)	**Sgn-jump** (random freq.)
Average	-0.0002	0.0000	-0.0001	-0.0000
Std. Dev.	0.3531	1.3238	0.3382	0.37821
Skewness	-0.5575	3.5688	2.1686	-0.0119
Kurtosis	21.6833	8.3542	22.5825	28.0288

VI. Conclusion: our class of models ...

(1) Matches both trajectorial and statistical properties of price dynamics

(2) Fits all markets

(3) Embeds risk factors (noise, spikes)

(4) Reproduces forecastable trends (drift, periodical jumps)

(5) Is Markovian

(6) Can be easily estimated and simulated

References

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Andrea Roncoroni is Assistant Professor of Finance at ESSEC Business School (France). He is regular lecturer at University Paris Dauphine, Bocconi University, and the Italian Stock Exchange. He holds PhD's in Applied Mathematics and Finance. His research interests cover quantitative modeling and risk management in energy markets. He has consulted for private companies (Gaz de France, Fideuram Capital Asset Management) and public institutions (International Energy Agency, Italian Authority for Electricity and Gas). He is author of the book "Implementing Models in Quantitative Finance: Methods and Cases" (with G. Fusai), Springer-Verlag, forthcoming 2006.

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