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Markov chains - Assignment 7

Exercise 1

Suppose that $h(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2), x \in \mathbb{R}$ is our target density (i.e. the density of a $N(0,1)$-distribution), and let $g(x) = \frac{1}{2}\exp(-|x|), x \in \mathbb{R}$ be the auxiliary density for the rejection sampling.

(a) Show that $h(x)/g(x) \le \sqrt{\frac{2}{\pi}\, e}$.

(b) Verify that random variable $X = WZ$ has density g given that Z and W are independent random variables with $Z \sim \mathrm{Exp}(1)$ and $P(W=-1) = 0.5$, $P(W=1) = 0.5$.

(c) Write down an algorithm to generate samples from h using rejection sampling based on g. At this, assume that your random number generator can only produce $U(0,1)$-pseudo random numbers.

Exercise 2

(a) Use Theorem 3.9 to generate samples from a Gamma distribution (with parameters $\lambda > 0$ and $r > 1$) by means of a quotient of uniform random variables.

(b) Repeat part (a) for a t-distribution with $r \in \mathbb{N}$ degrees of freedom.

Exercise 3

Generate samples from a χ^2-distribution with $r = 4$ degrees of freedom

(a) using exercise 2 (with $\chi^2(r) = \Gamma(\frac{1}{2}, \frac{r}{2})$),

(b) using sums of independend exponentially distributed random variables and

(c) using the respresentation as a sum over squared $N(0,1)$-random variables.

Implement all three methods and compare the results my means of the sample mean and variance for sample sizes $n = 10^2, 10^3, 10^4$ and 10^5.