

# **VALUATION OF ELECTRICITY FORWARD CONTRACTS: THE ROLE OF DEMAND AND CAPACITY**

**(Work in Progress)**

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# COMMODITY FORWARD PRICING LITERATURE

## 1.- “Traditional Literature”: COST-OF-CARRY MODEL

Convenience yield: Brennan & Schwartz, *JF*; 1985; Schwartz, *JF*, 1997,...

**Problem: Electricity is NON-STORABLE**

## 2.- LONG / SHORT-TERM MODEL (Schwartz & Smith, *MS*, 2000)

⇒ Lucía & Schwartz, *RDR* 2002. Electricity (NordPool)

⇒ Villaplana, 2003. SS model + JUMP. Role of Jump Risk Premium.

## 3.- “Equilibrium Model”, where **electricity spot price is linked to some fundamental economic state variables.**

Eydeland and Geman (1998 & 1999), Geman and Vasicek (2001), Pirrong & Jermakyan (2002), Barlow (*Math. Finance*, 2002),...

## IMPLICATIONS OF THE PAPER:

### **1.- VALUATION ELECTRICITY DERIVATIVES. MODEL.**

Eydeland & Geman, 2000; Pirrong & Jermakyan, 2002; Barlow, *Math. Fin.* 2002;

### **2.- ECONOMIC DETERMINANTS OF FORWARD RISK**

**PREMIUM.** Bessembinder & Lemmon, *JF* 2002; Longstaff & Wang, *JF*, 2004

### **3.- ANALYSIS OF INVESTMENT BEHAVIOR IN ELECTRICITY MARKETS.**

Joskow & Tirole, 2004; IEA, 2003; IDEI-CEPR conference Jan. 2004...

Discussion: whether market-based regulation will be able to provide enough generation capacity in the system or not.

Under-investment  $\Rightarrow$  very high prices and rationing.

“Long-term power system adequacy (...) remains an open question”, Pérez-Arriaga, 2003

# GOAL

PRESENT A **FRAMEWORK** FOR THE ANALYSIS OF THE EFFECT OF **SUPPLY** AND **DEMAND** CONDITIONS ON **DERIVATIVE PRICES**

$$\text{ELECTRICITY SPOT PRICE}_t = \varphi (\text{DEMAND}_t, \text{GENERATION CAPACITY}_t)$$

$\Rightarrow \varphi (\bullet)???$

$\Rightarrow$  HOW TO MODEL STATE VARIABLES???

## GOAL (cont.)

- DEVELOP A MODEL FOR THE VALUATION OF ELECTRICITY FUTURES (AND OTHER DERIVATIVES).
  - CAPTURES **THE ROLE OF DEMAND AND “SUPPLY”**
  - **CLOSED FORM**, ALLOW TO EXTRACT RISK-NEUTRAL PARAMETERS FROM TRADED CONTRACTS.
- TO COMPLEMENT/EXTEND THEORETICAL RESULTS AND EMPIRICAL EVIDENCE ON THE BEHAVIOR OF **RISK PREMIUM** IN POWER MARKETS:

$$RP_t \equiv F(t, T, S) - E_t^P(S_T)$$

## SUPPLY CONDITIONS ARE IMPORTANT, BUT...

- **Bessembinder y Lemmon, *Journal of Finance*, 2002:**

Equilibrium Model.  $P = a \left( \frac{D}{N} \right)^{c-1}$ . N (generators) constant  $\Rightarrow$   
cannot take into account supply shocks.

### Forward Risk Premium: Skewness

Volatility of Demand & Convexity Supply Function. No valuation.

- **Longstaff y Wang, *Journal of Finance*, 2004:**

Forward Risk Premium PJM. “Demand and Capacity matter”. But, they assume: Constant Capacity. No Valuation.

- **Pirrong y Jermakyan, 2000:**

Demand + Marginal Fuel. Endogenous Demand Risk Premium (calendar time). Computational Cost.

•**Barlow, *Math. Fin.*, 2002:**

Demand as non-linear O-U process. No Supply. No Valuation

•**Skantze y Ilic, 2001:**

Supply as Residual. No Pricing.

Exist empirical evidence on the importance of supply shocks (and demand-supply relationship):

Explain spot price fluctuations

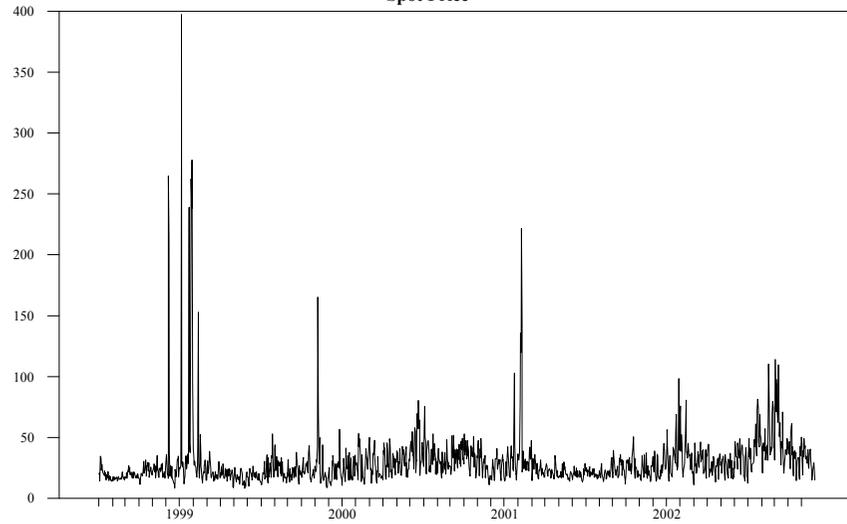
Also explains fluctuations of forward prices

Krapels (2000), Birnbaum et al. (2000), Kollberg et al. (1999),...

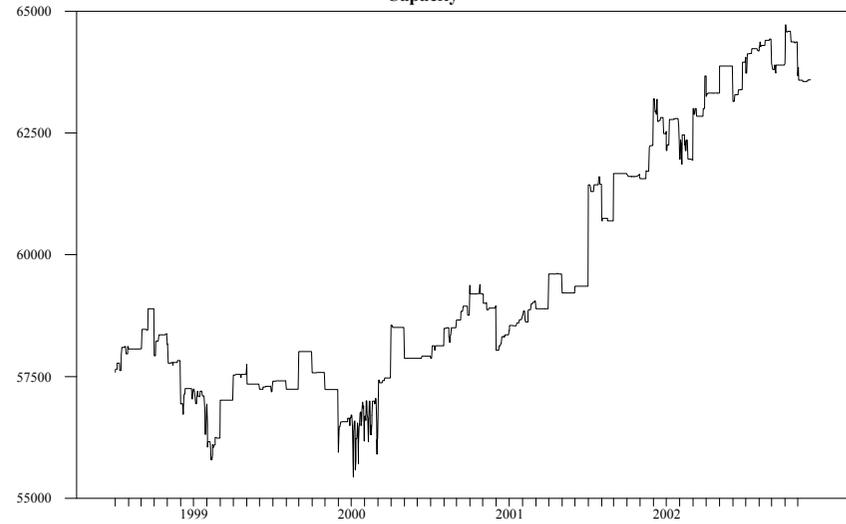
# PJM

1/1/1999-31/5/2003

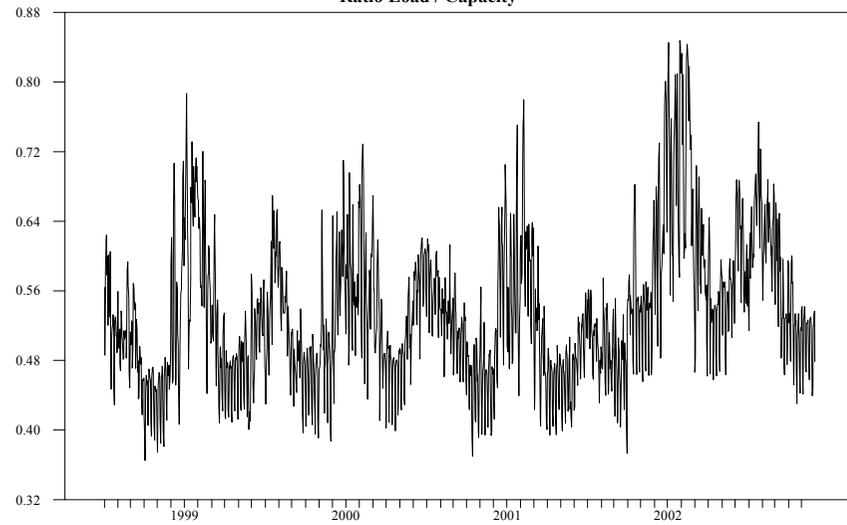
### Spot Price



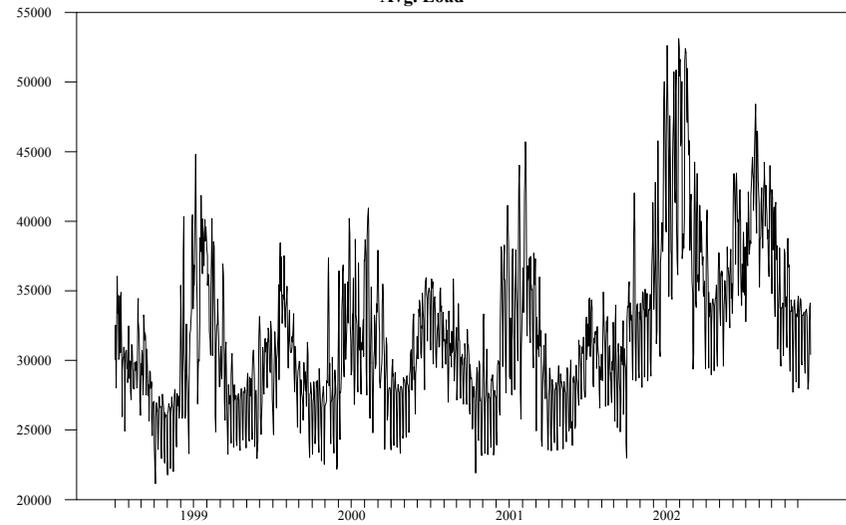
### Capacity



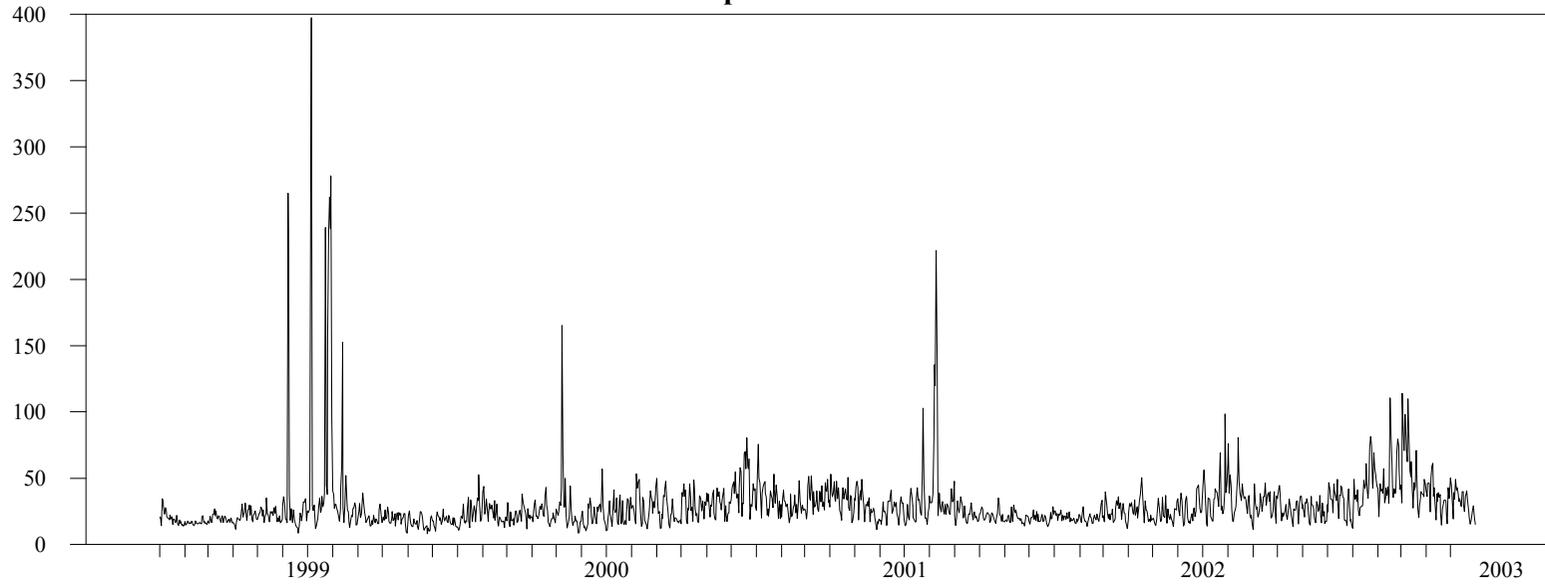
### Ratio Load / Capacity



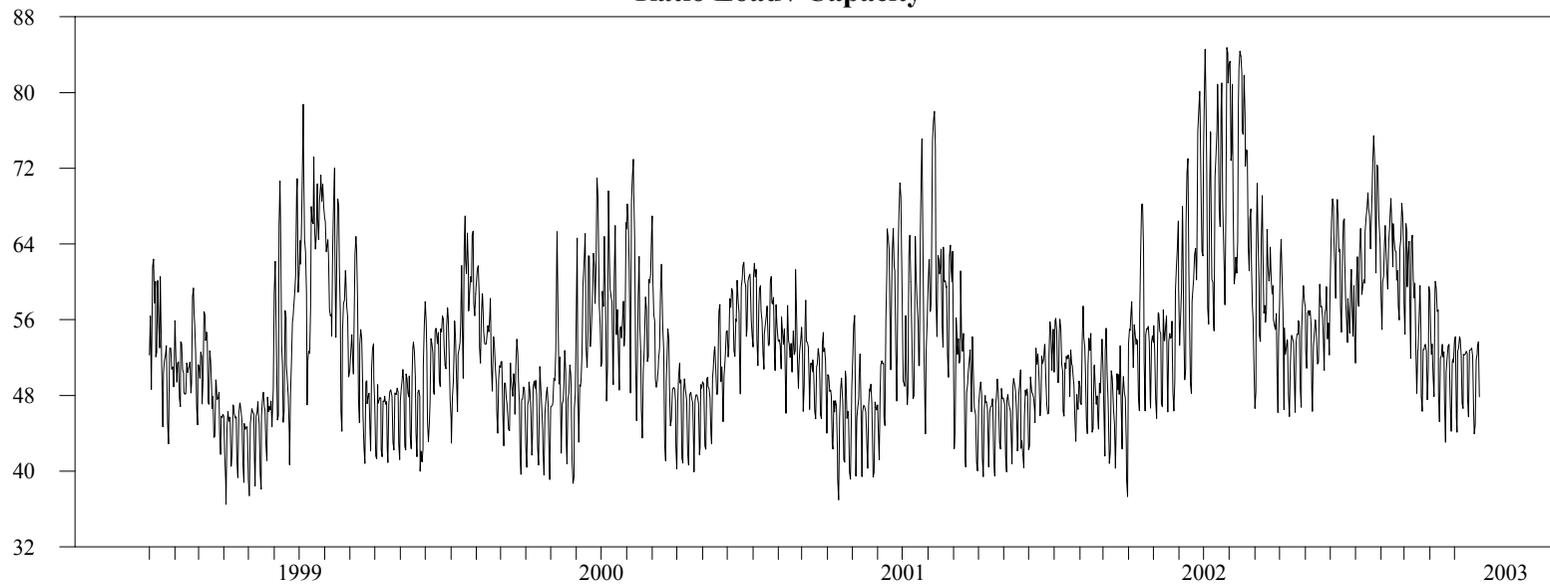
### Avg. Load



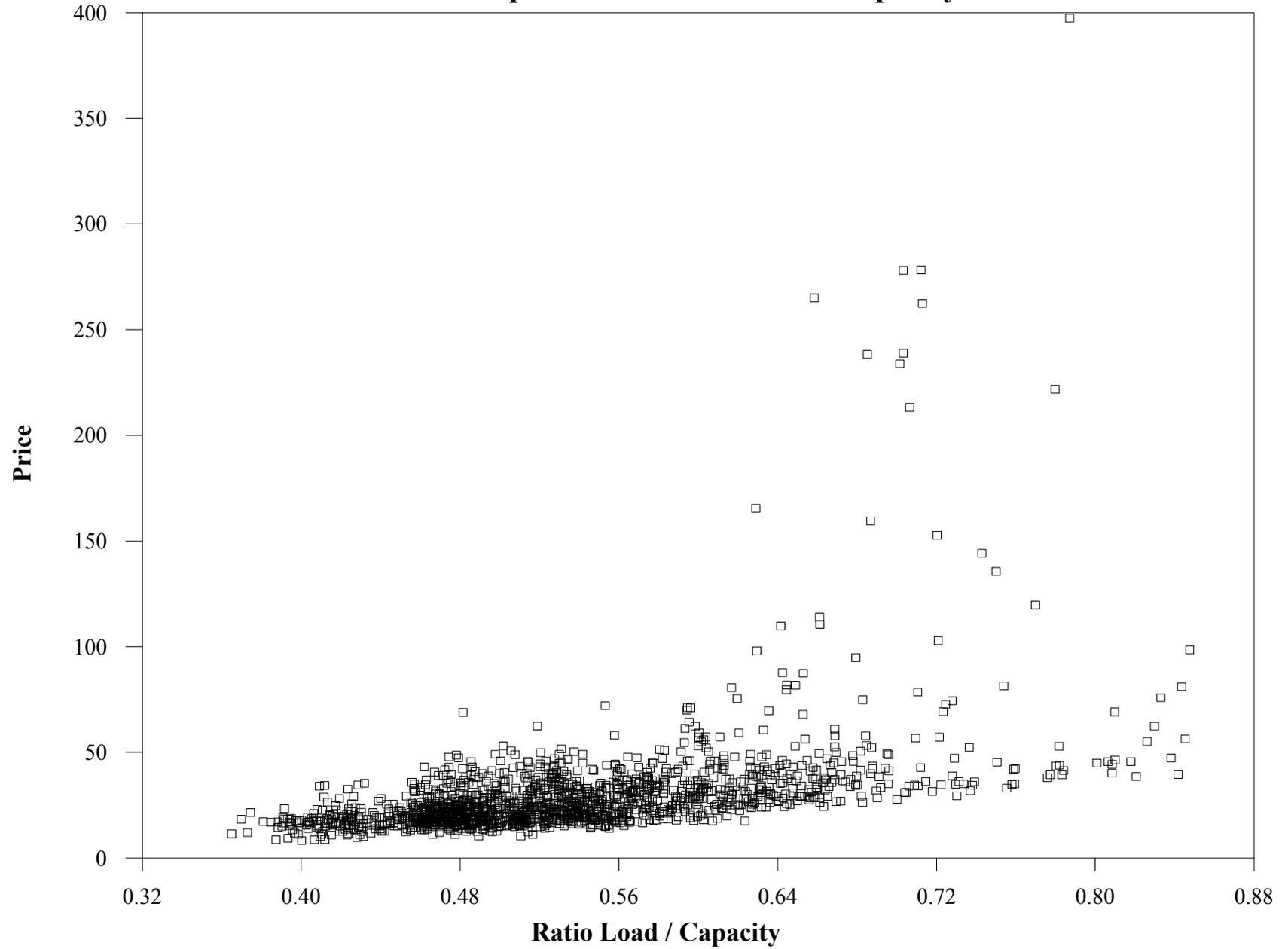
**Spot Price**



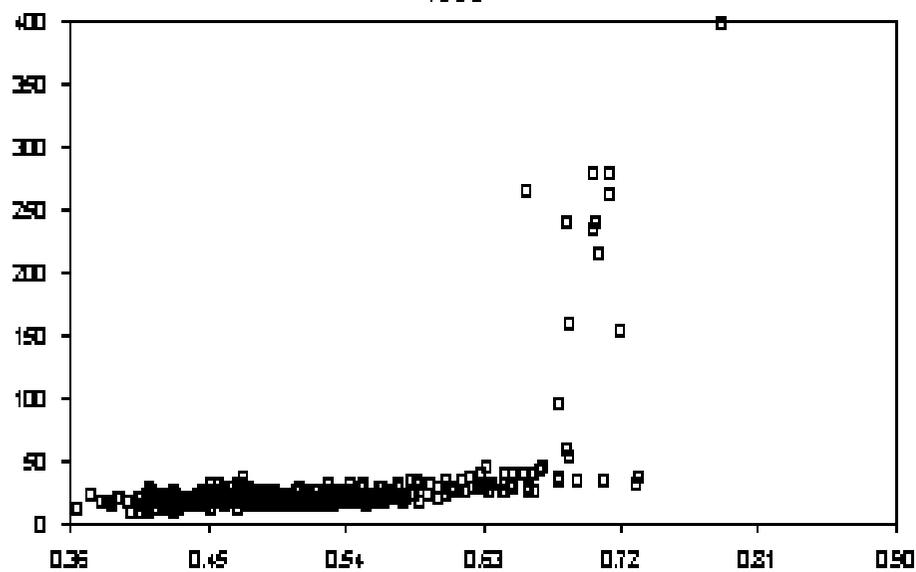
**Ratio Load / Capacity**



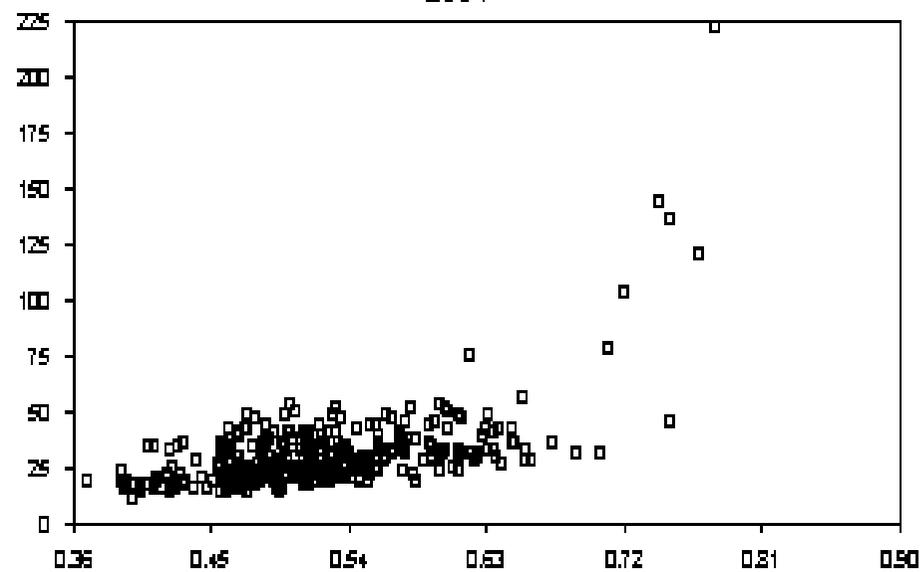
**Relationship: Price & Ratio Load/Capacity**



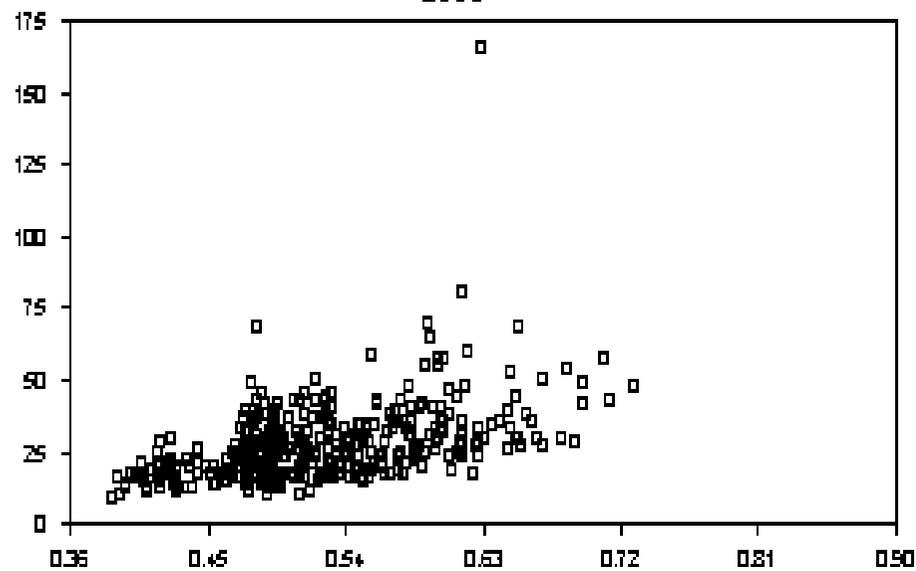
1999



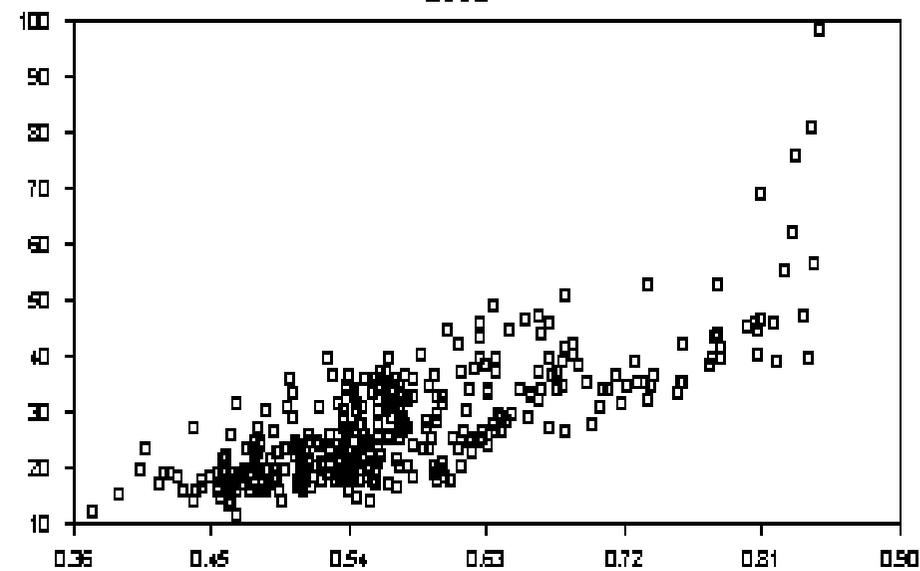
2001



2000



2002



$$\text{ELECTRICITY SPOT PRICE}_t = \varphi (\text{DEMAND}_t, \text{GENERATION CAPACITY}_t)$$

$\Rightarrow \varphi (\bullet) ???$

**•CONSTRAINTS:**

**•*EMPIRICAL OBSERVATION***

**•*VALUATION OF FINANCIAL ASSETS:***

AFFINE (JUMP-DIFFUSIONS) PROCESSES (Duffie et al.,  
*Econometrica*, 2000)

$\Rightarrow$  *Log-spot price linear function of state variables*

$\Rightarrow$  HOW DO WE MODEL STATE VARIABLES???

AFFINE (JUMP-DIFFUSIONS) PROCESSES

$\Rightarrow \varphi(\bullet)???$

Bessembinder and Lemmon, *Journal of Finance*, 2002

1-period model;  $N_P$  power producers;

-Power production cost function:  $TC_i = F + \frac{a}{c} (Q_{P_i})^c$

-Profit:

$$\Pi_{P_i} = P^W Q_{P_i}^W + P^F Q_{P_i}^F - F - \frac{a}{c} (Q_{P_i})^c$$

-Clearing Condition Spot Market:

$$N_P \cdot Q_{P_i}^W = Q^D$$

-Equilibrium Price:

$$P^W = a \left( \frac{Q^D}{N_P} \right)^{c-1}$$

Therefore for a given generic cost function

$$TC_i = F + f(Q_{P_i}) \rightarrow \Pi_{P_i} = P^W \cdot Q_{P_i}^W - F - f(Q_{P_i})$$

$$\max_{Q_{P_i}^W} \Pi_{P_i} \Rightarrow P^W - f'(Q_{P_i}^W) = 0 \Rightarrow Q_{P_i}^W = f'(P^W)^{-1}$$

And the resulting clearing condition and equilibrium price are:

$$N_P \cdot Q_{P_i}^W = Q^D \Rightarrow N_P \cdot f'(P^W)^{-1} = Q^D$$

$$\Rightarrow P^W = f'\left(\frac{Q^D}{N_P}\right)$$

We may think on a generic cost function  $f$  (non-observable) that gives us as a result the empirical observed (and estimated) function  $\varphi(\bullet)$

An important point is we consider generation capacity as an stochastic variable.

**So the model can be seen as an “extension” of B&L specification. Since we take a *general  $f$*  and we allow  $N_p$  to be stochastic.**

We could also introduce some variable related to cost of inputs (gas, oil,...). Technically the methodology is flexible enough to extend the model in such a way (forward price of gas, see Pirrong & Jermakyan, 2002).

## MAIN CHARACTERISTICS **DEMAND** PROCESS:

- SEASONALITY:  $g(t)$
- MEAN-REVERSION
- SEASONAL VOLATILITY

$$D_t = g(t) + X_t$$

$$g(t) = B0 + B2 \cdot t + D1 \cdot labora_t + C1 \cdot \sin\left((t + C2) \cdot \frac{2\pi}{365}\right) + C3 \cdot \sin\left((t + C4) \cdot \frac{4\pi}{365}\right)$$

$$X_t = B1 \cdot X_{t-1} + \sigma_x^s \cdot \varepsilon_t$$

$$\sigma_x^s = QS1 + QS2 \cdot spring_t + QS3 \cdot fall_t + QS4 \cdot summer_t$$

	<b>PJM (U.S.)</b>				<b>NORDPOOL (Scandinavia)</b>			
	<b>Constant Volatility</b>		<b>Seasonal Volatility</b>		<b>Constant Volatility</b>		<b>Seasonal Volatility</b>	
	<b>Model</b>		<b>Model</b>		<b>Model</b>		<b>Model</b>	
Parameter	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.
B0	26645,59	271,19	26904,66	123,11	9670,01	91,04	10101,38	103,45
B1	0,598	0,02	0,544	25,76	0,802	47,99	0,790	42,95
B2	2,59	0,35	2,05	7,00	2,97	21,68	2,26	17,69
D1	4479,42	182,30	4374,31	27,35	4,68	0,07	11,17	0,17
C1	-4380,63	153,69	4063,42	157,76	-3073,72	-45,32	-3011,19	-52,52
C2	65,70	2,82	-3402,35	-1636,51	257,64	214,54	258,91	225,81
C3	-4246,03	191,87	-3974,79	-22,81	-	-	-	-
C4	113,02	1,13	660,71	719,38	-	-	-	-
STDV	2404,42	182,30			797,43	48,65		
QS1			1939,42	30,36			1023,62	21,21
QS2			-180,93	-2,23			-259,82	-4,53
QS3			64,90	0,75			-174,81	-2,92
QS4			1708,53	9,05			-527,99	-9,80
<i>LL</i>		-10879,26		-10776,19		-8999,47		-8944,78
<i>SC</i>		21822,22		21637,32		18062,07		17973,74

## GENERATION CAPACITY VARIABLE:

- Mean- Reverting
- It could be seasonal (deterministic)
- We may allow the the average level to change (deterministic or stochastic)
- We also allow the possibility of JUMPS: outages, imports constraints (transmission congestion).

Just for exposition reason we present two models:

$\varphi(\bullet)$  is the same in both models

Particular specification has been empirically compared against other possible specifications.

**MODEL A: Long-term is constant**

**MODEL B: Stochastic Long-term (Supply and Demand)  
Variables**

## MODEL A (Under Objective Probability Measure)

$$P_t = C_t^\gamma \cdot \beta \cdot e^{\alpha \cdot D_t}$$

$$D_t = g(t) + X_t$$

$$dX = -k_x X dt + \sigma_x^{seas.} dZ_x$$

$$dc = k_c (\theta_c - c) dt + \sigma_c dZ_c + J_c(\eta_{J,c}) d\Pi(\lambda_c)$$

$$dZ_x dZ_c = \rho dt$$

$$\gamma < 0; \alpha > 0$$

## Model A Under Risk-Neutral Probability Measure

$$dX = k_x (\theta_x^* - X) dt + \sigma_x^{seas} dZ_x^*$$

$$dc = k_c (\theta_c^* - c) dt + \sigma_c dZ_c^* + J_c^* (\eta_{J,c}^*) d\Pi(\lambda_c)$$

$$dZ_x^* dZ_c^* = \rho dt$$

where

$$\theta_x^* = -\frac{\phi_x \cdot \sigma_x^{seas}}{k_x} \quad \text{and} \quad \theta_c^* = \theta_c - \frac{\phi_c \cdot \sigma_c}{k_c}$$

**ASSUMPTION:** Constant Market Price of Risk.

Can be relaxed as long as we maintain the AJD structure.

# Pricing Formula

## (Forward Contracts; Characteristic Function)

State Variables: **Affine Jump-Diffusion Processes.**

$$F(x, c, t, T) = \exp \left\{ f(T) + \gamma \cdot c_t \cdot e^{-k_c(T-t)} + \alpha \cdot x_t \cdot e^{-k_x(T-t)} + A(T-t) \right\}$$

where

$$\begin{aligned} A(T-t) = & \theta_c^* \gamma (1 - e^{-k_c \tau}) - \frac{\phi_x \cdot \sigma_x^{seas} \cdot \alpha}{k_x} (1 - e^{-k_x \tau}) + \frac{(\sigma_x^{seas})^2 \alpha^2}{4k_x} (1 - e^{-2k_x \tau}) \\ & + \frac{\sigma_c^2 \gamma^2}{4k_c} (1 - e^{-2k_c \tau}) + \frac{\rho \sigma_x \sigma_c \gamma \alpha}{k_x + k_c} (1 - e^{-(k_x + k_c) \tau}) + \frac{\lambda_c}{k_c} \ln \left( \frac{\gamma \eta_{J,c}^* e^{-k_c \tau} - 1}{\gamma \eta_{J,c}^* - 1} \right) \end{aligned}$$

## FORWARD RISK PREMIUM

$$\begin{aligned} RP_t &\equiv \ln F(t, T, P) - \ln E_t^P(P_T) = \\ &= -\gamma \frac{\phi_c \sigma_c}{k_c} (1 - e^{-k_c \tau}) - \alpha \frac{\phi_x \sigma_x^{seas}}{k_x} (1 - e^{-k_x \tau}) + \frac{\lambda_c}{k_c} J^{RP} \end{aligned}$$

Forward Risk Premium will be higher (periods/markets) :

- Higher Volatility Demand
- Higher “Convexity Demand” and “Convexity Supply”:  $\alpha, \gamma$
- “Mkt. Price Demand risk” and “Mkt. Price Supply Risk”

Previous evidence PJM: summer months

Pirrong & Jermakyan (2002), Villaplana (2003), Bessembinder & Lemmon (JF, 2004),...

## MODEL B (Under the Objective Probability Measure)

$$P_t = C_t^\gamma \cdot \beta \cdot e^{\alpha \cdot D_t}$$

### DEMAND

$$D_t = s(t) + \xi_t^D + \chi_t^D$$

$$d\xi_t^D = \mu_D dt + \sigma_\xi dZ_\xi$$

$$d\chi_t^D = -k_\chi^D \chi_t^D dt + \sigma_\chi^D dZ_{\chi,D}$$

### SUPPLY

$$C_t = \theta_t + \chi_t^C$$

$$d\theta_t = \mu_\theta dt + \sigma_\theta dZ_\theta$$

$$d\chi_t^C = -k_C \chi_t^C dt + \sigma_\chi^C dZ_{\chi,C}$$

## MODEL UNDER THE RISK-NEUTRAL MEASURE

$$P_t = C_t^{\gamma_c} \cdot \beta \cdot e^{\gamma_D \cdot D_t}$$

$$\Rightarrow \ln P_t = [\ln \beta + \gamma_D \cdot g(t)] + \gamma_c (\theta_t^c + \chi_t^c) + \gamma_D \cdot (\xi_t^D + \chi_t^D)$$

$$D_t = g(t) + \xi_t^D + \chi_t^D$$

$$d\xi_t^D = \mu_D^* dt + \sigma_{\xi,D} dZ_{\xi,D}^*$$

$$d\chi_t^D = -(k_D \chi_t^D + \phi_D) dt + \sigma_{\chi,D} dZ_{\chi,D}^*$$

$$\ln C_t = c_t = \theta_t^c + \chi_t^c$$

$$d\theta = \mu_\theta^* dt + \sigma_\theta dZ_\theta^*$$

$$d\chi_t^c = -(k_c \chi_t^c + \phi_c) dt + \sigma_{\chi,c} dZ_{\chi,c}^*$$

$$\ln F(t, T, X) = f(T) + A(\tau) + \gamma_c \cdot \theta_t + \gamma_c \cdot c_t \cdot e^{-k_c \cdot \tau} \\ + \gamma_D \cdot \xi_t^D + \gamma_D \cdot \chi_t^D \cdot e^{-k_D \cdot \tau}$$

$$A(\tau) = \boxed{\mu_\theta^* \cdot \gamma_c \cdot \tau} - \frac{\phi_c \gamma_c}{k_c} \cdot (1 - e^{-k_c \tau}) + \boxed{\mu_D^* \cdot \gamma_D \cdot \tau} + \frac{\phi_D \cdot \gamma_D}{k_D} (1 - e^{-k_D \tau}) \\ + \frac{\sigma_\theta^2 \cdot \gamma_c^2}{2} \tau + \frac{\sigma_c^2 \cdot \gamma_c^2}{4k_c} (1 - e^{-2k_c \tau}) + \frac{\sigma_\xi^2 \cdot \gamma_D^2}{2} \tau + \frac{\sigma_D^2 \cdot \gamma_D^2}{4k_D} (1 - e^{-2k_D \tau})$$

## EMPIRICAL ANALYSIS: PJM market

We estimate MODEL A.

Daily Data:

Spot Prices, Demand, Available Capacity & 1-Month Forward Prices.

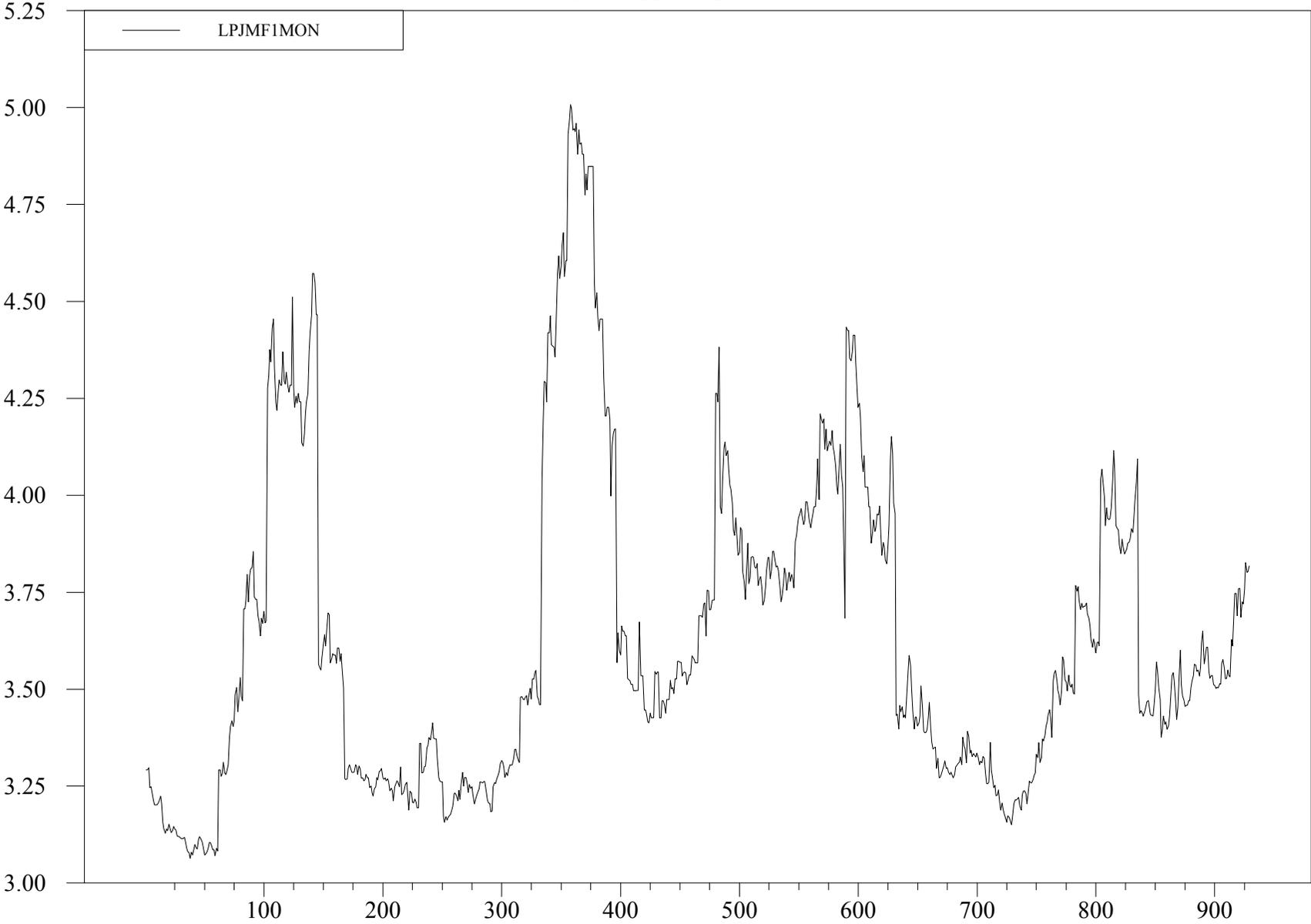
Period: 1999 – 2002

Step 1: Estimate the parameters under the empirical probability measure

Step 2: Extract risk-neutral parameters from Forward prices

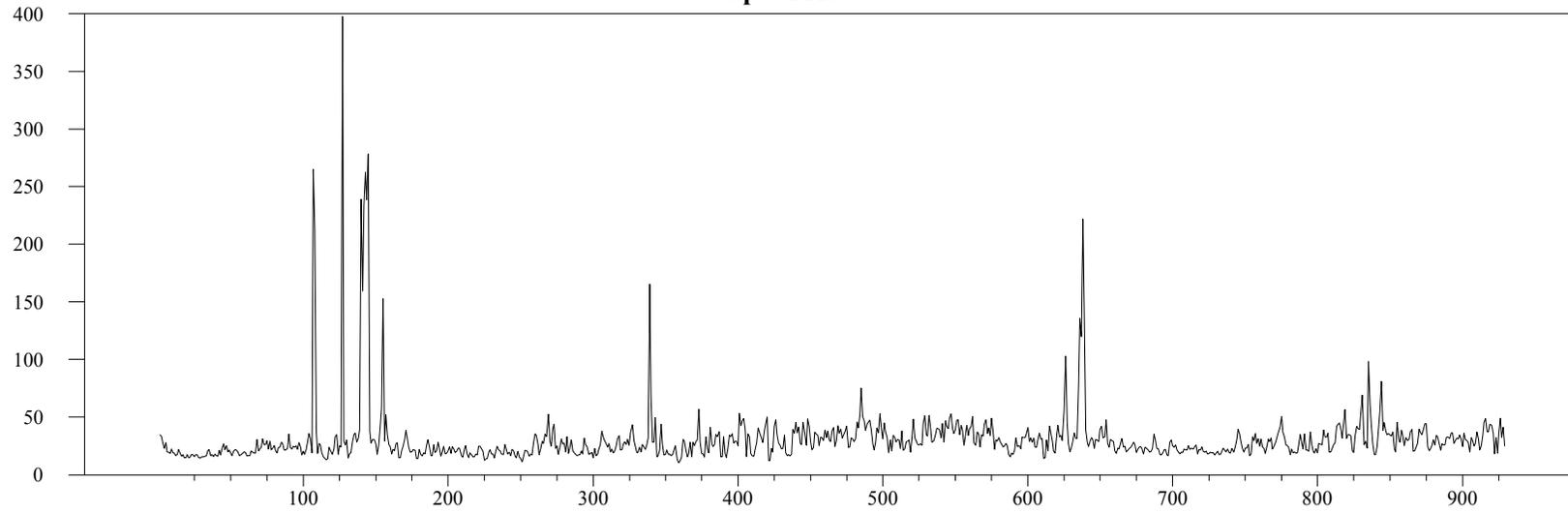
# Daily 1-month Log-Forward Price. PJM

Jan 1999 - March 2003

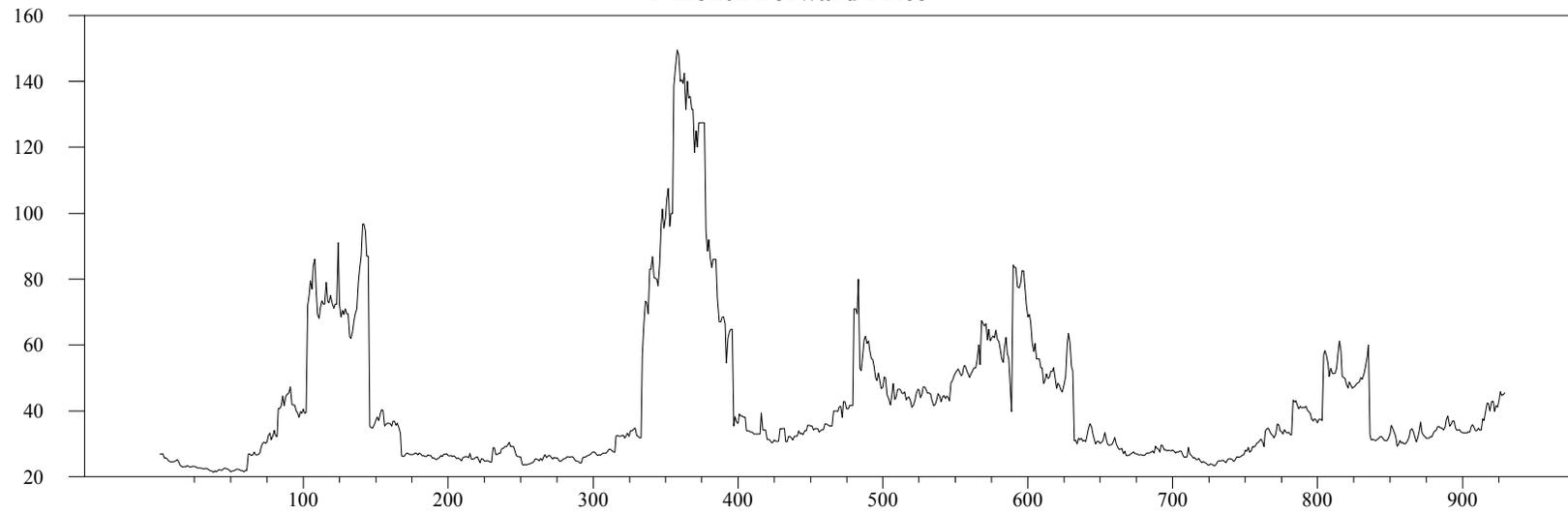


# PJM. Jan 1999 - March 2003

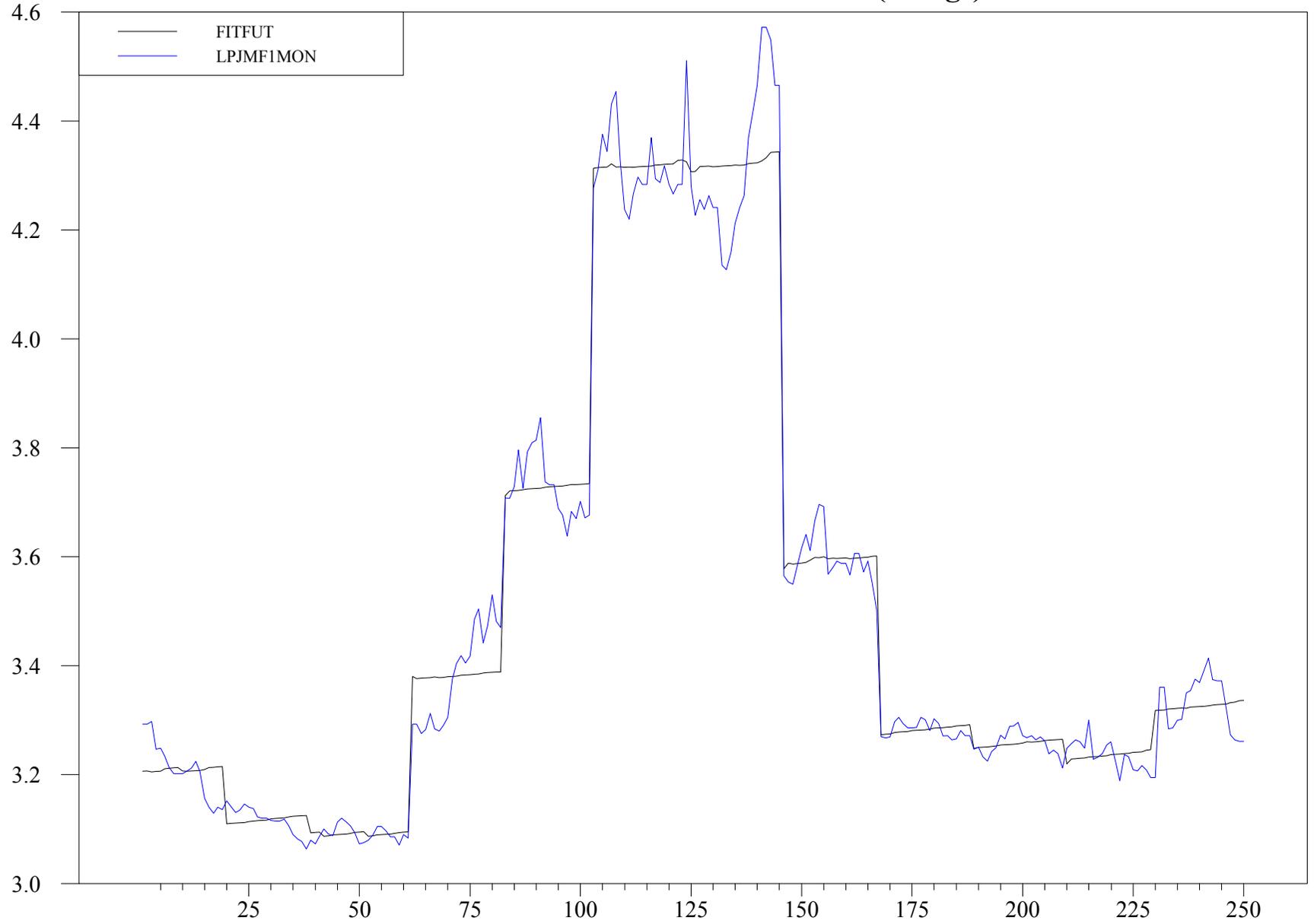
## Spot Price



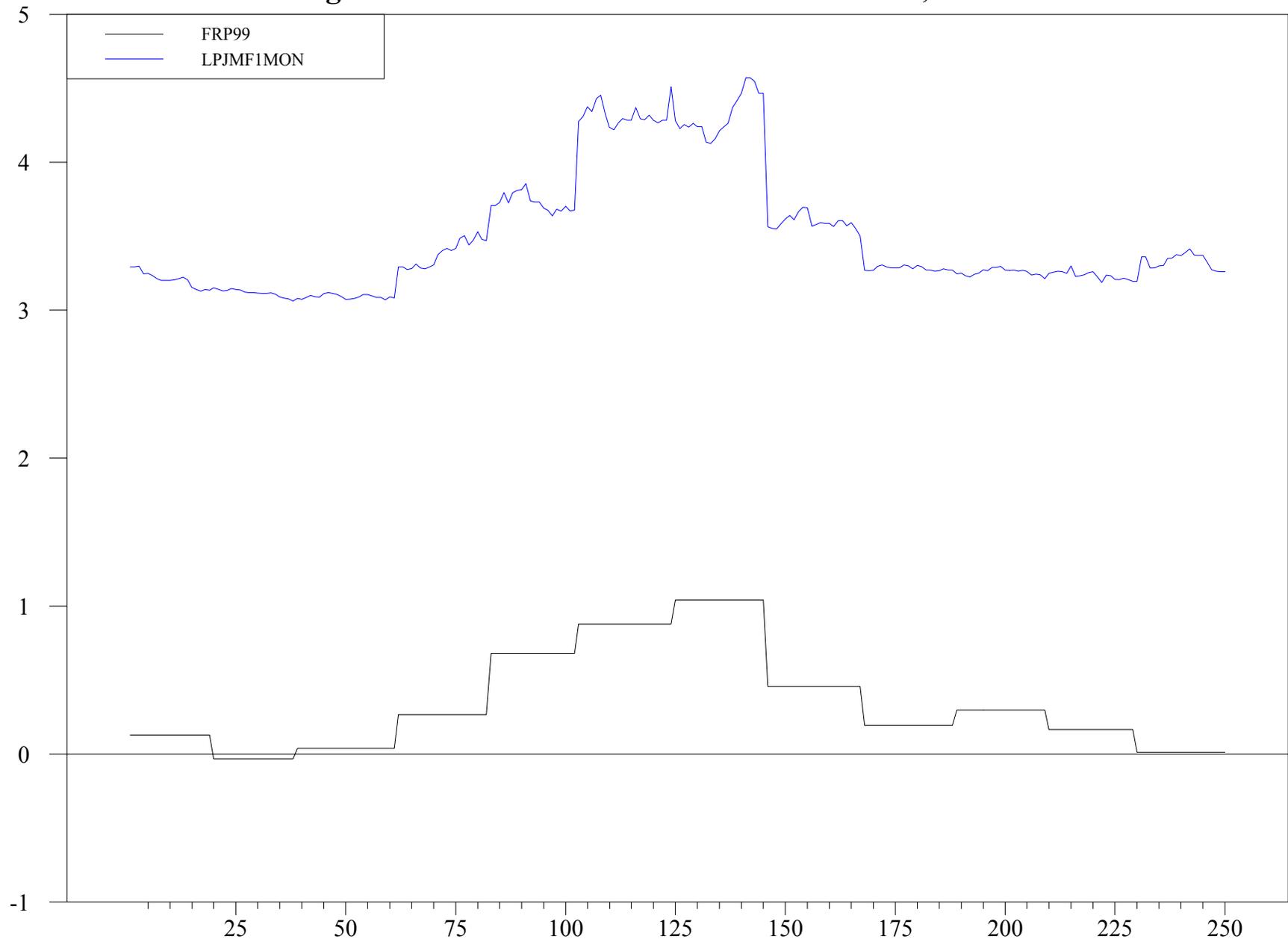
## 1-month Forward Price



**Model vs. Observed Forward Price 1999 (in logs)**

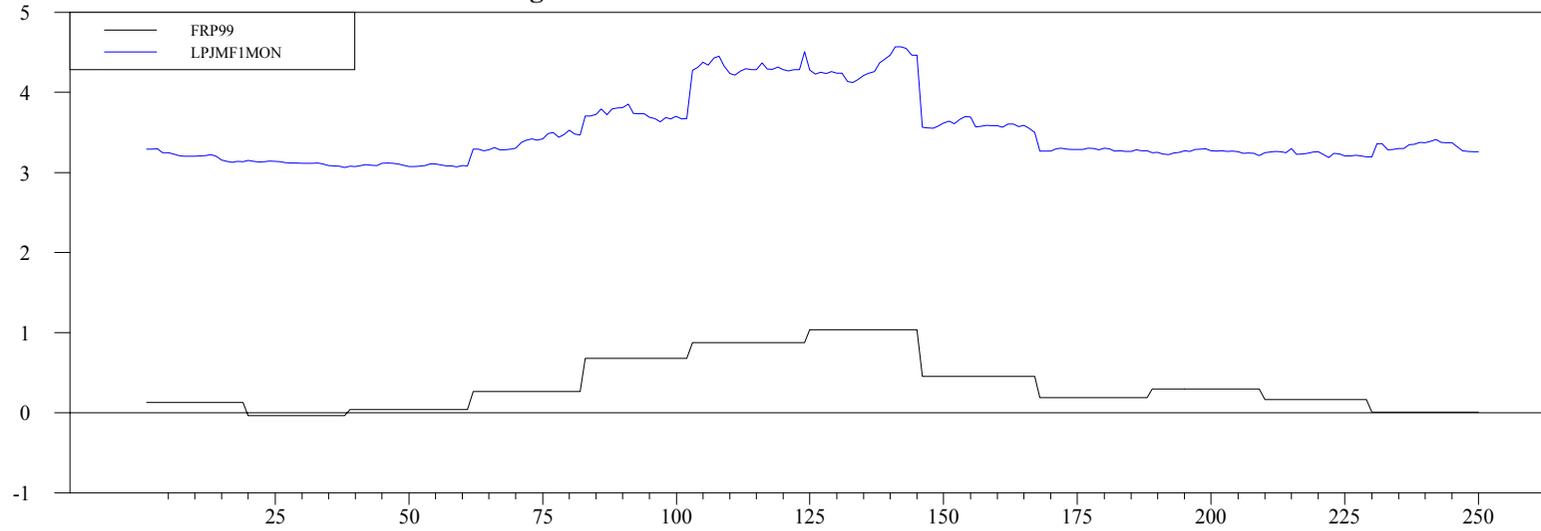


# Log-Forward Price & Forward Risk Premium, 1999

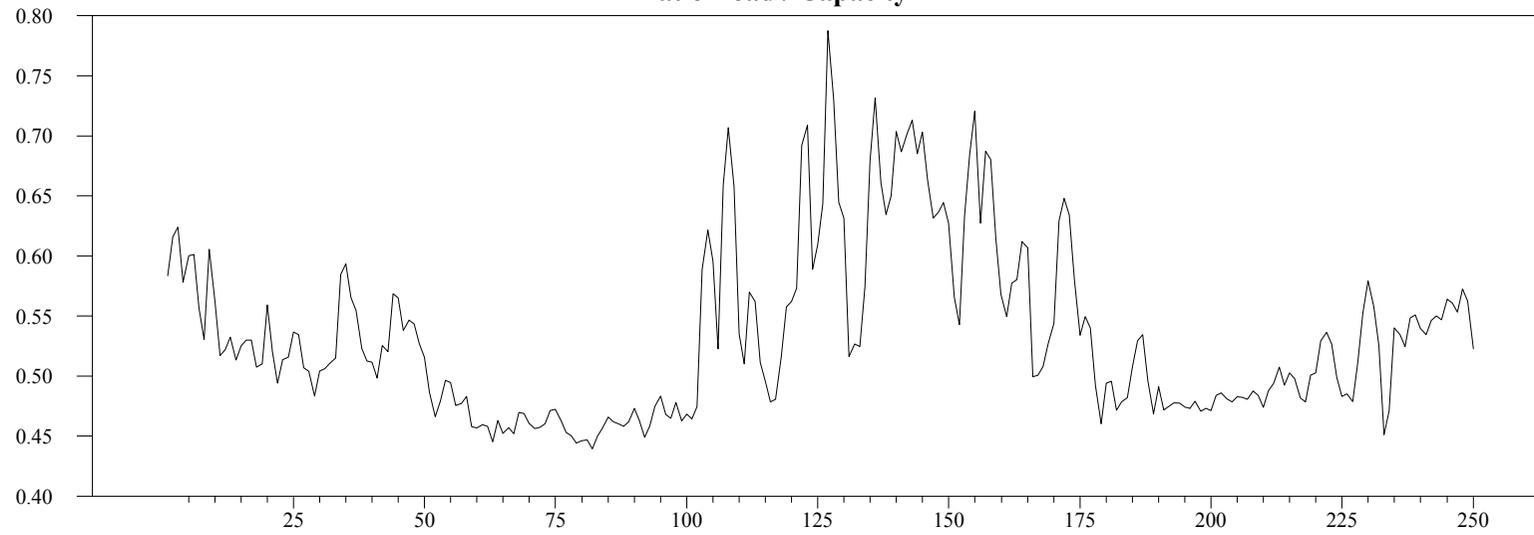


# PJM. 1999

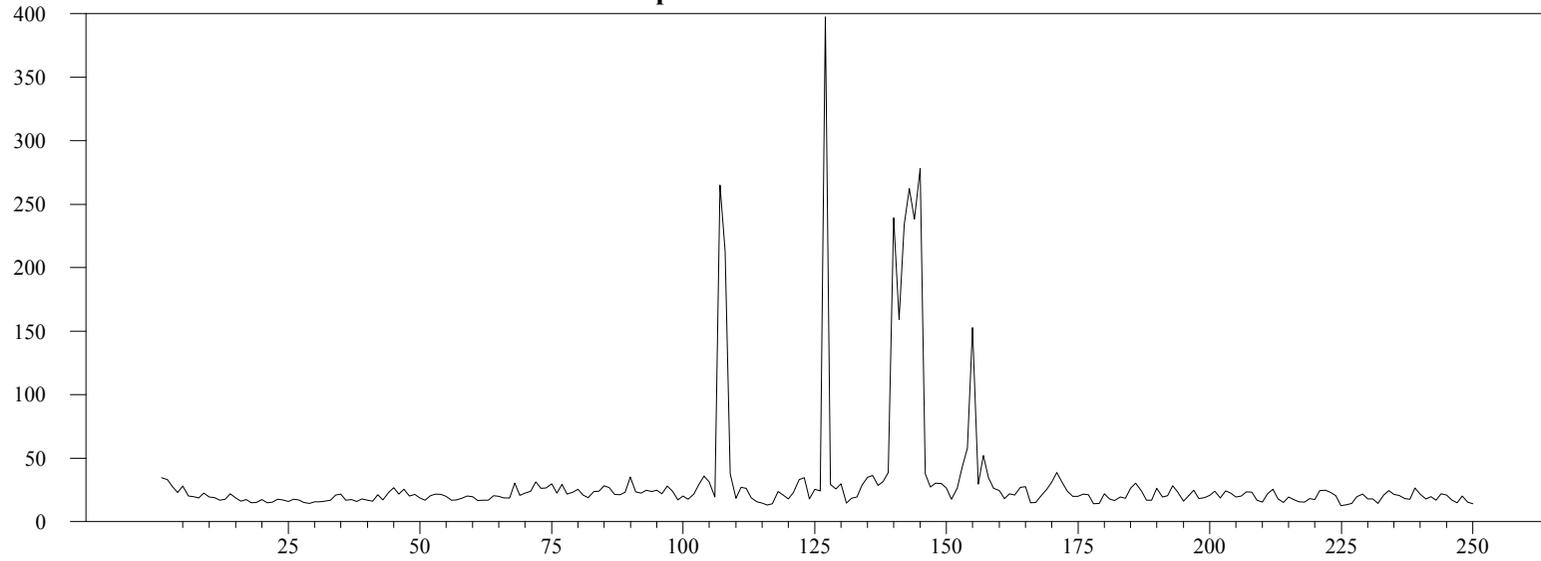
## Log-Forward Price & Forward Risk Premium



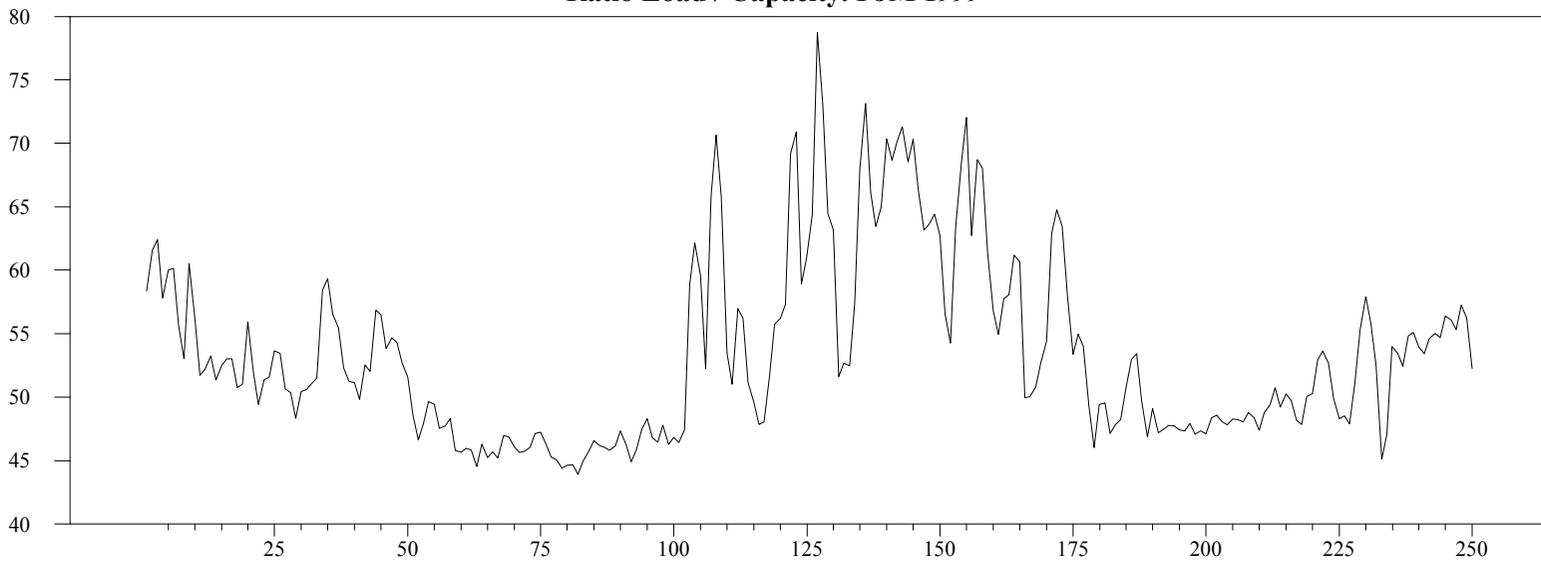
## Ratio Load / Capacity



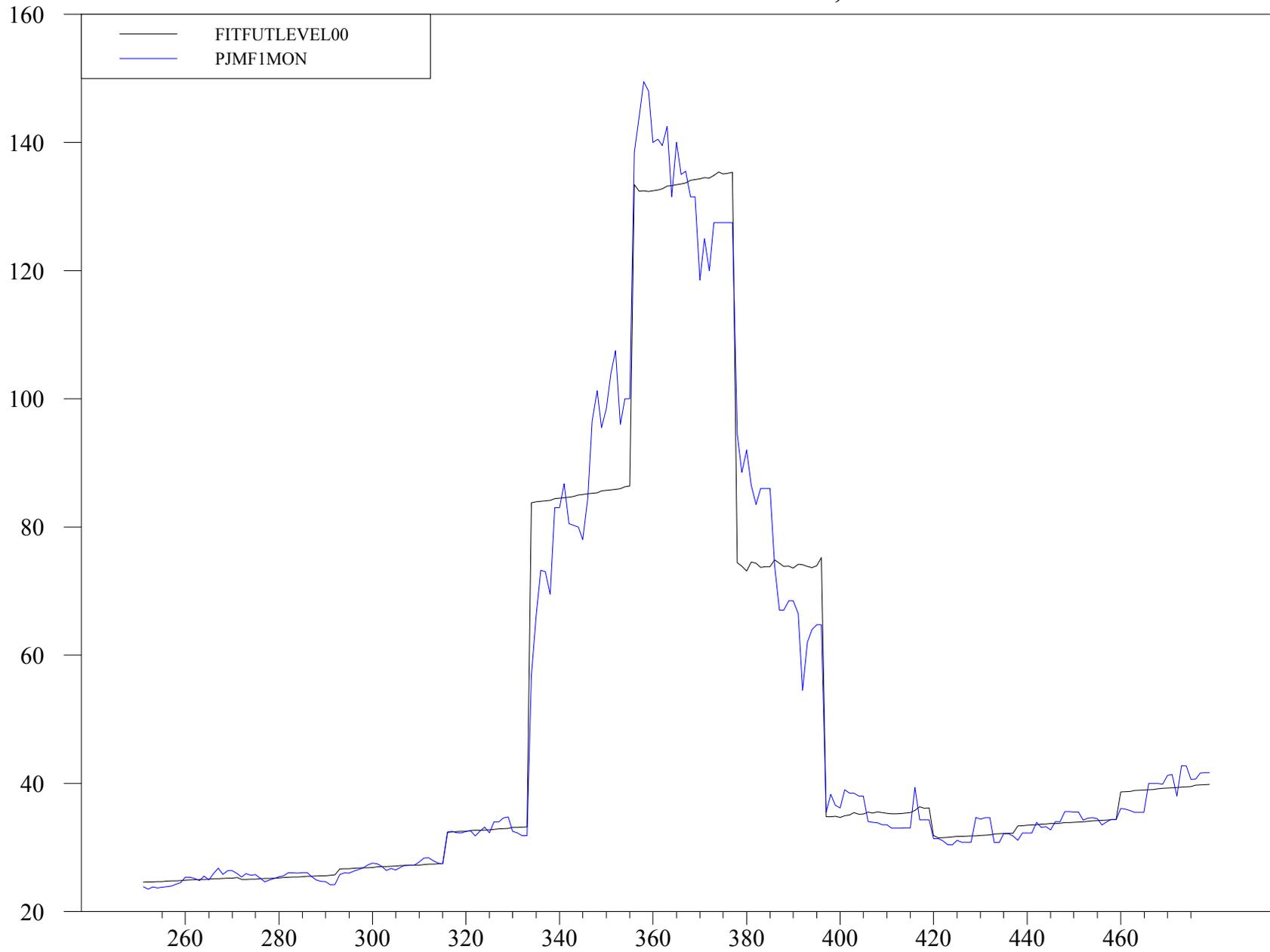
**Spot Price. PJM 1999**



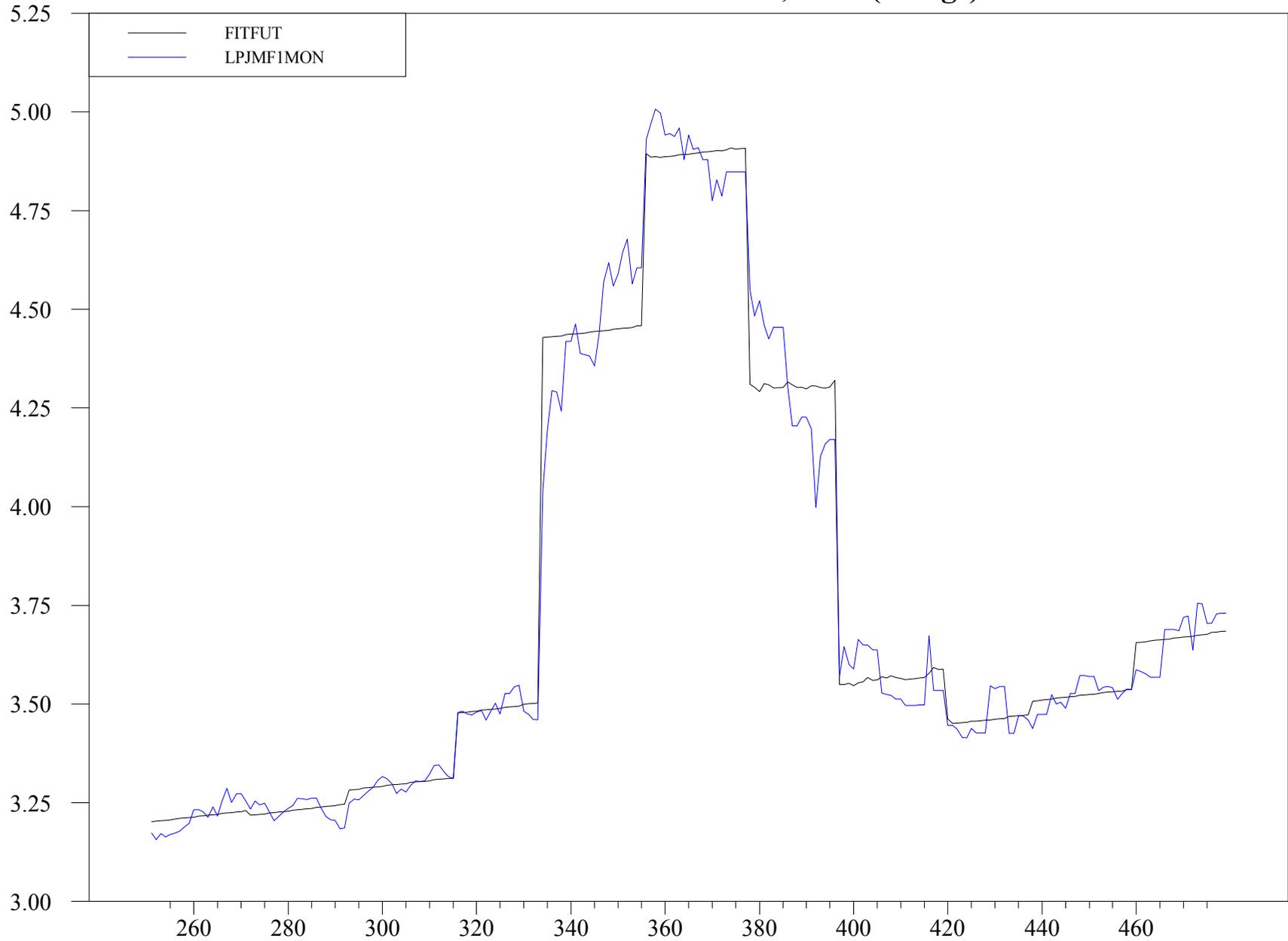
**Ratio Load / Capacity. PJM 1999**



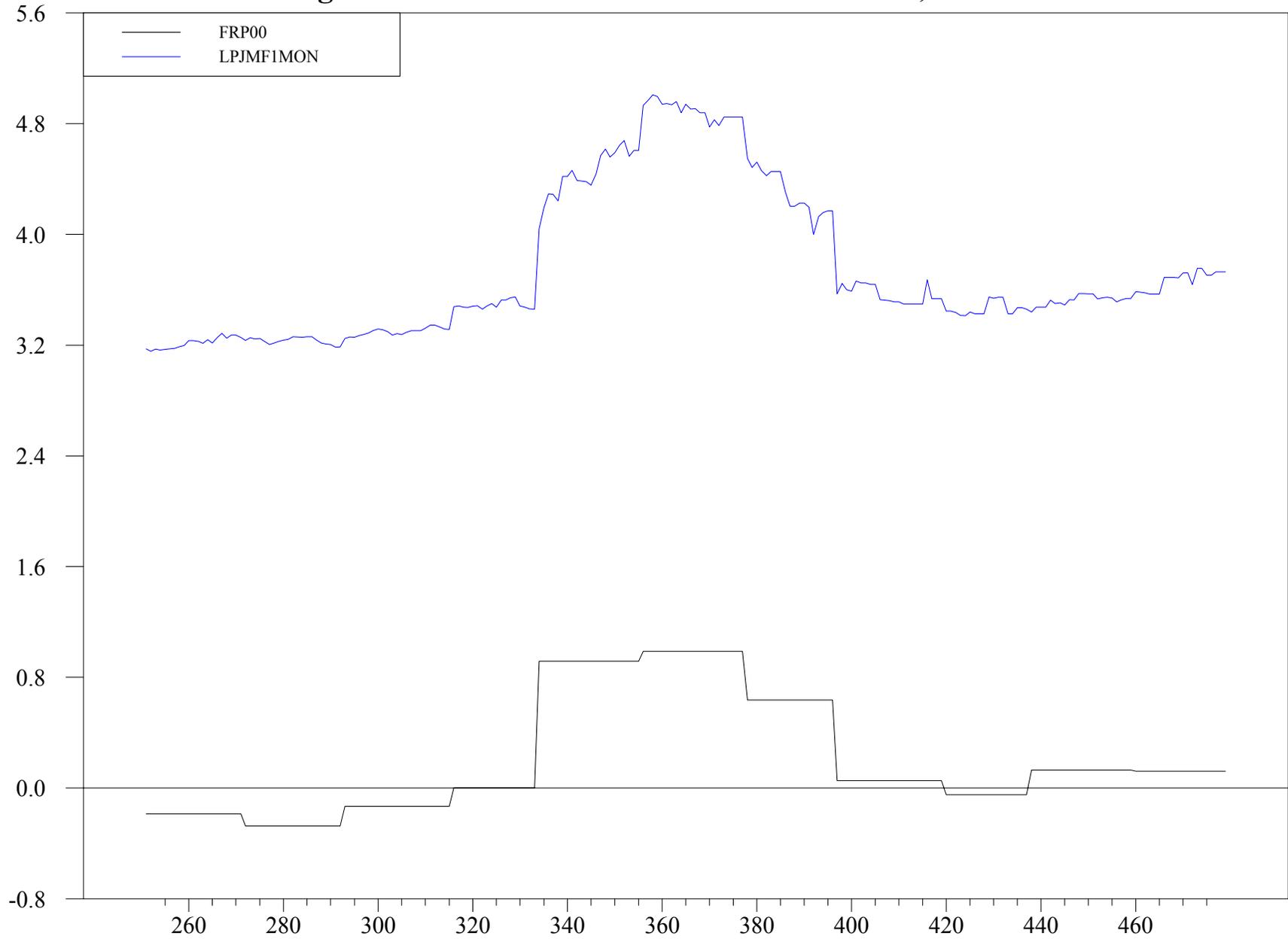
# Model vs. Observed Forward Price, 2000



**Model vs. Observed Forward Price, 2000 (in logs)**



# Log-Forward Price & Forward Risk Premium, 2000



## EMPIRICAL RESULTS:

The model captures the observed pattern of 1-month forward prices

When we extract the risk premium component, we find it to be significant and being determined by economic risks

We find that the seasonal risk premium is clearly related to the economic determinants.

Seasonality in Risk premium is not only generated by demand volatility.

The result obtained by empirically estimating the proposed model corroborates the results by Bessembinder & Lemmon (*JF*, 2002) and those of Longstaff & Wang (*JF*, 2004). Economic determinants of the forward risk premium.

# CONCLUSIONS

- WE HAVE PRESENTED A GENERAL FRAMEWORK THAT TAKES INTO ACCOUNT DEMAND AND SUPPLY (“Generation Capacity”) VARIABLES
- WE HAVE DERIVED A NEW PRICING FORMULA IN CLOSED FORM (we extend previous papers, Barlow, *Math. Fin.*, 2002)
- OUR MODEL IS A “FIRST STEP” ON THE ANALYSIS OF THE EFFECT OF DEMAND AND SUPPLY CONDITIONS ON DERIVATIVES PRICES
- PROVIDES INTUITION ON FORWARD RISK PREMIUM BEHAVIOR: “cross-market” and “time-series”