

Logic Gates and Boolean Functions

and first steps with CircuitVerse

Boolean Function (Scalar Valued)

- Function of the form

$$f : B^n \rightarrow B$$

with

$$B := \{0, 1\}$$

- We can interpret 0 as "false" and 1 as "true"

NOT Gate

Function

$$f : B \rightarrow B, x \mapsto f(x)$$

with

$$f(x) \equiv 1 - x$$

x	$\neg x$
0	1
1	0

AND Gate

Function

$$\wedge : B^2 \rightarrow B, (a, b) \mapsto a \wedge b$$

with

a	b	$a \wedge b$
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate

Function

$$\vee : B^2 \rightarrow B, (a, b) \mapsto a \vee b$$

with

a	b	$a \vee b$
0	0	0
0	1	1
1	0	1
1	1	1

XOR Gate (Exclusive Or)

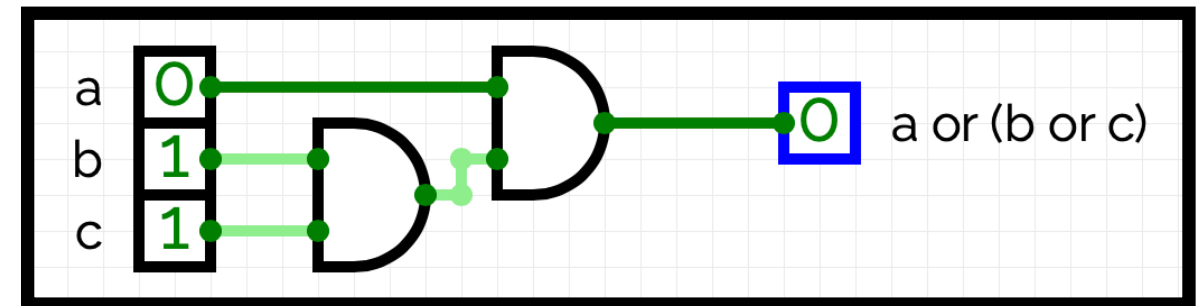
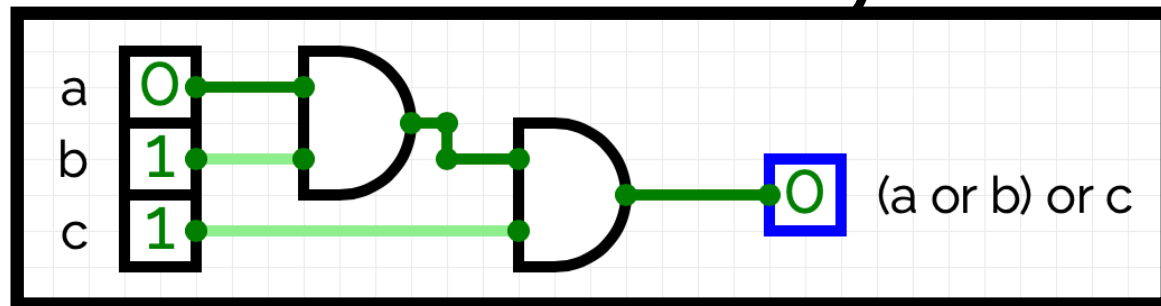
Function

$$\dot{\vee} : B^2 \rightarrow B, (a, b) \mapsto a \dot{\vee} b$$

with

a	b	$a \dot{\vee} b$
0	0	0
0	1	1
1	0	1
1	1	0

Associativity



$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

we can (for example) define

$$a \wedge b \wedge c := (a \wedge b) \wedge c$$

Convention: Operator Precedence

<i>Operator</i>	<i>Precedence</i>
$\neg$	4
$\wedge$	3
$\dot{\vee}$	2
$\vee$	1

$$a \vee (b \wedge c) = a \vee b \wedge c$$

Example

$$\neg a \wedge b \vee a \wedge \neg b$$

$$= ((\neg a) \wedge b) \vee (a \wedge (\neg b))$$

a	b	$\neg a$	$\neg b$	$\neg a \wedge b$	$a \wedge \neg b$	$\neg a \wedge b \vee a \wedge \neg b$
<i>0</i>	<i>0</i>	<i>1</i>	<i>1</i>	<i>0</i>	<i>0</i>	<i>0</i>
<i>0</i>	<i>1</i>	<i>1</i>	<i>0</i>	<i>1</i>	<i>0</i>	<i>1</i>
<i>1</i>	<i>0</i>	<i>0</i>	<i>1</i>	<i>0</i>	<i>1</i>	<i>1</i>
<i>1</i>	<i>1</i>	<i>0</i>	<i>0</i>	<i>0</i>	<i>0</i>	<i>0</i>