Modelling Electricity Spot Prices

A Regime-Switching Approach

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Energie
braucht Impulse

Agenda

- Model Overview
- Daily Price Process
- Hourly Profile Process
- Backtesting
- Applications
- Outlook

Electricity Spot Prices

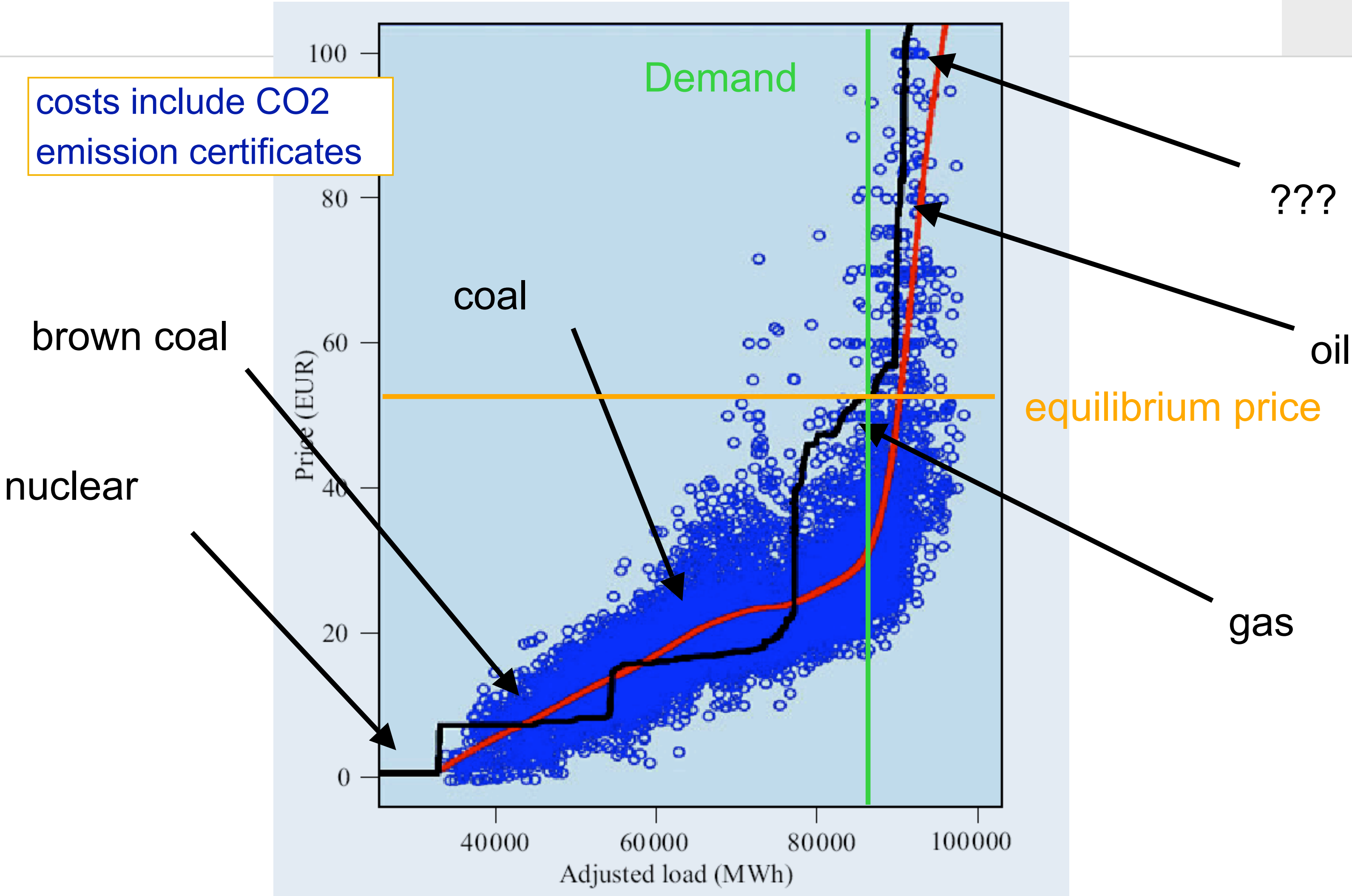
Features

- seasonality (yearly, weekly, daily)
- spikes

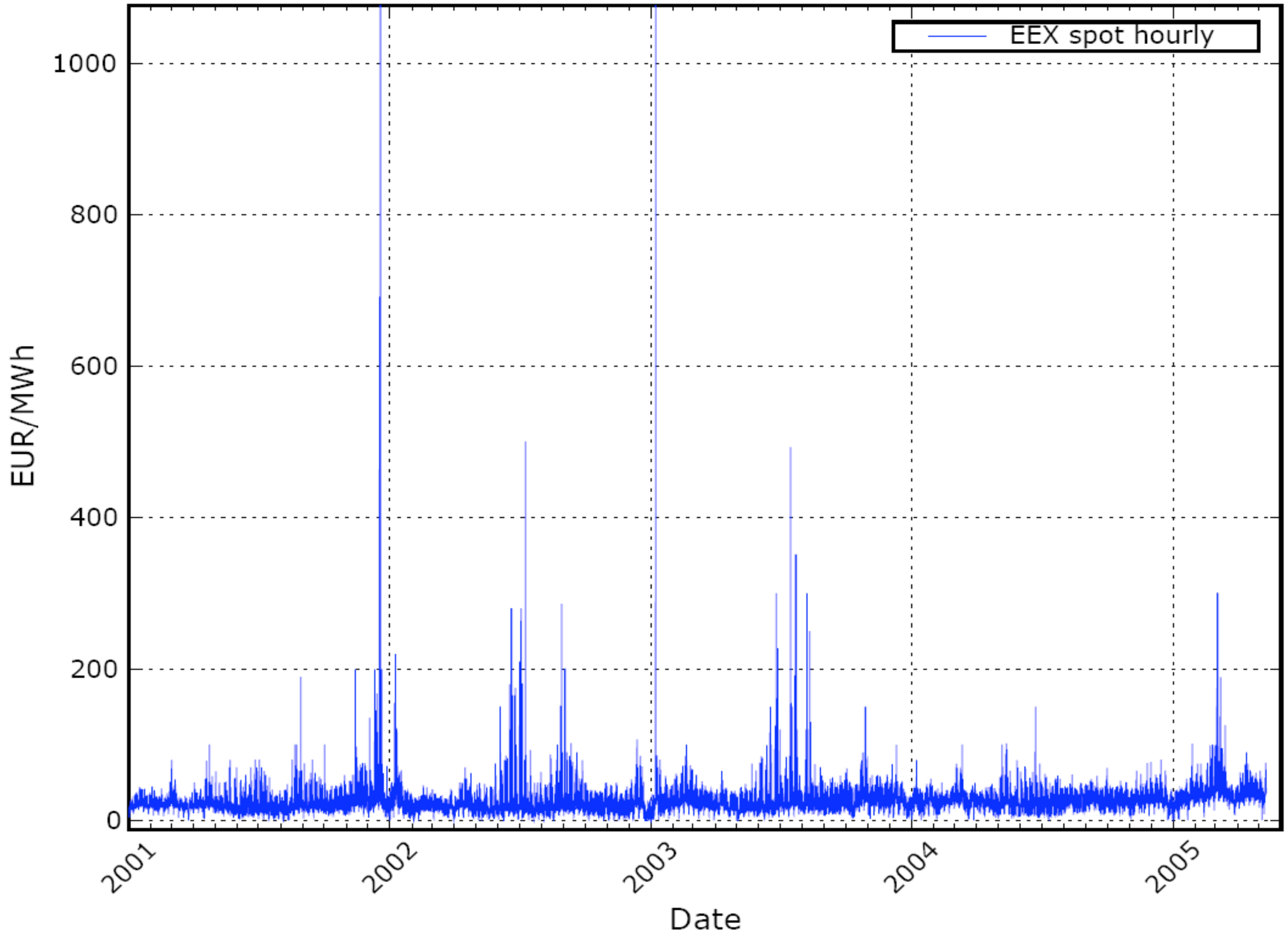
Explanation

- power not efficiently storable => no cash-and-carry arbitrage
- inelastic demand curve
- seasonal weather-dependent demand pattern
- events can cause market shocks
  (plant outages, low water levels, extreme temperature)

Marginal Costs of Generation



Historical Hourly Spot Prices EEX



Fundamental and Stochastic Approaches

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Fundamental

- model system generation and load
- price = marginal generation costs
- needs fuel prices and data about generation capacity
- many sources of uncertainty (generation, import/export, …)

Stochastic

- view power prices as time series
- choose appropriate stochastic process
- calibrate to price data
- needs only prices as input data

Hybrid

- use both approaches (e.g. SMaPS[1])

Here we concentrate on the stochastic approach!

[1] M.Burger, B.Klar, A.Müller, G.Schindlmayr
A spot market model for pricing derivatives in electricity markets, Quantitative Finance 4 (2004) 109-122

Model Overview

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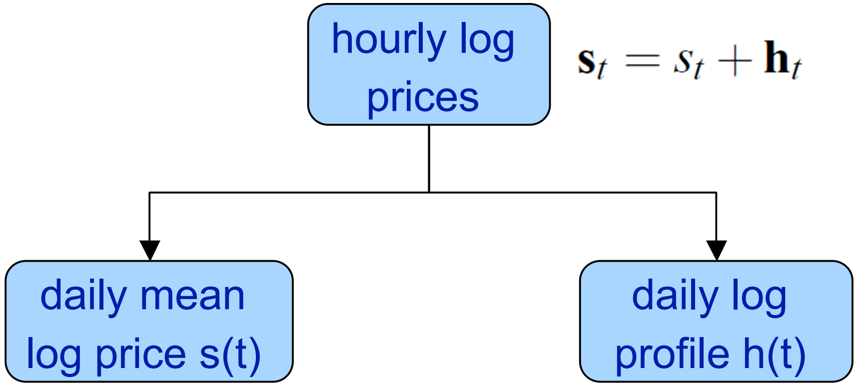
Notation:

- $\mathbf{S}_t = (S_t^1, \ldots, S_t^{24})$: Vector of hourly spot prices on day t
- $\mathbf{s}_t = \log \mathbf{S}_t$: vector of logarithmic hourly spot prices
- $s_t = \frac{1}{24}\sum_{i=1}^{24} \mathbf{s}_t$: mean logarithmic price



$\mathbf{s}_t = s_t + \mathbf{h}_t$

regime-switching AR(1)-process

PCA-decomposition + ARMA-process

$s_t = \frac{1}{24}\sum_{i=1}^{24} \mathbf{s}_t$

$$\sum_{i=1}^{24} \mathbf{h}_t = 0$$

Different processes for business days and non-business days

Daily Price Process: Seasonality

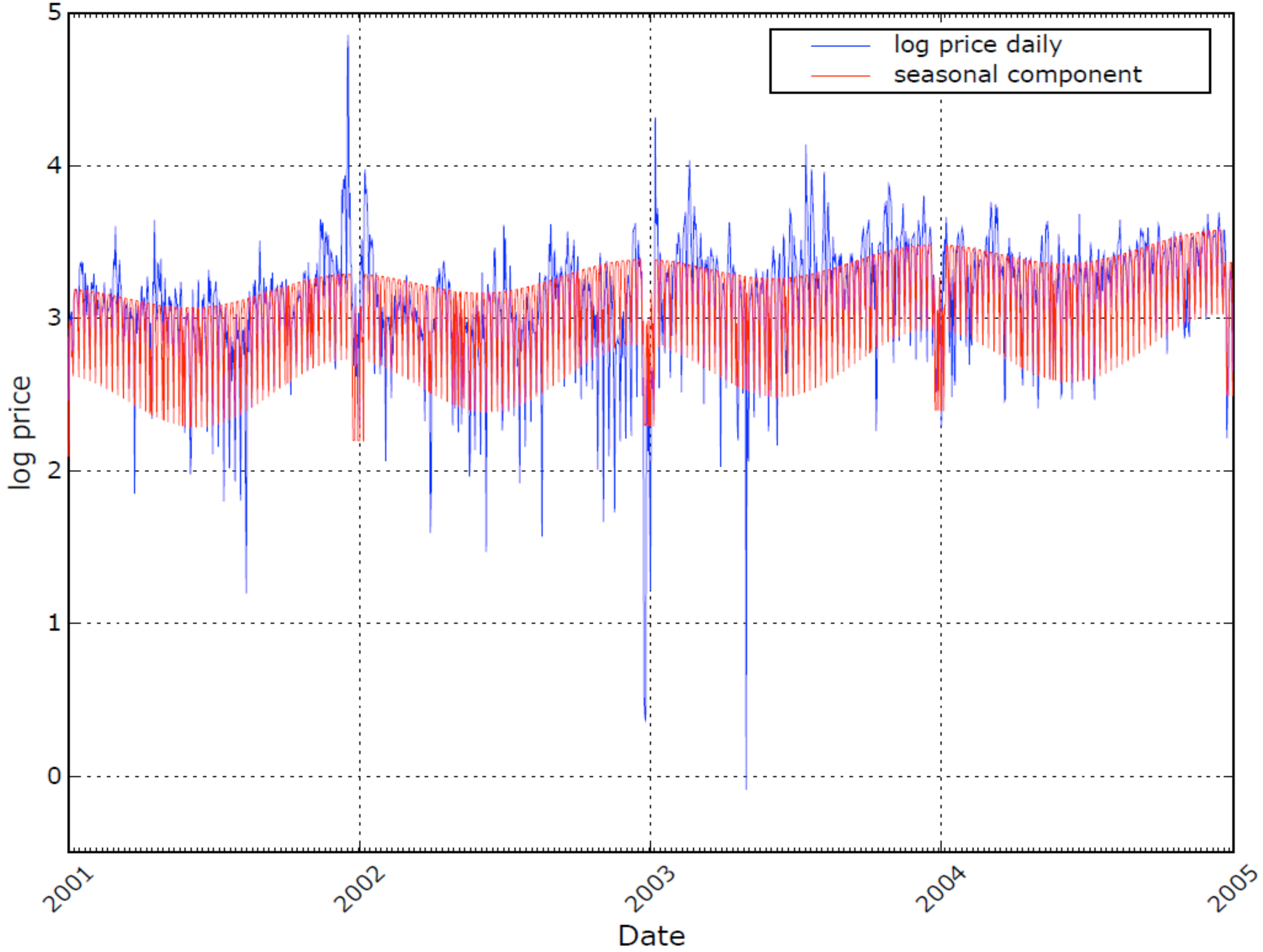
Seasonal component:

- dummy variables for weekdays, holidays, vacation periods (1,..,N_d)
- sin/cos regressors for yearly seasonality
- linear trend

$$
\begin{aligned}
s_t \; = \; & \sum_{d=1}^{N_d} \mathbf{1}_{J_d}(t)\beta_d^A + \mathbf{1}_{J_d^{DT}}(t)\cos(2\pi t/365)\beta_d^B + \mathbf{1}_{J_d^{DT}}(t)\sin(2\pi t/365)\beta_d^C + \mathbf{1}_{J_d^{DT}}(t)t\beta_d^D \\
& + \mathbf{1}_{J_d^{DT}}(t)\mathbf{1}_{J^{VP}}(t)\beta_d^E + y_t,
\end{aligned}
$$

coefficient	**description**
β_d^A	mean level
β_d^B, β_d^C	Amplitudes for yearly seasonality
β_d^D	deterministic drift
β_d^E	price effect of vacation period

Daily Price Process: Seasonal Component EEX



Daily Price Process: Regime-Switching AR(1)

Model:

- $y_k - \mu_{r_k} = \phi_{r_k} \left(y_{k-1} - \mu_{r_{k-1}} \right) + \sigma_{r_k} \varepsilon_k$
- r_k = regime at time k
- transition matrix (for two regimes):

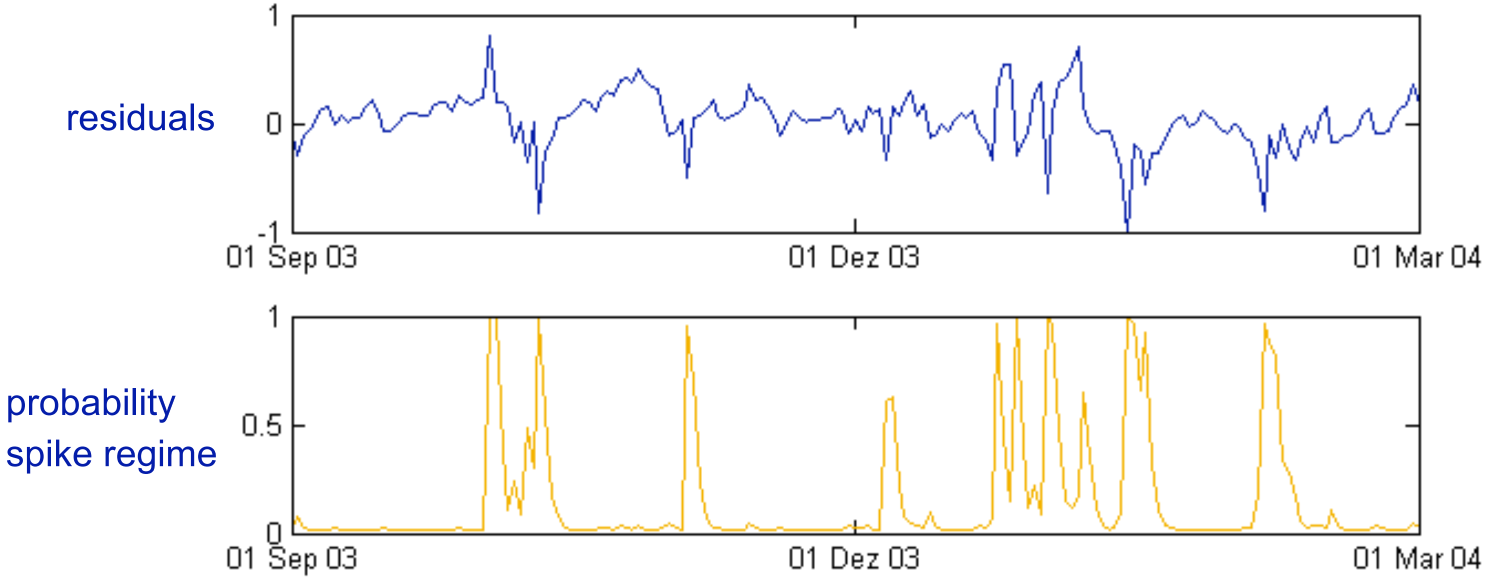
$$P = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix}$$

- calibration: Hamilton filter (max. likelihood optimization)
- example: EEX (Jan 01 – May 05)

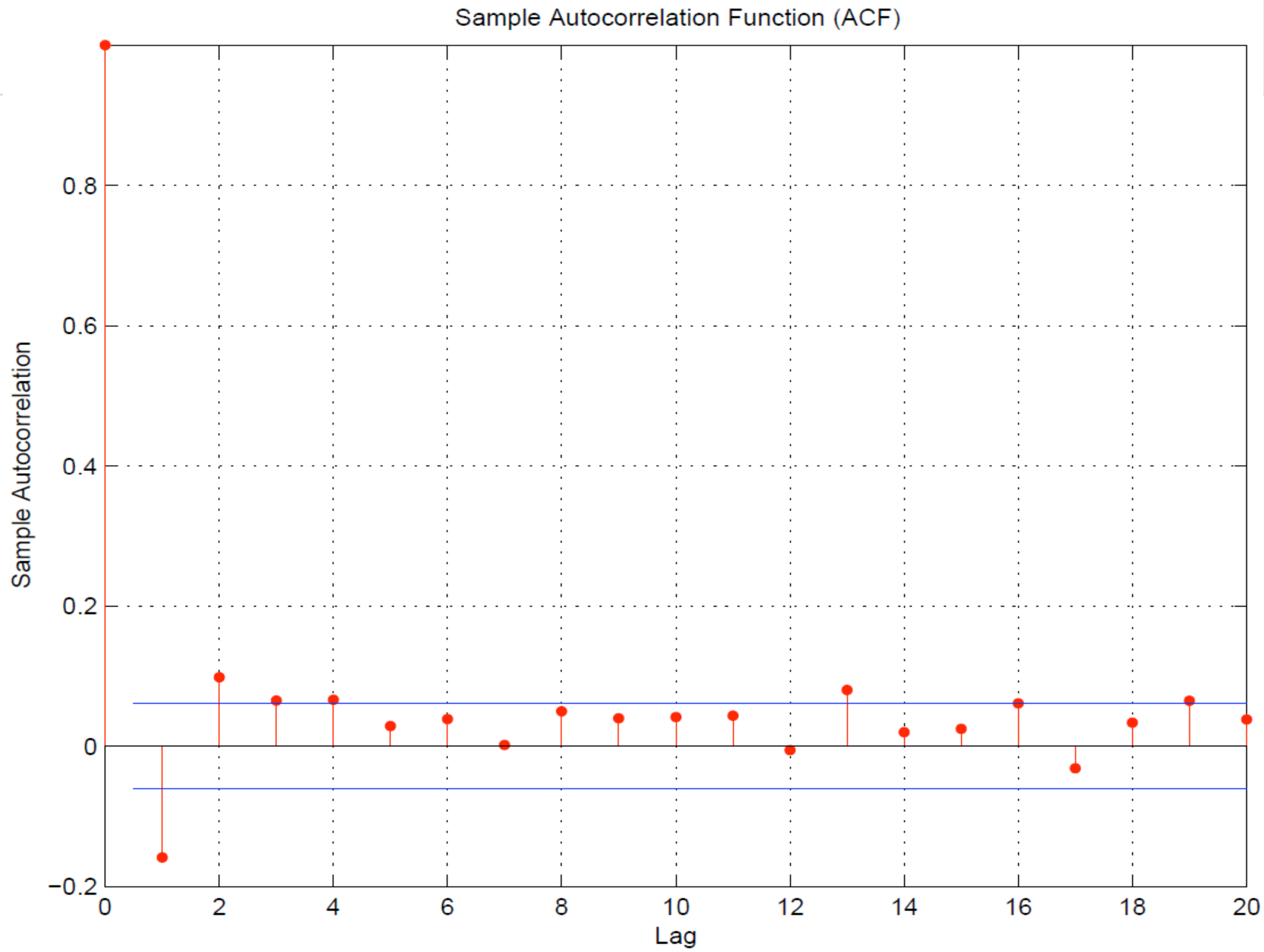
	μ	ϕ	σ
regime 1	-0.004	0.74	0.11
regime 2	-0.02	0.68	0.30

$$P = \begin{pmatrix} 0.94 & 0.28 \\ 0.06 & 0.72 \end{pmatrix}$$

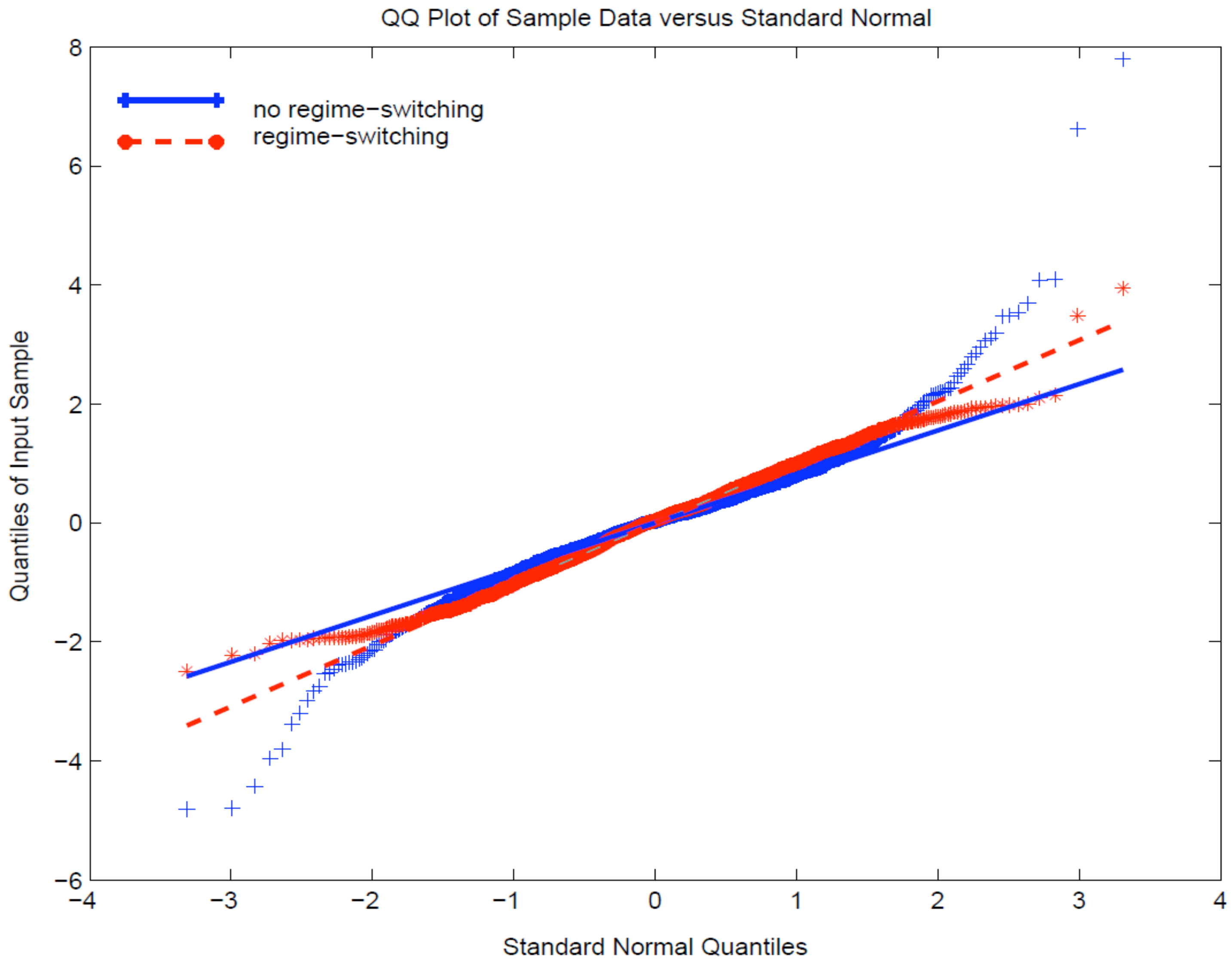
Daily Price Process: Regime Identification



Daily Price Process: Autocorrelation of Residuals



Daily Price Process: Q-Q Plot of Residuals



Hourly Profiles: PCA Decomposition

Regression: seasonal component

$$\mathbf{h}_t = \hat{\mathbf{h}}_t + \Delta\mathbf{h}_t$$

PCA decomposition for 24h-residuals

$$\Delta\mathbf{h}_t = \mathbf{Z}\mathbf{P}^T$$

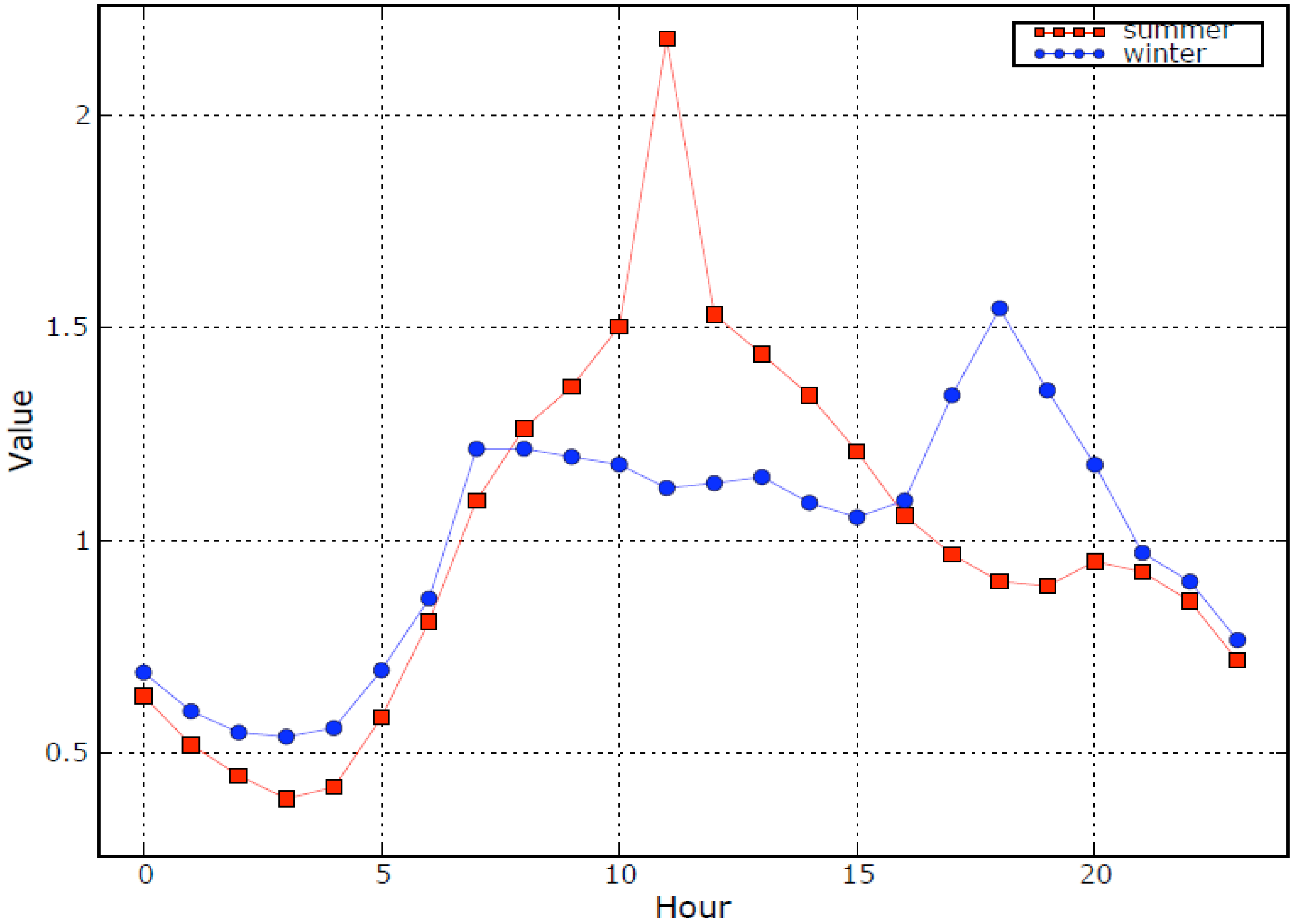
$$\mathbf{Z} = (\Delta\mathbf{h})\,\mathbf{P}$$

stochastic model for factor loads

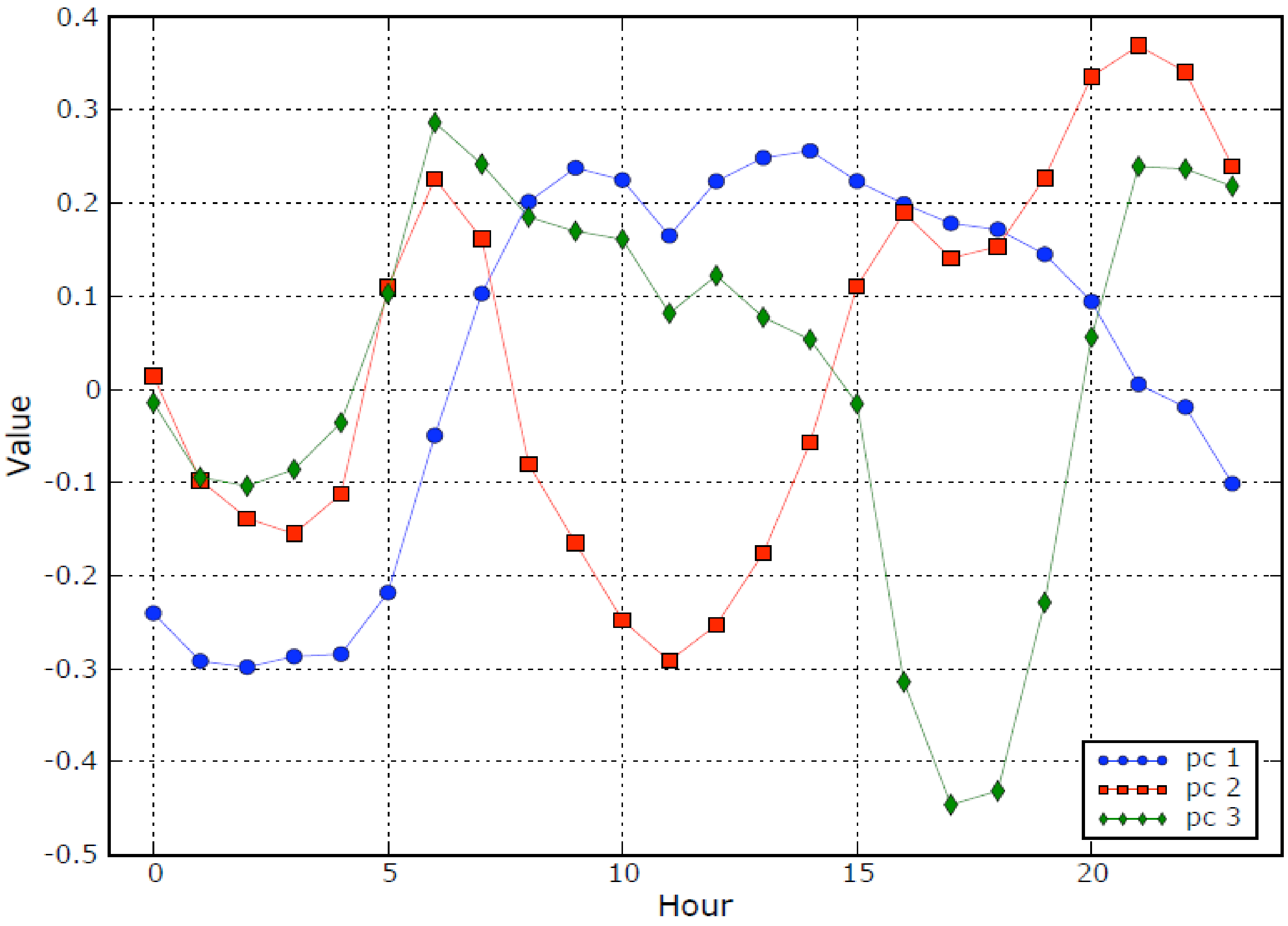
ARMA(p,q) for z^i_k

For spike regime: take random historical profile according to season and weekday

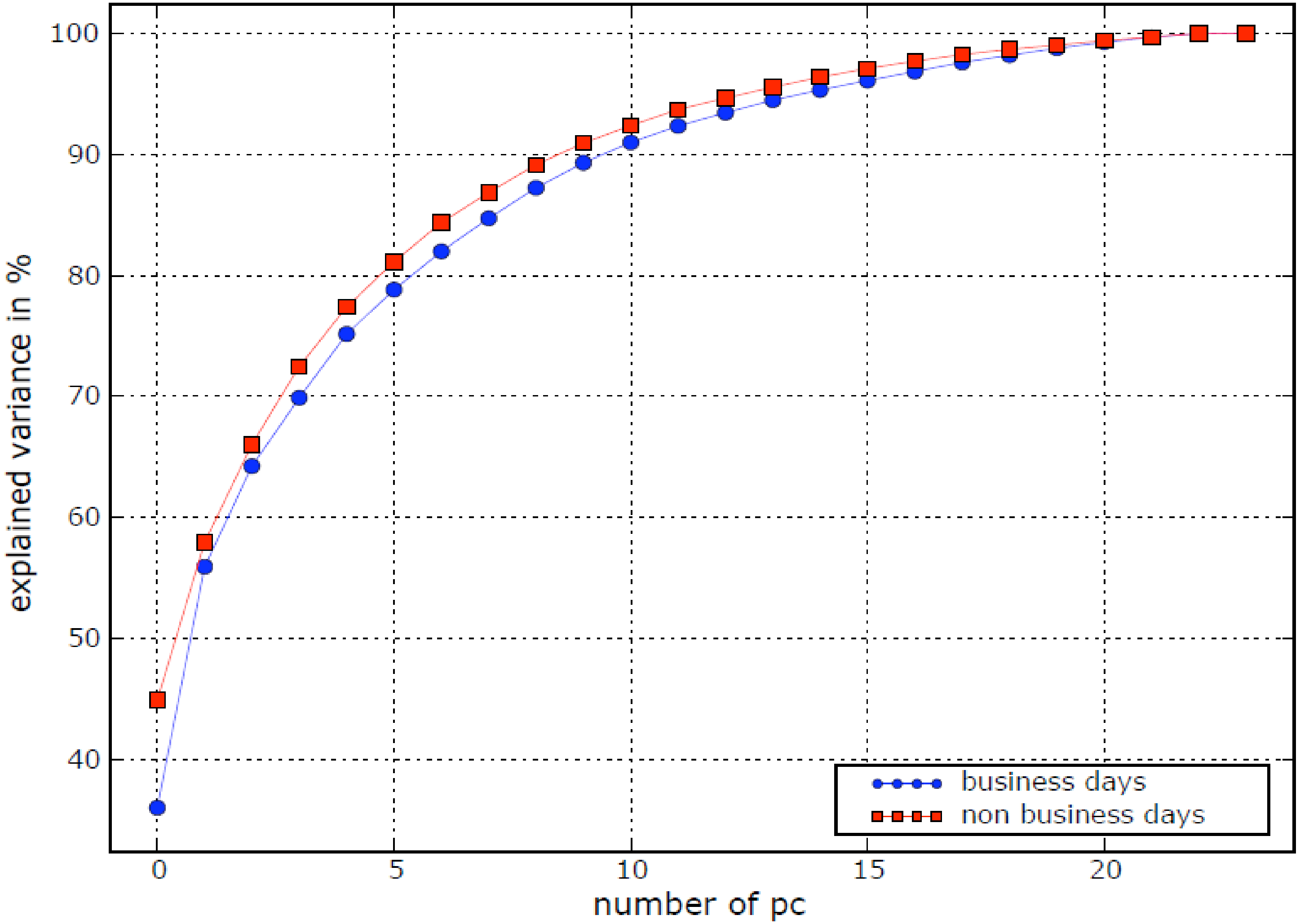
Hourly Profiles: Seasonal Component



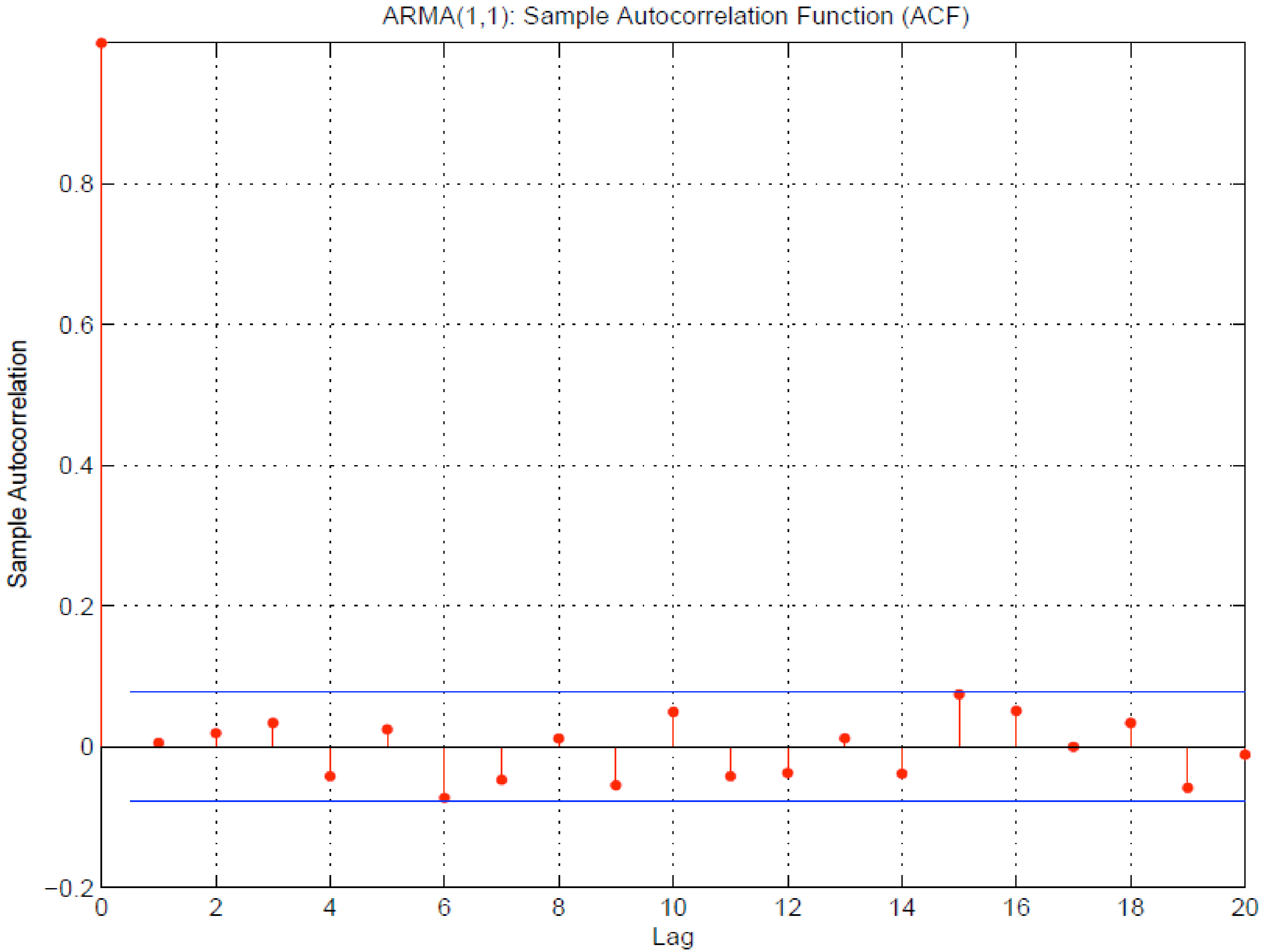
Hourly Profiles: Principal Component Vectors



Hourly Profiles: PCA Explained Variance



Hourly Profiles: Factor Load Autocorrelation of Residuals



The Long-Term Dynamics

Model: $\mathbf{s}_t = f(t) + y_t + \mathbf{h}_t + l_t$

- f(t) : seasonal (deterministic) component
- y_t : regime-switching process
- $\mathbf{h}_t$: hourly profile process
- l_t : long term process $\quad l_{t+1} = (\mu_t - \tfrac{1}{2}\sigma_l^2) + \sigma_l \varepsilon_t^l$

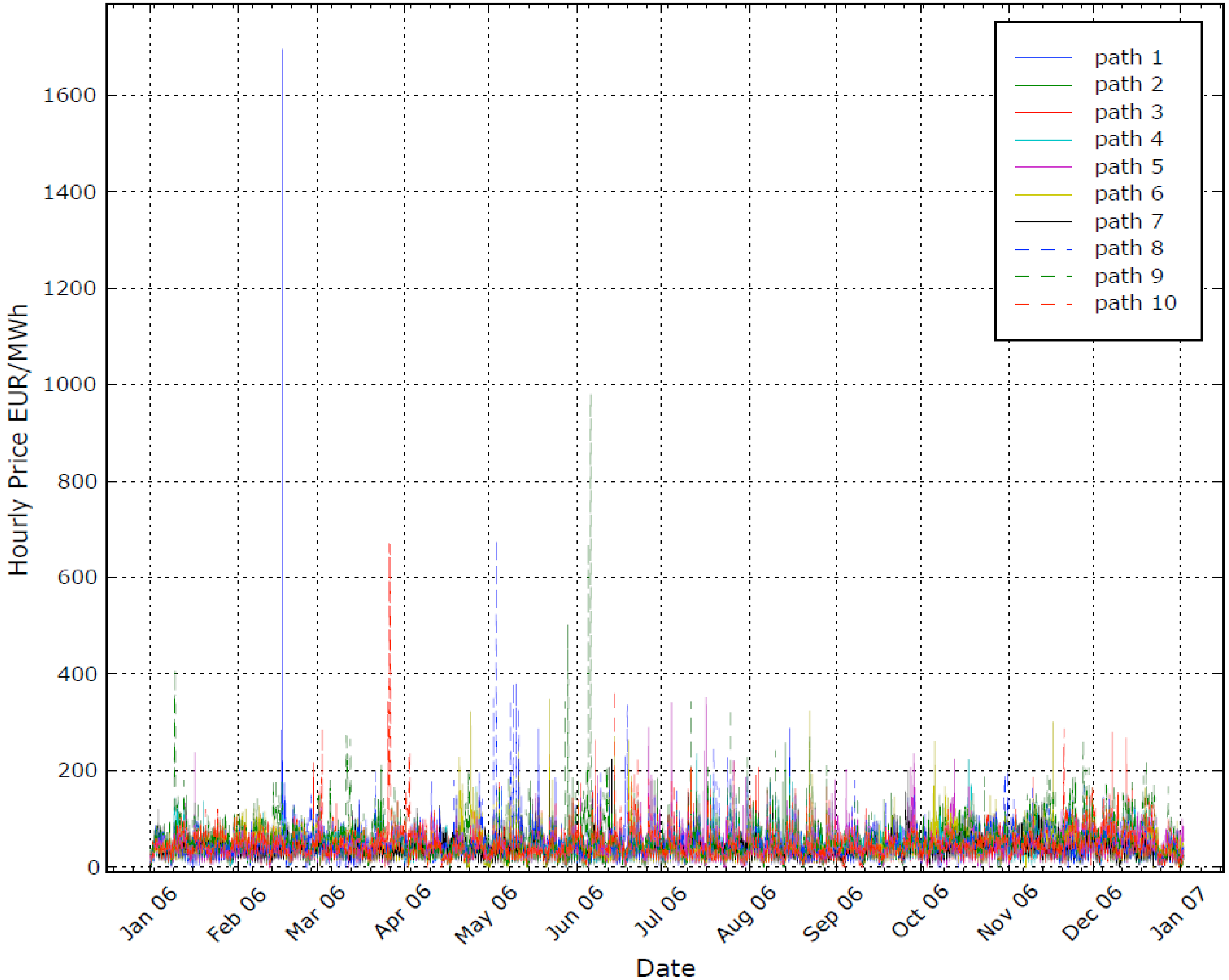
Future price: for T>>t

- short term dynamics: $\quad \mathbf{E}_t[y_T] \approx \mathbf{E}[y_T] \quad \mathbf{E}_t[\mathbf{h}_T] \approx \mathbf{E}[\mathbf{h}_T]$
- long term dynamics $\quad F_{t,T} = \mathbf{E}_t[S_T] \approx C(T)\mathbf{E}_t[\exp(l_t)]$
- long-term approximation: Black's future price model

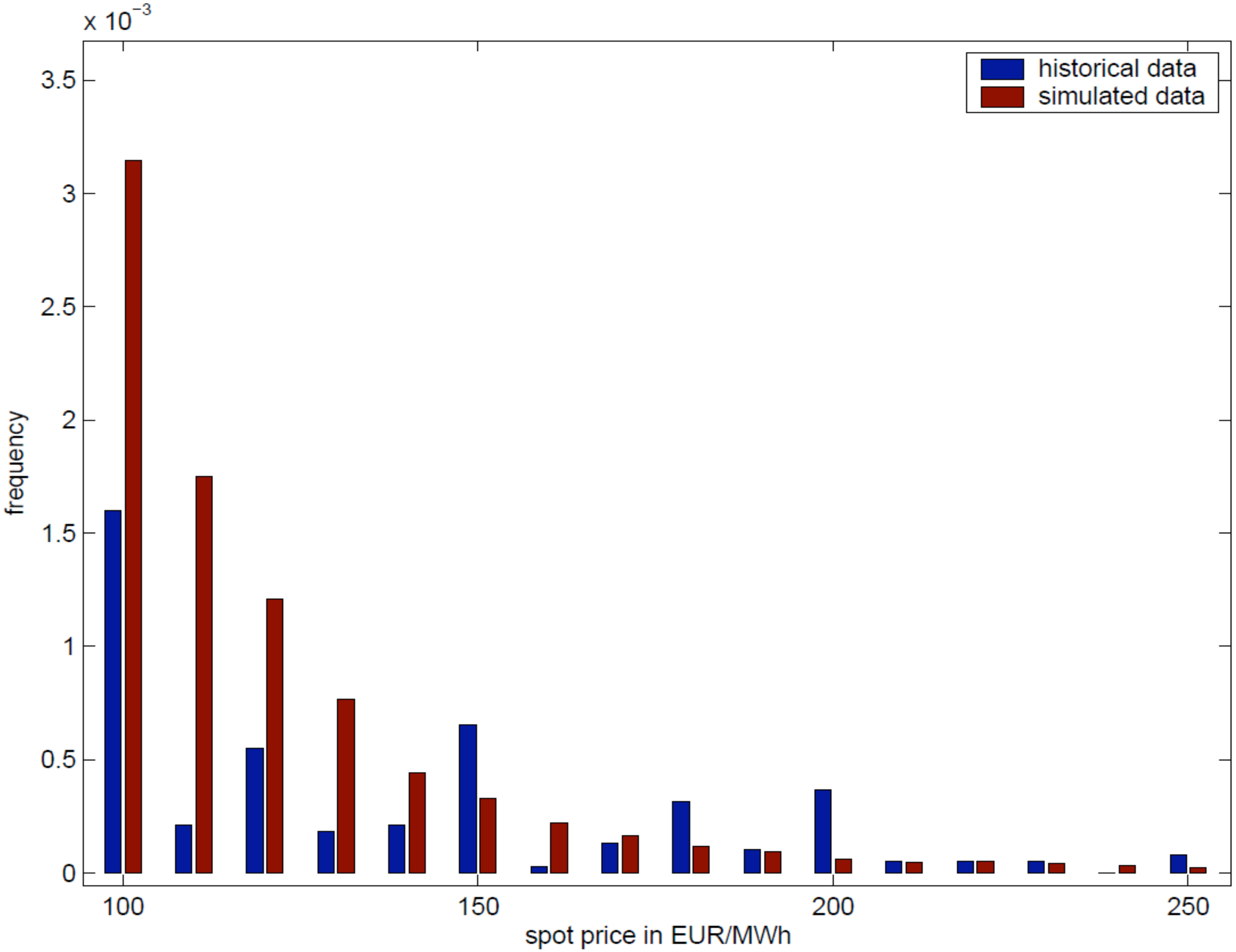
Calibration

- historical volatility or implied volatility (depending on application)

Simulation Results: Sample Paths

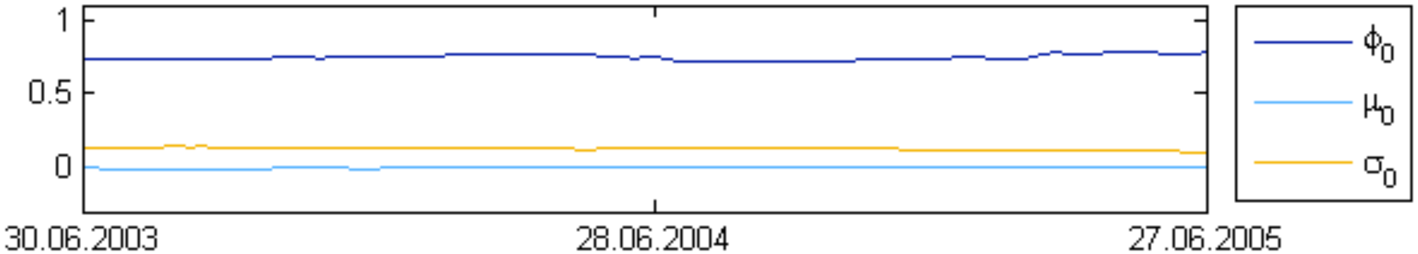


Simulation Results: Histogram Prices 100-250 €/MWh

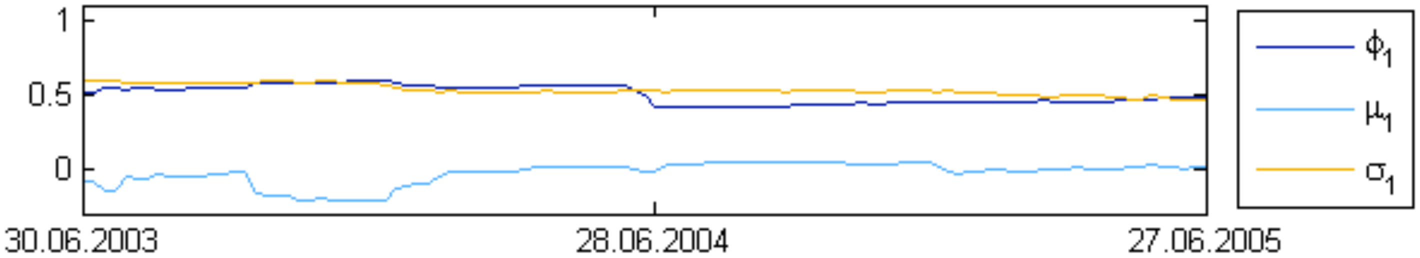


Backtesting: Calibration Stability

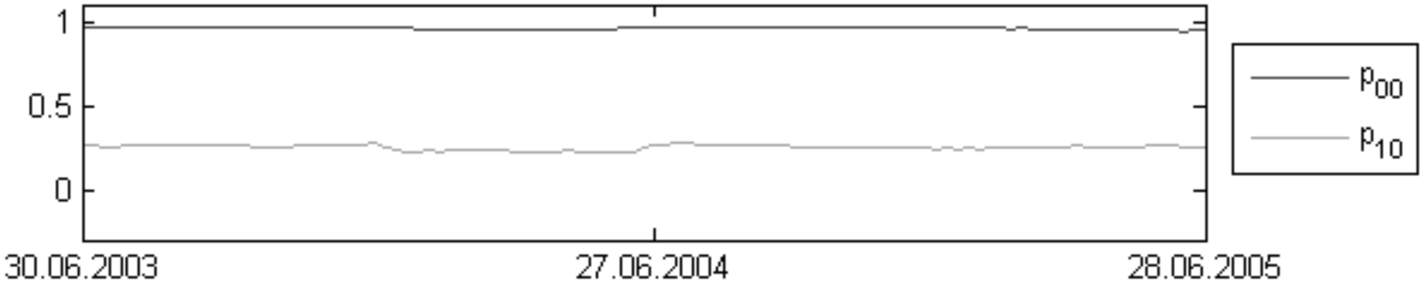
moving two-year calibration period



parameter base regime



parameter spike regime



transition probabilities

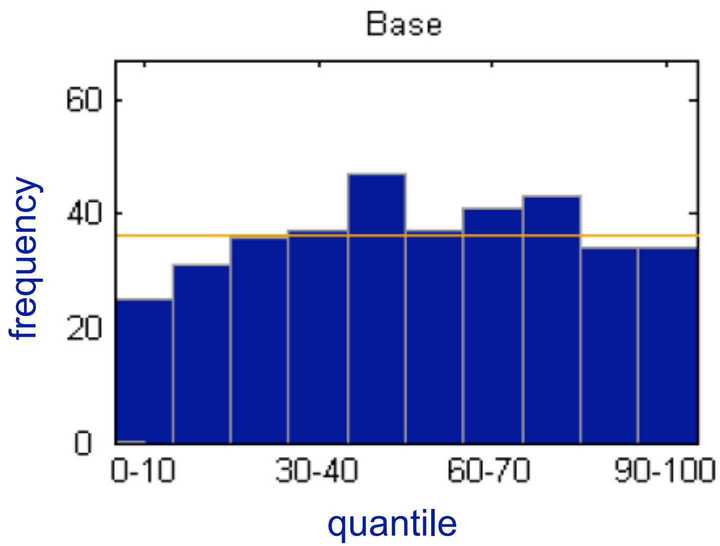
Backtesting: 1-Day-Forecasting Quality

	Regime-Switching-Model				
	❶ $2R$	❷ $3R$	❸ 2^B1^{nB}	❹ 2^B2^{nB}	❺ 3^B1^{nB}
	Business Days				
MAE	4,520	4,492	4,157	4,132	4,130
RMSE	6,877	6,839	6,429	6,455	6,385
MAPE	11,488	11,436	10,476	10,360	10,430
	Non-Business Days (excl. Holidays)				
MAE	3,106	3,130	3,615	3,678	3,716
RMSE	3,905	4,032	4,635	4,719	4,746
MAPE	11,562	11,562	13,537	13,708	13,892
	Holidays				
MAE	4,250	5,396	5,535	5,446	4,764
RMSE	5,597	6,249	6,308	6,295	5,626
MAPE	18,017	23,315	23,803	23,769	20,580

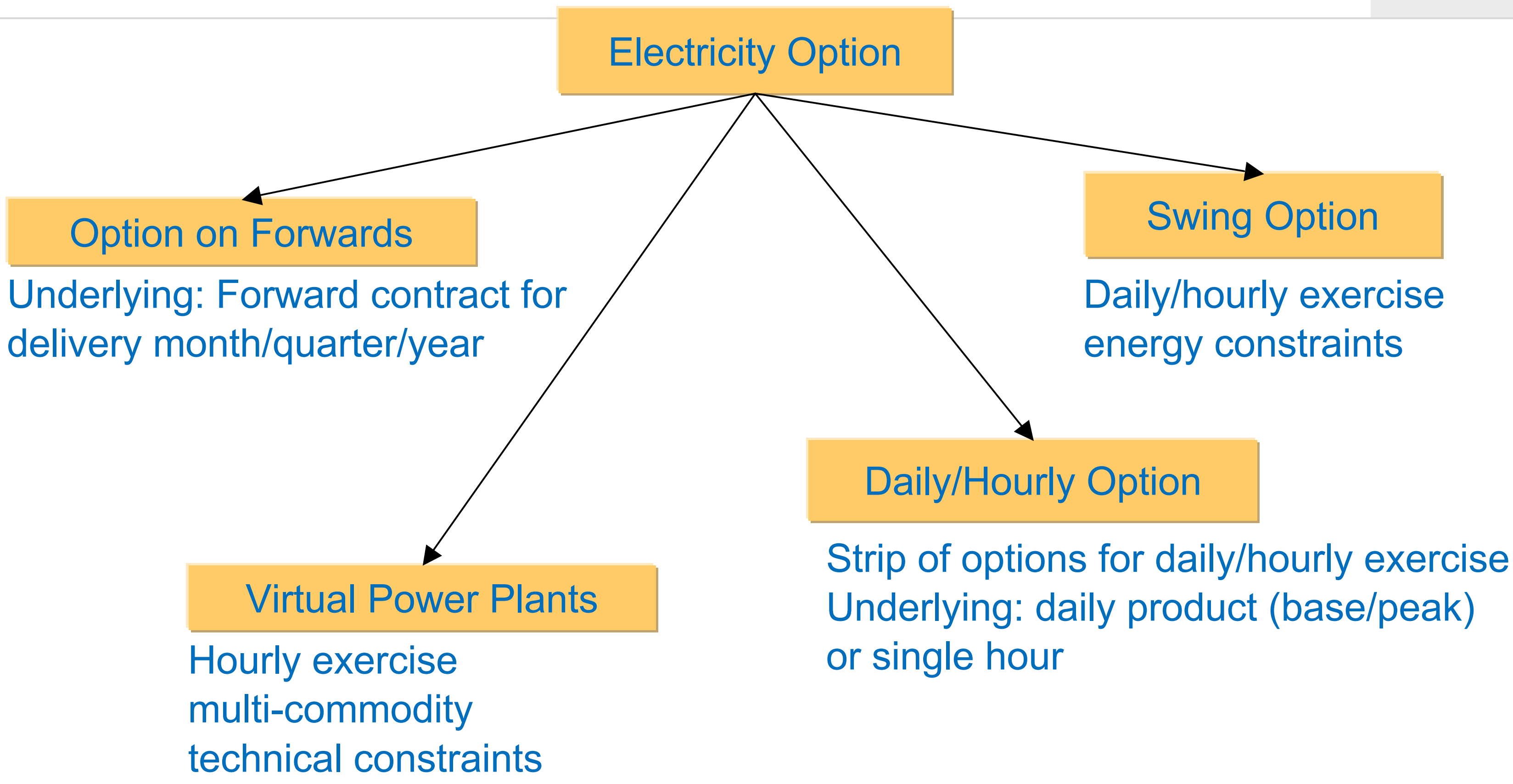
Backtesting: Quantile-Statistics

How do the probability distributions compare?

- histogram to analyze, how often the real spot price falls into which quantile of the model distribution
- period: 01.07.2004 – 30.06.2005
- calibration off-sample (uses data from 01.01.2001- 30.06.2005)



Applications: Option Pricing



Applications: Option Pricing and Hedging

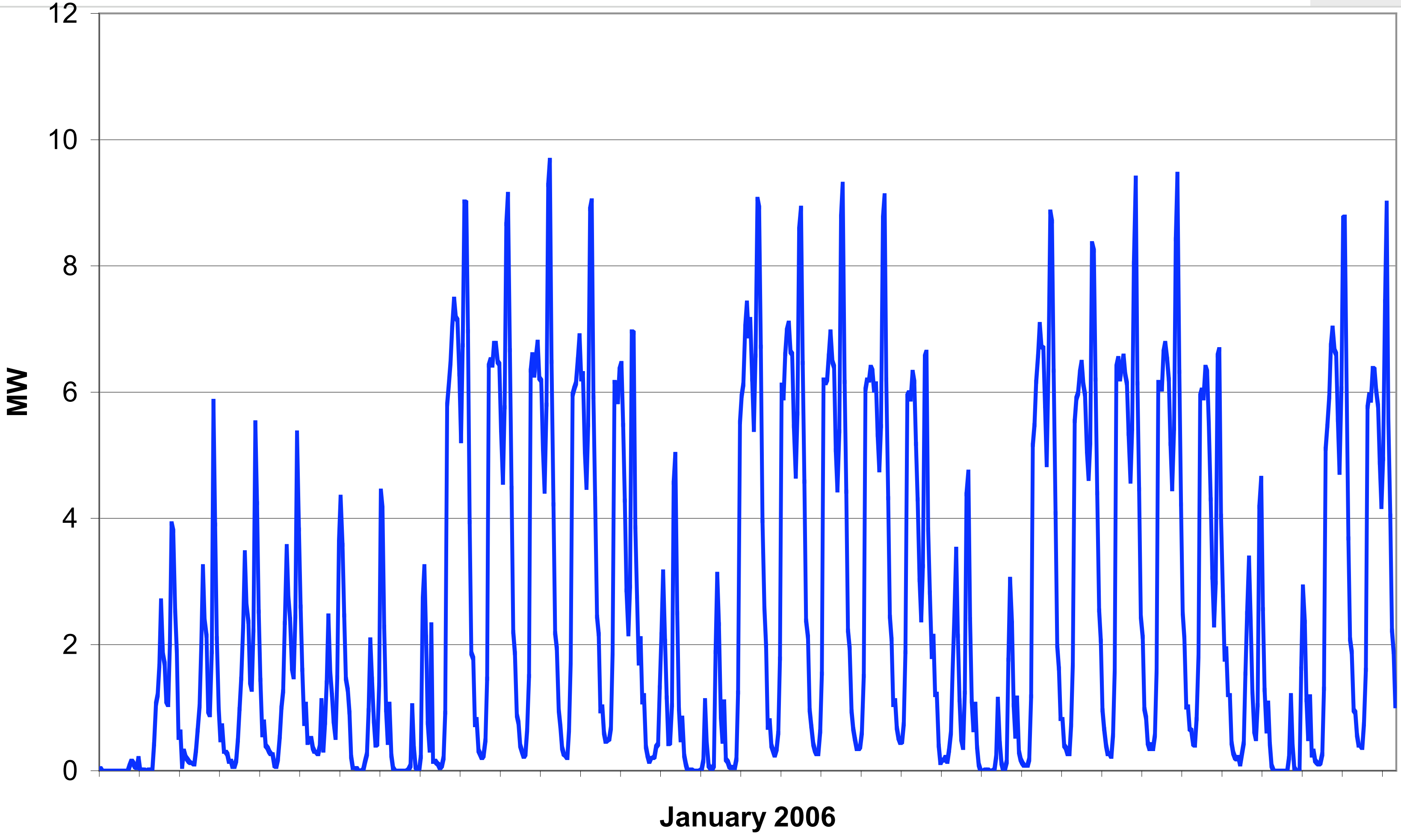
Hourly call option

- period: 01/01/2006 – 01/01/2007
- strike: 60 €/MWh
- capacity: 10 MW

Pricing results

- price: 380.000 €
- inner value: 190.000 €
- profit-at-risk (95%): 160.000 €
- mean exercise 16 GWh (1600 h)

Applications: Mean Exercise Schedule



Applications: Hedging Strategies

energetic hedging

- calculate mean exercise schedule
- sell energetic equivalent base and peak contracts

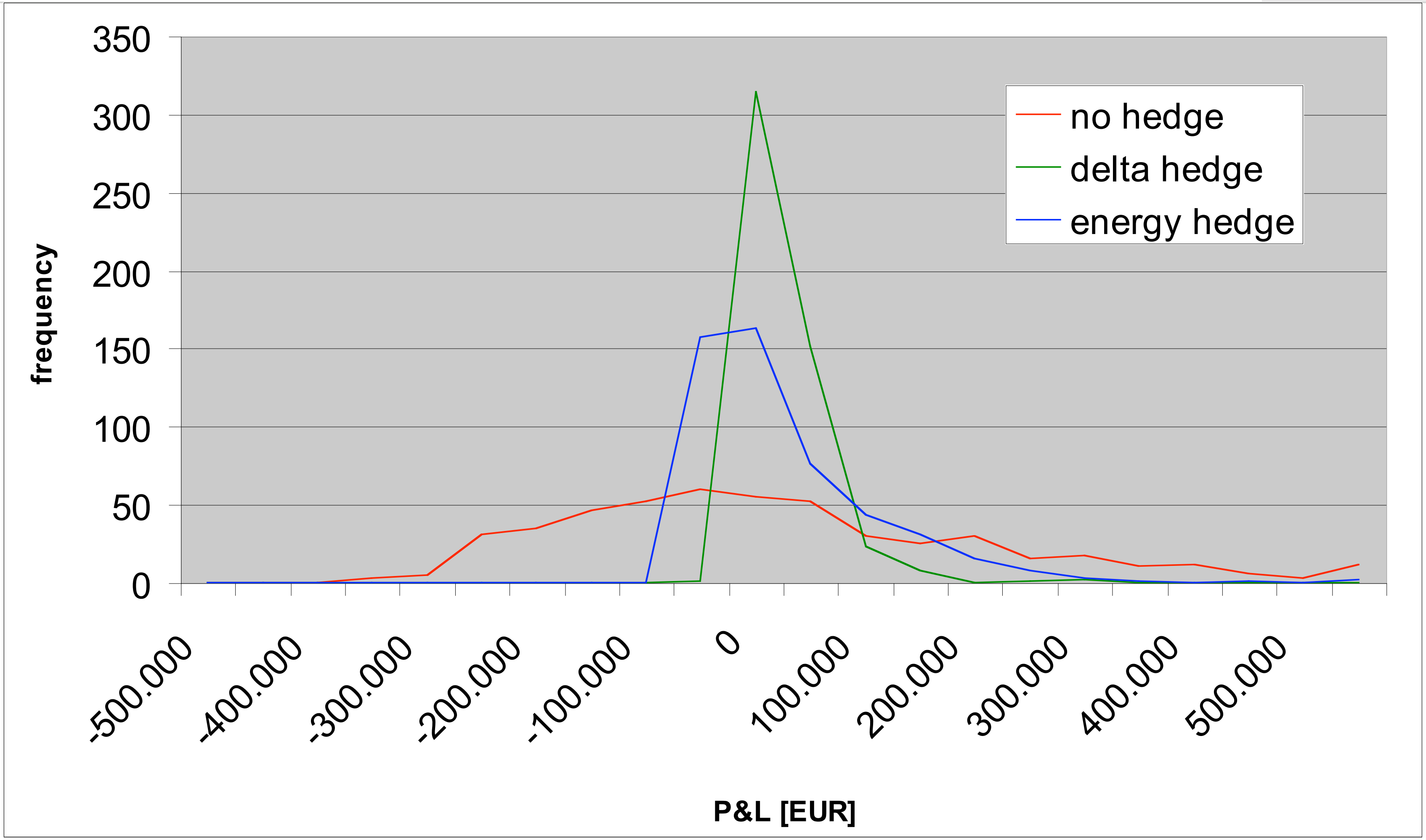
delta hedging

- calculate delta sensitivities with respect to base/peak forward prices
- construct delta neutral portfolio

variance-minimizing hedge

- calculate hedge ratios by minimizing portfolio variance

Applications: Analyzing Hedging Strategies



Outlook

- better coupling of business days and non-business days
- improve dynamics of hourly profiles, especially during spike regime
- integration of spot and future price models
- multi-commodity model: integrate fuel and CO_2 prices