# Imperfect competition, demand uncertainty and capacity adjustment

A microeconomic model of the firm and some macroeconomic implications

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### Abstract:

In this paper, a theoretical model of the price versus quantity adjustment of the firm is developed. The model is characterized by short-run capacity constraints, uncertainty about demand and imperfect competition on the product market. The microeconomic model is complemented by aggregation over firms. The aggregate model exemplifies the prominent role of capacity utilization as business cycle indicator and yields a variant of an accelerator model for the capacity adjustment. The demand and cost multipliers depend on the share of firms with capacity constraints, and the price adjustment is determined by unit labour costs, capacity utilization and the market structure.

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# 1 Introduction

The analyses of market structure and macroeconomic fluctuations are strongly related through the price-setting behaviour of firms. First, macroeconomic evidence on cyclical fluctuations of economic activity and the analysis of price vs. quantity adjustments of firms can reveal insights about the competitive situation on the markets.<sup>1</sup> For instance, the probably most important macroeconomic business cycle indicator is capacity utilization. However, in perfectly competitive industries, there is no reason, why firms should underutilize their capacities in case of demand shocks.

Second, the analysis of market structure and price adjustment can help to a better understanding of the propagation of macroeconomic shocks.<sup>2</sup> For instance, the classical and the Keynesian model differ mainly with respect to the underlying model of the price adjustment. In classical models, an immediate or very fast adjustment of prices and permanent market clearing is assumed. Keynesian models, in contrast, emphasize the relevance of price rigidities, market disequilibria and quantity reactions. Both models yield opposite policy implications; therefore, the analysis of price adjustment is important for macroeconomic policy.

A convenient framework to analyse price vs. quantity adjustments is monopolistic competition. Imperfect competition provides a prerequisite to a general theory of price adjustment.<sup>3</sup> The theory of price adjustment is dominated by the idea that prices adjust in the presence of excess demand or supply on the market. However, this mechanism is essentially ad hoc and does not reflect optimizing behaviour. In addition, it requires a disequilibrium interpretation of the price adjustment which is hardly compatible with perfect competition. It is known that monopolistic competition, by itself, cannot explain why aggregate demand movements affect output.<sup>4</sup> However, monopolistic competition of real and nominal quantitites.

The main imperfection which is analysed in this paper is a dynamic adjustment of capacities. In the theoretical model, it is assumed that capacities adjust only with a delay with respect to changes of costs and demand, thus under uncertainty about demand.<sup>5</sup> The analysis of a dynamic adjustment permits the consistent introduction of capacity constraints and uncertainty into the model. The analysis of dynamics in terms of adjustment delays simplifies the theoretical analysis of the model; it reduces the dynamic decision problem of the firm into a sequence of static decision models which can be solved stepwise.

<sup>&</sup>lt;sup>1</sup>See Hall (1986) and Carlton (1989).

<sup>&</sup>lt;sup>2</sup>See Mankiw (1985), Hall (1986,1988), Blanchard, Kiyotaki (1987) and Solow (1998).

<sup>&</sup>lt;sup>3</sup>See Arrow (1959) and Barro (1972).

<sup>&</sup>lt;sup>4</sup>See Blanchard, Kiyotaki (1987).

<sup>&</sup>lt;sup>5</sup>For the analysis of a delayed adjustment, see e.g. Kydland, Prescott (1982). The model here is basically a variant of the model of Hall (1986,1988).

As a first step, the short-run price and quantity adjustment of the firm is analysed within a model of monopolistic competition on the product market and short-run capacity constraints. The short-run model already yields cost and demand multipliers depending on capacity utilization. It also yields a Phillips-curve type of price adjustment function depending on unit labour costs and capacity utilization. The implied price and quantity adjustment is asymmetric during the business cycle.

An extended model introduces a delayed adjustment of prices and employment with uncertainty about demand and still predetermined capacities and production technology. This medium-run model provides a framework to analyse short-run price rigidities combined with constraints on the adjustment of quantities. The model can account for prolonged delivery lags and labour hoarding during recessions, i.e. it is consistent with a procyclically varying productivity of labour. In addition, it introduces a dynamic adjustment of prices vs. quantities with respect to demand shocks, demand expectations and capacity constraints. The theoretical model yields an error correction mechanism for prices and employment; the only required assumption about expectation formation is a positive autocorrelation of demand shocks.

The capacity adjustment is analysed in the long-run model. For the capacity decision, the optimal response of prices, output and employment with respect to demand shocks is taken into account. The model yields an accelerator mechanism for the capacity adjustment. Optimal capacities are determined by production costs and expected demand shifts. The model can be understood as an error correction model for investment; capacities adjust to achieve an optimal utilization in expected values. The substitution decision depends on relative factor costs and factor utilization. It is shown that the inefficiency associated with a delayed adjustment of capacities and demand uncertainty exhibits the same effects on optimal capacities, the capital-labour ratio and average prices as higher capital costs.

The model of the firm is complemented by aggregation. The microeconomic relations at the firm level are explicitely translated into macroeconomic relations between the aggregates. In addition, the equilibrium shares of firms with supply and demand constraints are determined. The aggregate model exemplifies the prominent role of capacity utilization as business cycle indicator; capacity utilization determines both the adjustment of prices and employment in the medium run and capital investment in the long run. The price adjustment depends on costs (supply shocks) and capacity utilization (demand shocks) according to a short-run Phillips curve mechanism. The extend of price vs. quantity adjustments is determined by the share of firms with capacity constraints, i.e. the medium-run demand and cost multipliers are regime dependent. This implies an asymmetric price and quantity adjustment with respect to demand and cost shocks during the business cycle.

The long-run capacity adjustment is determined by costs and demand expectations through a flexible accelerator mechanism for investment. This introduces a source of instability into the aggregate adjustment. Within the model, the short run and the long run are distinguished by the flexibility of capacities, not by the stickyness of prices as in standard Keynesian models. The model also provides a framework to discuss the impact of demand uncertainty and competition (market structure) on the adjustment of prices and quantities. The only departures from the standard model are a delayed adjustment with uncertainty about demand and monopolistic competition on the product market.

# 2 Assumptions

In the theoretical model, a strong separability of the dynamic structure of the firm's decisions is assumed: In the short run, output, employment and prices are endogenous. In the long run, the firm decides on investment and the production technology, i.e. capacities and the production technology adjust only with a delay with respect to demand and cost changes, thus under uncertainty about demand.<sup>6</sup> As an extention, a delayed adjustment of prices and employment is discussed.

The advantage of these assumptions, as compared with the assumption of an immediate adjustment, is that disequilibria and adjustment dynamics can be explicitely analysed. In most adjustment models, dynamics are analysed under the assumption of non-linear adjustment costs. However, it is difficult to find examples for adjustment costs which can account for the observed slow adjustment of many economic variables. On the other hand, changing decision variables necessarily takes time, and even a short delay between the decision to change capacities or the price and the realization of a demand shock can introduce considerable uncertainty. In addition, the analysis of the dynamic adjustment in terms of adjustment delays and uncertainty reduces the dynamic decision problem of the firm to a sequence of static decision models which can be solved stepwise:

- In the basic model, the determination of output, prices and employment takes place in the short run with predetermined capacities and production technology.
- In the extended model, the adjustment of prices and employment takes place in the medium run, thus under uncertainty about demand.
- Capacities and the production technology are determined in the long run; therefore both variables can be treated as predetermined for the shortand medium-run adjustment. The investment decision takes the expected optimal adjustment of output, prices and employment into account.

 $<sup>^6{\</sup>rm For}$  a discussion of adjustment delays, see Kydland, Prescott (1982).

The theoretical analysis is carried out within a framework of imperfect competition on the product market.<sup>7</sup> Monopolistic competition provides a useful framework for the analysis of the price vs. quantity adjustment. It is firstly a prerequisite for the analysis of price *change*.<sup>8</sup> Price adjustment with respect to excess demand or supply requires a disequilibrium interpretation which is hardly compatible with perfect competition. In addition, the introduction of capacity constraints and demand uncertainty implies a monopolistic adjustment of the firms operating on the market at least in the short run. Finally, fixed costs of production and increasing returns to scale associated with for instance innovations or advertising provide further arguments for a monopolistic market structure.<sup>9</sup>

Within the microeconomic analysis, a market is defined by the supply of a single firm and the demand for the firm's product. In the sequel, an aggregation procedure is discussed to derive implications for macroeconomic relations. In order to distinguish demand shifts, the price elasticity of demand and demand uncertainty, a log-linear demand curve is assumed,<sup>10</sup>

$$\ln YD = \eta \cdot \ln p + \ln Z + \varepsilon, \quad \eta < -1, E(\varepsilon) = 0, \operatorname{Var}(\varepsilon) = \sigma_{\varepsilon}^{2}. \tag{1}$$

The time and firm indices are omitted to simplify the notation. Demand YD is determined by the price p, exogenous demand shifts Z and a demand shock  $\varepsilon$ . The demand shift Z stands for aggregate demand and the market share of the firm which is determined by, for instance, the prices of other firms, consumer preferences and the quality of the firm's product. The assumption of a constant price elasticity of demand  $\eta$  is not important for the results. It should be seen as a reference case or a local approximation which permits a convenient discussion of the deviations caused by demand uncertainty, capacity constraints and imperfect competition.

In the basic model, it is assumed that the realized value of the demand shock  $\varepsilon$  is known at the time of the output, price and employment decision, but is not known at the time of the investment decision. In the extended version, there is uncertainty about  $\varepsilon$  at the time of the price and employment decision. For convenience, it is assumed that the distribution of  $\varepsilon$  can be approximated by a continuous probability distribution function (p.d.f.) which is completely characterized by its expected value  $E(\varepsilon)$  and its variance  $\sigma_{\varepsilon}^2$ . In addition,  $\varepsilon$  exhibits positive autocorrelation which is exploited by the firm for the adjustment. Further assumptions about expectation formation are not required to derive the qualitative properties of the adjustment process.<sup>11</sup>

<sup>&</sup>lt;sup>7</sup>See e.g. Barro (1972) and Dixit, Stiglitz (1977).

 $<sup>^{8}</sup>$ See Arrow (1959) and Barro (1972).

<sup>&</sup>lt;sup>9</sup>See for instance Kamien, Schwarz (1975), Cohen, Levin (1989), Scherer, Ross (1990) and Aghion, Howitt (1992).

<sup>&</sup>lt;sup>10</sup>Log-linear demand curves can be derived from CES utility functions (Deaton, Muellbauer, 1980), and log-linear relations permit an easy aggregation over firms (Lewbel, 1992).

<sup>&</sup>lt;sup>11</sup>For instance, the solution of non-linear adjustment cost models usually requires the strong and inconsistent assumption of static expectations.

Supply YS is determined by a short-run limitational production function with capital K and labour L as inputs,

$$YS = \min(YC, YL) = \min(\pi_k \cdot K, \pi_l \cdot L), \quad \pi_l = \pi_l(k, \theta), \pi_k = \pi_k(k, \theta).$$
(2)

YC are capacities, YL is the employment constraint and  $\pi_l$ ,  $\pi_k$  are the productivities of labour and capital. It is assumed that the capital stock as well as the factor productivities are predetermined in the short run; they are determined by the long-run investment decision. Adjustment delays for the capital stock arise from planning, decision, delivery and installation lags for investment, the assumption of short-run fixed production coefficients corresponds to a putty-clay technology.<sup>12</sup> The factor productivities are determined by the capital-labour ratio k and production efficiency  $\theta$ . The factor prices are assumed to be exogenous at the firm level.

These assumptions imply constant marginal costs within the capacity limit in the short run. Note that this strong linearity is not required, nothing of importance depends on it. However, constant marginal costs of production appear as a plausible first approximation for the short-run adjustment with fixed capacities. In addition, it permits an easy definition of capacities from the production side alone. It should also be seen as a reference case which permits to discuss the effects of capacity constraints more clearly.

## 3 Output, prices and employment

#### 3.1 Imperfect competition and capacity constraints

As the starting point, the short-run adjustment of output, prices and employment is discussed. The model corresponds to the standard framework of imperfect competion, extended with short-run capacity constraints.<sup>13</sup> The optimization problem of the firm can be written as

$$\max_{\to p, Y, L} p \cdot Y - w \cdot L - c \cdot K \quad \text{s.t.} \quad Y \le \{YC, YL, YD\}.$$
(3)

Supply and demand are determined according to eqs. (1) and (2) above. w are wages and c are the user costs of capital. Since the capital stock and the factor productivities are predetermined, the first order condition is

$$p \cdot (1+1/\eta) \cdot (1-\lambda_{YC}) \cdot \pi_l - w = 0. \tag{4}$$

 $\lambda_{YC}$  is the shadow price of the capacity constraint; it is zero in case of sufficient capacities. For the optimal solution, two cases can be distinguished:

 $<sup>^{12}</sup>$ The analysis of a dynamic adjustment of capacities has a long tradition in empirical investment models, see Jorgenson (1963) and Jorgenson, Stephenson (1967). The assumption of a putty-clay technology became common with the work of Bischoff (1971).

<sup>&</sup>lt;sup>13</sup>The model is basically a variant of the model of Hall (1986,1988).

1. In case of sufficient capacities  $\lambda_{YC} = 0$ , the optimal price, output and employment result from

$$p(w) = \frac{w}{\pi_l \cdot (1 + 1/\eta)},$$
 (5)

$$\ln Y(w) = \eta \cdot \ln p(w) + \ln Z + \varepsilon \quad \text{and} \quad L(w) = Y(w)/\pi_l.$$
 (6)

The optimal price is determined by unit labour costs and the price elasticity of demand, output results from introducing this price into the demand function, and employment is the labour input required to produce this output. The firm suffers from underutilization of capacities.

2. In case of capacity shortages  $\lambda_{YC} \neq 0$ , output, employment and the price result from

$$Y = YC, \quad L(YC) = YC/\pi_l, \tag{7}$$

$$\ln p(YC) = (\ln YC - \ln Z - \varepsilon)/\eta.$$
(8)

Optimal output is equal to the capacity constraint, employment is again given as the corresponding labour requirement, and the optimal price results from solving the demand function for p at YD = YC. Insufficient capacities restrain output and employment, the firm increases the price.

There is exactly one value of the demand shock  $\varepsilon = \overline{\varepsilon}$  which distinguishes these cases,

$$\overline{\varepsilon} = \ln YC - \eta \cdot \ln p(w) - \ln Z. \tag{9}$$

The most important characteristics of the model are a minimum price p(w) and a capacity limit YC. The supply curve is horizontal (flat) within the borders of capacity and vertical (steep) at the capacity limit. The optimal price is determined either by unit labour costs and the degree of competition on the market or by the relation of the levels of demand and capacity; optimal output and employment are determined either by unit labour costs and the level of demand or by capacities.

<u>Figure 1</u> gives a visual impression of the model. For a negative demand shock  $\varepsilon_1 < \overline{\varepsilon}$ , the price is determined by unit labour costs and the mark-up is determined by the price elasticity of demand. The firm suffers from underutilization of capacities. For a positive demand shock  $\varepsilon_2 > \overline{\varepsilon}$ , insufficient capacities restrain output and the firm increases the price.  $\varepsilon = \overline{\varepsilon}$  is the borderline which distinguishes these cases. In the short run, the firm adjusts with respect to demand either by changing the price (in the capacity constrained case), or by changing output and employment (in the unconstrained case).

Note the implied asymmetry of the price and quantity adjustment. For demand increases, the adjustment of output and employment is bounded by capacities and the price rises instead. For demand reductions, the price adjustment is bounded by marginal costs and the price elasticity of demand and output and employment are reduced instead, i.e. the model supplies an argument for a downward rigidity of prices in recessions. A similar asymmetry





results for cost changes: In case of capacity constraints, output, prices and employment remain unchanged; in case of sufficient capacities, output, prices and employment adjust; real wages change only in case of capacity constraints and with changes of productivity and competition.

Finally, a large variance of demand shocks requires a high frequency of price and quantity adjustments. In addition, in case of a high capacity utilization, price adjustments should be frequent and quantity adjustments should be rare; in case of a low capacity utilization, quantity adjustments should be frequent and price adjustments should be rare. That means the model provides a theoretical foundation for the relevance of capacity utilization as a business cycle indicator. It yields clear testable hypotheses about the effects of capacity utilization on the direction and the variability of price vs. quantity adjustments with respect to demand and cost shocks.

The microeconomic model of the firm also provides a consistent basis for aggregation. If firms differ only with respect to the realization of the demand shocks  $\varepsilon$ , the microeconomic minimum condition of supply and demand of the firms can be explicitly translated into a macroeconomic relation between the average (expected) values of demand and supply and the variance of demand shocks  $\sigma_{\varepsilon}^2$ . For instance, if the distribution of  $\varepsilon$  is approximated by the normal distribution, the aggregate relation exhibits the same functional form as the microeconomic relation, except for a change of the normalizing constant which is determined by the variance of demand shocks,<sup>14</sup>

$$\ln \mathcal{E}(YD) = \mathcal{E}(\ln YD) + 0.5 \cdot \sigma_{\varepsilon}^{2} = \eta \cdot \ln p + \ln Z + 0.5 \cdot \sigma_{\varepsilon}^{2}.$$
 (10)

E is the expectation operator,  $n \cdot E(YD)$  is aggregate demand, n is the number of firms. If costs, prices and demand shifts differ between firms, the normalizing constant is determined by the variance of the logarithm of demand at the micro level.<sup>15</sup> In addition, the aggregate counterpart of the microeconomic minimum condition can accurately be approximated by a CES-type function of aggregate output  $n \cdot E(Y)$  in terms of aggegate capacities  $n \cdot E(YC)$  and aggregate demand  $n \cdot E(YD)$ ,

$$E(Y)^{1/\rho} \approx E(YD)^{1/\rho} + E(YC)^{1/\rho}, \quad \rho < 0.$$
 (11)

 $\rho$  can be interpreted as a mismatch parameter (mismatch between demand and capacities) with  $\partial E(Y)/\partial \rho < 0$  and  $\lim_{\rho \to 0} E(Y) = \min[E(YD), E(YC)]$ .  $\rho$  is completely determined by the covariance of capacities and demand at the micro level.<sup>16</sup> The aggregate multipliers, i.e. the elasticities of aggregate output with respect to capacities and demand can be calculated from eq. (11) as

$$\frac{\partial \mathcal{E}(Y)}{\partial \mathcal{E}(YD)} \cdot \frac{\mathcal{E}(YD)}{\mathcal{E}(Y)} = \left\{\frac{\mathcal{E}(YD)}{\mathcal{E}(Y)}\right\}^{1/\rho} = \operatorname{prob}(YD < YC)$$
(12)

and correspondingly for capacities. These elasticities approximate the regime probabilites, i.e. the shares of firms within the respective regime. The aggregate model implies that the demand and cost multipliers depend on the business cycle. In boom situations with a high capacity utilization and a large share of firms with capacity constraints, prices adjust with respect to demand with only small output and employment effects and only small effects from cost changes. In recession periods with a large share of firms with sufficient capacities, quantities (output and employment) adjust with respect to demand and cost changes, and prices adjust only with respect to costs. The microeconomic case dependency of cost and demand effects corresponds to demand and cost multipliers depending on the regime shares at the macro level; the share of firms exhibiting capacity constraints is determined by the relation of aggregate demand and aggregate capacities.

The aggregate model also implies an augmented short-run Phillips curve mechanism for the price adjustment: Prices adjust with respect to unit labour cost (supply shocks) and capacity utilization (demand shocks), i.e. the model replicates certain stylized facts of the business cycle. If in addition aggregate demand depends on employment, the model yield the usual Keynesian multiplier but only within the borders of capacities,<sup>17</sup> i.e. the model exhibits

<sup>&</sup>lt;sup>14</sup>See Smolny (1993) and Stoker (1993) for a discussion.

<sup>&</sup>lt;sup>15</sup>The variance of the logarithm of demand is determined by the variances and correlations of demand shocks  $\varepsilon$ , demand shifts Z and prices (costs).

<sup>&</sup>lt;sup>16</sup>See Smolny (1993). It can be shown that  $\rho$  is determined by a nearly linear relation in terms of the standard deviation of  $\ln YD - \ln YC$  within the empirically relevant range.

<sup>&</sup>lt;sup>17</sup>In addition, labour supply constraints and endogenous wage adjustments might constrain the multiplier.

both classical and Keynesian features. From a macroeconomic viewpoint, the relevant variable is the share of firms with sufficient capacities.

The model extends the standard formulation of imperfect competition by introducing medium-run capacity constraints. Capacity constraints firstly increase the realism of monopolistic price setting, i.e. capacity constraints provide a microeconomic foundation of a monopolistically competitive market structure. A competitive firm is a special case of the model: It always chooses full utilization of capacities at the market price which is equal to marginal costs. This implies that underutilization of capacities during the business cycle indi*cates* a monopolistically competitive market structure. Second, capacity constraints are a reasonable assumption for the short-run adjustment of output, employment and prices. Adjusting capacities necessarily takes time, and investment models were among the first which introduced a dynamic adjustment into economic analyses of firm behaviour. Finally, the combination of imperfect competition and capacity constraints yields reasonable macroeconomic effects for the determination of the short-run multiplier and the price adjustment during the business cycle. The model exhibits both classical and Keynesian features without recurrence to price rigidities; capacity constraints imply an asymmetric adjustment of prices and quantities with respect to positive and negative shocks.

## 3.2 Uncertainty and the price and employment adjustment

Now the model is extended to introduce uncertainty into the price and employment adjustment. It is assumed that prices and employment must be chosen in advance, thus under uncertainty about demand. Adjustment delays for employment can be justified with legal/contractual periods of notice and search, screening and training time.<sup>18</sup> The assumption that the firm sets price tags also appears reasonable,<sup>19</sup> and even a short delay between the decision to change the price and the realization of demand can introduce considerable uncertainty. In this model, output is determined in the short run as the minimum of demand and supply, i.e.

$$Y = \min(YD, YS). \tag{13}$$

The medium-run optimization problem is

$$\max_{\to L,p} p \cdot \mathcal{E}(Y) - w \cdot L - c \cdot K \tag{14}$$

s.t. eqs. (1) and (2) above. Expected output is determined as

$$E(Y) = E[\min(YD, YS)] = \int_{-\infty}^{\overline{\varepsilon}} YD \cdot f_{\varepsilon} d\varepsilon + \int_{\overline{\varepsilon}}^{\infty} YS \cdot f_{\varepsilon} d\varepsilon$$
(15)

<sup>&</sup>lt;sup>18</sup>See e.g. Blanchard, Diamond (1992) and Hamermesh, Pfann (1996).

<sup>&</sup>lt;sup>19</sup>See Carlton (1986,1989) and Blinder (1991).

 $f_{\varepsilon}$  is the p.d.f. of the demand shock  $\varepsilon$ . For small values of the demand shock, output is determined by demand (the first integral); for large values of  $\varepsilon$ , output is determined by supply (the second integral);  $\overline{\varepsilon}$  is defined as the specific value of the demand shock  $\varepsilon$  where demand equals supply,

$$\overline{\varepsilon} = \ln YS - \eta \cdot \ln p - \ln Z. \tag{16}$$

The first order conditions of the optimization problem with respect to prices and employment are given  $by^{20}$ 

$$\eta \cdot \int_{-\infty}^{\overline{\varepsilon}} YD \cdot f_{\varepsilon} d\varepsilon + \mathcal{E}(Y) = 0, \qquad (17)$$

$$p \cdot \int_{\overline{\varepsilon}}^{\infty} f_{\varepsilon} d\varepsilon \cdot (1 - \lambda_{YC}) \cdot \pi_l - w = 0.$$
 (18)

From eqs. (15), (16) and (17), it can be shown that the optimal  $\overline{\varepsilon}$  depends only on the price elasticity of demand  $\eta$  and demand uncertainty  $\sigma_{\varepsilon}$  (see appendix A, proposition A.1),

$$\overline{\varepsilon} = \overline{\varepsilon}(\eta, \sigma_{\varepsilon}). \tag{19}$$

 $\overline{\varepsilon}$  and  $\sigma_{\varepsilon}$  also determine the expected utilization of supply  $U_l := E(Y)/YS$  and the optimal probability of demand constraints,  $\operatorname{prob}(YD < YS)$  (see appendix A, proposition A.2). That means, utilization and the regime probabilities do not depend on costs, capacities and expected demand shifts Z. The economic intuition of this result is that (for given supply and costs) the elasticity of output with respect to the price is chosen equal to one: With higher prices, demand decreases with elasticity  $\eta$ ; expected output decreases with elasticity  $\eta$ , times the weighted probability that demand is less than supply; the expected share of output in the demand constrained case is chosen equal to the inverse of the absolute value of the price elasticity of demand,<sup>21</sup>

$$\operatorname{prob}_{w}(YD < YS) := \frac{\int_{-\infty}^{\overline{\varepsilon}} YD \cdot f_{\varepsilon} d\varepsilon}{\int_{-\infty}^{\overline{\varepsilon}} YD \cdot f_{\varepsilon} d\varepsilon + \int_{\overline{\varepsilon}}^{\infty} YS \cdot f_{\varepsilon} d\varepsilon} = -\frac{1}{\eta}.$$
 (20)

The firm chooses the price to achieve an optimal probability of supply constraints and an optimal utilization of supply. For optimal prices and employment, two cases can be distinguished:

1. In case of capacity constraints  $\lambda_{YC} \neq 0$ , supply and employment are determined from capacities and labour productivity,

$$Y = YL = YC$$
 and  $L(YC) = YC/\pi_l$ . (21)

The optimal price results from inserting capacities and the optimal  $\overline{\varepsilon}$  into the definition of  $\overline{\varepsilon}$  and solving for p,

$$\ln p(YC) = \left[ \ln YC - \ln Z - \overline{\varepsilon}(\eta, \sigma_{\varepsilon}) \right] / \eta.$$
(22)

<sup>&</sup>lt;sup>20</sup>Note that the value of the integrands in eq. (15) at  $\varepsilon = \overline{\varepsilon}$  are equal.

 $<sup>^{21}</sup>$ Inserting the definition of expected output, eq. (15), into the first order condition with respect to prices, eq. (17), yields eq. (20).

The price depends on capacities YC, expected demand shifts Z and the optimal  $\overline{\varepsilon}$ ; the elasticity of the price with respect to capacities and the demand shift is  $1/\eta$ ; the price does not depend on costs.

2. In case of sufficient capacities  $\lambda_{YC} = 0$ , the optimal price follows directly from the first order condition with respect to employment, eq. (18). The marginal costs of an additional unit of employment are equal to the wage rate w. Marginal returns are determined as the price, multiplied with the productivity of labour and multiplied with the probability that the additional unit of output can be sold; the mark-up of prices on unit labour costs is chosen equal to the inverse of the optimal probability of supply constraints,

$$\frac{w}{\pi_l \cdot p(w)} = \operatorname{prob}(YL < YD).$$
(23)

Since the optimal probability of supply constraints is competely determined by demand uncertainty  $\sigma_{\varepsilon}$  and the price elasticity of demand  $\eta$ , the price does not depend on capacities and expected demand shifts. The firm adjusts quantities with respect to demand. The optimal price can also be determined from the price elasticity of demand, unit labour costs and the expected utilization of employment (see appendix A, proposition A.3),

$$p(w) = \frac{w}{U_l \cdot \pi_l \cdot (1 + 1/\eta)}.$$
(24)

The inefficiency associated with demand uncertainty and a delayed adjustment exhibits the same effect as higher wage costs. Supply and employment result from inserting this price and the optimal  $\overline{\varepsilon}$  into the definition of  $\overline{\varepsilon}$  and solving for supply YL and employment L,

$$YL(w) = \eta \cdot \ln p(w) + \ln Z + \overline{\varepsilon}(\eta, \sigma_{\varepsilon}) \quad \text{and} \quad L(w) = YL/\pi_l.$$
(25)

The immediate adjustment (or the absence of uncertainty) is contained as the limiting case  $\sigma_{\varepsilon} \to 0$ . Without uncertainty,  $U_l \to 1$ , and the firm can achieve full utilization of employment. Introducing uncertainty reduces the expected utilization of employment and exhibits the same effect on prices and employment as higher variable costs. Figure 2 gives a visual impression of the model.  $f_{YD}$  is the p.d.f. of demand. For small values of L and YL, the probability that the marginal unit of labour will be used is large; the marginal returns of labour exceed marginal costs. For higher values of YL, the probability that demand exceeds supply decreases, and the marginal return of labour decreases, a unique optimum is therefore assured. If capacities restrain supply, the firm increases the price to achieve the optimal probability of supply constraints and the optimal utilization of supply.

The model extends the standard formulation of monopolistic competition by introducing uncertainty about demand and medium-run capacity constraints. The assumption of a delayed adjustment of prices and employment enhances the economic interpretation of the model. Ex ante, the firm sets prices and employment under uncertainty about demand, i.e. the firm chooses one point in



the  $\{p, Y\}$ -diagram (see figure 3). Uncertainty increases the optimal price and reduces employment through the costs of underutilization of employment. Relevant for the price setting is a capacity limit  $YS = YL \leq YC$  and a minimum price p(w) which is determined by unit labour costs, the price elasticity of demand and demand uncertainty. In case of sufficient capacities, there is a clear correspondence of income distribution shares, the price elasticity of demand and the probability of demand constraints; in case of capacity constraints, the relation of the demand shift Z and capacities YC determines the optimal price. The optimal regime probabilities are still determined by uncertainty and the price elasticity of demand.

Ex post, rationing of demand or underutilization of employment can occur. For a positive demand shock  $\varepsilon = \varepsilon_1$ , the firm cannot satisfy all customers (delivery lags), for a negative demand shock  $\varepsilon = \varepsilon_2$ , underutilization of capacities and labour hoarding occur. Short-run demand shocks can be identified from the utilization of the production factors. The short-run demand situation can be identified from the utilization of employment, the medium-run business-cycle situation can be identified from the utilization of capacities.

The model also provides a framework for the analysis of the price and quantity adjustment during the business cycle. Suppose the stochastic process generating the demand shocks is autocorrelated. The firm exploits this autocorrelation when forming demand expectations for the future. Then, a short-run demand shock affects output and the utilization of labour and capital today. The adjustment of the firm depends on the availability of capacities: In case of capacity constraints (in boom periods), the firm adjusts the price



and employment remains unchanged; with sufficient capacities (in recession periods), the price remains unchanged and the firm adjusts employment. The model can be understood as an error correction model: Prices and employment adjust to achieve an optimal utilization; if the actual utilization differs from the optimum, prices and/or employment adjust.

The dynamic formulation of the model provides clear testable hypotheses about the microeconomic effects of capacity utilization on the direction and the variability of price vs. quantity adjustments with respect to demand and cost shocks (see <u>table 1</u>).<sup>22</sup> It also provides a hypothesis about effects of demand uncertainty and the price elasticity of demand on the price and quantity adjustment.<sup>23</sup> Demand uncertainty increases the variance of output and therefore increases average costs. Prices should be higher and employment should be lower. In addition, uncertainty increases the necessity of price and employment adjustments; it becomes more difficult to achieve a high utilization of capacities and employment. A low price elasticity of demand  $|\eta|$ , i.e. less competition should result in higher prices and less employment. In combination with fixed costs of price adjustments, less competition should favour quantity adjustments against price adjustments in case of demand shocks. That means, the model also yields testable hypotheses about market-structure effects on

 $<sup>^{22}</sup>$ Empirical studies of the price adjustment at the firm level are rare, but in Smolny (1998b), the expected effects of capacity utilization on the direction and the volatility of price and quantity adjustments are confirmed by an empirical analysis with a large panel of firm-level data from West German manufacturing.

<sup>&</sup>lt;sup>23</sup>For a more detailed discussion, see Barro (1972), Hall (1986), Carlton (1986), Blanchard, Kiyotaki (1987) and Smolny (1998a).

|            | U | Z | $ \eta $ | $\sigma_arepsilon$ | w | $\pi_l$ |
|------------|---|---|----------|--------------------|---|---------|
| $\Delta y$ | + | + | +        | —                  | — | +       |
| $\Delta L$ | + | + | +        | —                  | — | ?       |
| $\Delta p$ | + | + | _        | +                  | + | _       |
| $\sigma_y$ | — |   | _        | +                  |   |         |
| $\sigma_L$ | _ |   | _        | +                  |   |         |
| $\sigma_p$ | + |   | +        | +                  |   |         |

Table 1: Market structure and capacity utilization

price rigidities and price vs. quantity adjustments.

The extention of the model also enhances the macroeconomic interpretation of the effects of imperfect competition and capacity constraints.<sup>24</sup> The assumption of a delayed adjustment of prices introduces demand uncertainty, price rigidities and rationing of demand (prolonged delivery lags) in the short run. It also permits a discussion of wage-price patterns and a staggered price setting for the analysis of the aggregate price adjustment.<sup>25</sup> The assumption of a delayed adjustment of employment permits an interpretation of the procyclical development of labour productivity in terms of optimal labour hoarding during recessions. The assumption of a slow adjustment of prices and quantities introduces dynamics into the multiplier process and can explain the persistence of disequilibrium situations. Finally, the assumption of autocorrelated demand shocks introduces expectations into the analysis of the dynamic adjustment of prices and quantities.

## 4 Capacities and capital-labour substitution

In the long run, the firm decides on capacities and the production technology. Since there is uncertainty about the demand shock  $\varepsilon$ , the realized future values of output, prices and employment are not known at the time of the investment decision. However, the firm knows the decision rule for those variables: They are given by the solutions of the short- and medium-run optimization problems as discussed above. The capacity adjustment is firstly analyzed within the model of the short-run adjustment of output, employment and prices; the deviations caused by a delayed adjustment of prices and employment are discussed afterwards.

 $<sup>^{24}{\</sup>rm The}$  aggregate counterparts of the microeconomic relations can again be derived from the aggregation procedure discussed above.

 $<sup>^{25}</sup>$ See e.g. Blanchard (1987).

## 4.1 Demand uncertainty and capacity adjustment

The firm maximizes expected profits which depend on expected sales, expected employment, the wage rate and capital costs. The decision variables are the capital stock K and the capital-labour ratio k. It is assumed that the production function is characterized by constant returns to labour and capital.<sup>26</sup> In the short run, output, prices and employment are determined from eqs. (5)-(8) above. Output is determined by demand in case of sufficient capacities and by capacities in case of sufficient demand. Sales result from introducing the corresponding prices, employment is given by the corresponding labour requirement. The optimization problem is

$$\max_{\to K,k} \int_{-\infty}^{\overline{\varepsilon}} \left( p(w) - \frac{w}{\pi_l} \right) \cdot Y(w) \cdot f_{\varepsilon} d\varepsilon + \int_{\overline{\varepsilon}}^{\infty} \left( p(YC) - \frac{w}{\pi_l} \right) \cdot YC \cdot f_{\varepsilon} d\varepsilon - c \cdot K.$$
(26)

 $\varepsilon$  and  $f_{\varepsilon}$  capture the uncertainty about demand at the time of the investment decision. The first order condition with respect to the capital stock K is<sup>27</sup>

$$\int_{\overline{\varepsilon}}^{\infty} \left[ p(YC) \cdot (1 + 1/\eta) - w/\pi_l \right] \cdot \pi_k \cdot f_{\varepsilon} d\varepsilon - c = 0.$$
(27)

Marginal costs are given by the user costs of capital c. Marginal returns to capital are achieved only, if capacities become the binding constraint for output, i.e. if  $\varepsilon > \overline{\varepsilon}$ . They are given by the price, minus the price reduction of a marginal increase in output, minus wage costs in the capacity constrained case. A unique optimum exists, p(YC) is decreasing in YC and K.<sup>28</sup> The following properties can be derived. The optimal value of  $\overline{\varepsilon}$  depends only on the price elasticity of demand, the variance of demand shocks and relative factor costs (see appendix B, proposition B.1),

$$\overline{\varepsilon} = \overline{\varepsilon} \left( \eta, \sigma_{\varepsilon}, \frac{c}{\pi_k} \frac{\pi_l}{w} \right).$$
(28)

 $\overline{\varepsilon}$  and  $\sigma_{\varepsilon}$ , in turn, determine both the probability of demand constraints prob(YD < YC) and the expected utilization of capacities  $U_c := E(Y)/YC$  (see appendix B, proposition B.2). Higher relative capital costs increase optimal utilization and reduce the probability of demand constraints; with high fixed costs, the firm chooses a higher probability of capacity constraints. More competition, i.e. a higher absolute value of the price elasticity of demand  $|\eta|$  also increases optimal utilization and reduces the probability of demand constraints. Both, higher relative capital costs and more competition increase the ratio between marginal costs and marginal returns of capital. More uncertainty reduces optimal utilization, since it becomes more difficult to achieve a higher utilization, and the probability of demand constraints increases.<sup>29</sup>

 $<sup>^{26}</sup>$ This assumption is not necessary for the qualitative results. It should be seen as a reference case or a local approximation which simplifies the discussion.

<sup>&</sup>lt;sup>27</sup>The value of both integrands in eq. (26) at  $\varepsilon = \overline{\varepsilon}$  is equal.

 $<sup>^{28}\</sup>mathrm{The}$  integrand is equal to 0 at the lower border of the integral.

 $<sup>^{29}\</sup>sigma_{\varepsilon}$  affects the relation between average p(YC) and p(w).

Both, the expected utilization of capacities and the regime probabilities do not depend on expected demand shifts Z and the level of factor costs. Note that equal regime probabilities, or equality of supply and demand in expected values has no specific meaning within the model and do not define an equilibrium. The equilibrium regime probability is defined by the optimal solution of the model, i.e. the optimal  $\overline{\varepsilon}$  which is determined by the price elasticity of demand, demand uncertainty and relative factor costs. The choice of capacities can be understood as the optimal choice of capacity utilization and regime probability.

Expected prices E(p) are determined as mark-up over labour and capital costs (see appendix B, proposition B.3), the *average* price depends also on the expected utilization of capacities (see appendix B, proposition B.4),

$$\frac{\mathrm{E}(p \cdot Y)}{\mathrm{E}(Y)} = \left(\frac{w}{\pi_l} + \frac{c}{U_c \cdot \pi_k}\right) / (1 + 1/\eta).$$
(29)

More uncertainty reduces the expected utilization of capacities; a lower utilization of capacities, in turn, exhibits the same effect on average prices as higher capital costs c. Finally, optimal capacities are determined as<sup>30</sup>

$$\ln YC = \eta \cdot \ln p(w) + \ln Z + \overline{\varepsilon}.$$
(30)

Optimal capacities depend loglinear on the demand shift Z, expected demand shifts increase all quantities proportionally and do not affect prices or relative quantities. This implies an accelerator mechanism for the capacity adjustment. Higher relative capital costs reduce capacities through the optimal value of  $\overline{\varepsilon}$ . A proportional increase in c and w leaves  $\overline{\varepsilon}$ , the regime probabilities and capacity utilization unchanged, but increases the price proportionally. Capacities decrease with elasticity  $|\eta|$ , the model exhibits linear homogeneity both in prices and quantities. Less competition reduces capacities through higher prices and through a lower optimal utilization, and more uncertainty reduces optimal capacifies through a lower utilization. Demand uncertainty exhibits the same effect on capacities and average prices as higher capital costs. The model without uncertainty is contained for  $\sigma_{\varepsilon} \to 0$  and  $U_c \to 1$ . Without uncertainty, the price is set as a mark-up over total costs and the mark-up is determined by the price elasticity of demand; optimal capacities and employment are given by the equality of demand YD, capacities YC and the corresponding employment constraint YL.

The second component of the investment decision concerns the choice of the optimal capital-labour ratio k. The capital-labour ratio, in turn, determines the productivities of labour and capital  $\pi_l, \pi_k$ . The optimal capital-labour ratio can be derived from differentiating eq. (26) with respect to k. The calculations are tedious but not difficult, and the result is intuitive. The optimal relation between the elasticities of the factor productivities of labour and capital with respect to the capital-labour ratio is chosen equal to the ratio of the corrected

 $<sup>^{30}</sup>$ Eq. (30) results from inserting eq. (28) into eq. (9) and solving for YC.

factor shares,<sup>31</sup>

$$-\frac{\frac{\partial \pi_k}{\partial k} \cdot \frac{k}{\pi_k}}{\frac{\partial \pi_l}{\partial k} \cdot \frac{k}{\pi_l}} = \frac{w \cdot U_c}{c} \frac{\pi_k}{\pi_l}.$$
(31)

Again, the inefficiency caused by uncertainty and a delayed adjustment exhibits the same effects as higher capital costs and favours substitution of labour against capital; the model without uncertainty is contained for  $\sigma_{\varepsilon} \to 0$  and  $U_c \to 1$ .

The assumption of a delayed adjustment of capacities and capital-labour substitution extends the deterministic model by introducing uncertainty and permits to analyse the resulting inefficiencies. Ex ante, the firm chooses capacifies and the factor productivities under uncertainty about demand. With uncertainty, optimal capacities and expected output are lower due to the costs of stochastic underutilization of capacities. Uncertainty also increases average prices and reduces the optimal capital-labour ratio through the effect on utilization. The optimal regime probabilities, the optimal utilization of capacities and the optimal capital-labour ratio do not depend on the level of costs and the level of demand. They are determined by relative costs, demand uncertainty and the price elasticity of demand. The model exhibits linear homogeneity both in prices and in quantities, thus yielding an accelerator model for investment. Depending on the adjustment speed of capacities with respect to demand expectations, this introduces a source of instability into the aggregate adjustment. Ex post, different regimes on the goods market and underutilization of capacities are possible. Since the demand multiplier depends on the share of firms with capacity constraints, the instability associated with the capacity adjustment is reduced. Firms exhibiting capacity constraints cannot increase output and employment in case of demand increases, and prices rise instead. Price increases reduce demand, employment and investment.

The model also provides a framework to analyse the price and quantity adjustment during the business cycle. Consider a positive demand shock. The short-run effects depends on capacity utilization: Firms with sufficient capacities increase output and employment, and capacity utilization increases; firms with capacity constraints increase only the price. If positive demand expectations persist, firms will, with a delay, increase capacities. The model can be understood as an error correction model for investment: Capacities adjust, if capacity utilization differs from the optimum. With higher capacities, output and employment increase further, while capacity utilization and prices should decrease. That means, demand shocks should exhibit an effect on prices, capacity utilization and regime proportions only in the short run.

A similar asymmetry results in case of cost shocks. Firms with capacity constraints should leave output, prices and employment unchanged in the short run; firms with sufficient capacities should increase the price which reduces

<sup>&</sup>lt;sup>31</sup>In case of a Cobb-Douglas production function, this relation is equal to the relative output elasticities of the factors, see appendix C. The appendix also contains the results for a CES production function

output and employment. In the long-run, capacities are reduced which in turn reduces employment and increases the price. The relevant variable for the aggregate adjustment is the share of firms with capacity constraints.

## 4.2 A three-step decision structure

The capacity adjustment can also be analysed in combination with uncertainty about demand for the price and employment adjustment. Let us assume uncertainty about the demand expectations at the time of the price and employment decision, i.e. uncertainty about the expected demand shift Z,

$$\ln Z = \ln \overline{Z} + z, \quad \mathcal{E}(z) = 0, \text{Var}(z) = \sigma_z^2.$$
(32)

z measures the difference of demand expectations at the time of the investment decision and the time of the price and employment decision. Prices and employment then depend on the realized value of z. In particular, employment and prices are determined either from eqs. (21) and (22) in the capacity constrained case or from eqs. (24) and (25) in the unconstrained case. There is exactly one value  $z = \overline{z}$  which distinguishes these cases,

$$\overline{z} = \ln YC - \ln Z - \overline{\varepsilon} - \eta \cdot \ln p(w).$$
(33)

Expected employment is determined as

$$E(L) = \int_{-\infty}^{\overline{z}} L(w) \cdot f_z dz + \int_{\overline{z}}^{\infty} L(YC) \cdot f_z dz$$
(34)

 $f_z$  is the p.d.f. of z. Expected output can be determined from expected employment and the expected utilization of employment,  $E(Y) = U_l \cdot \pi_l \cdot E(L)$ . Expected sales result as  $E(p \cdot Y) = U_l \cdot \pi_l \cdot E(p \cdot L)$ . Note that the expected utilization of employment is completely determined by the price elasticity of demand  $\eta$  and demand uncertainty at the time of the price and employment decision  $\sigma_{\varepsilon}$ , i.e. it is not stochastic and does not depend on the capacity decision.<sup>32</sup> The long-run optimization problem can be written as<sup>33</sup>

$$\max_{\to K} \int_{-\infty}^{\overline{z}} [(p(w) \cdot \pi_l \cdot U_l - w] \cdot L(w) \cdot f_z dz + \int_{\overline{z}}^{\infty} [p(YC) \cdot \pi_l \cdot U_l - w] \cdot L(YC) \cdot f_z dz - c \cdot K.$$
(35)

This formulation of the optimization problem shows that the solution of the model can be performed correspondingly to the basic model of section 4.1 above. The first order condition with respect to the capital stock is given by<sup>34</sup>

$$\int_{\overline{z}}^{\infty} [p(YC) \cdot (1+1/\eta) \cdot U_l - w/\pi_l] \cdot \pi_k \cdot f_z dz - c = 0.$$
(36)

<sup>32</sup>Note that  $\overline{\varepsilon}$  and the regime probabilities on the product market are also determined by  $\eta$  and  $\sigma_{\varepsilon}$ , i.e. they are also not stochastic and do not depend on the capacity decision.

 $<sup>^{33}</sup>$ See eq. (26) for comparison.

<sup>&</sup>lt;sup>34</sup>The value of both integrands in eq. (35) at  $z = \overline{z}$  is equal. Note that only L(w) and p(YC) are stochastic and only L(YC) and p(YC) depend on capacities.

Note that capacities affect output, prices and employment only if capacities are the binding constraint for employment. It can be shown that the optimal  $\overline{z}$  depends only on uncertainty about z, the price elasticity of demand  $\eta$  and relative factor costs (see appendix D, proposition D.1),

$$\overline{z} = \overline{z} \left( \eta, \sigma_z, \frac{c}{\pi_k} \frac{\pi_l}{w} \right). \tag{37}$$

 $\overline{z}$  and  $\sigma_z$ , in turn, determine the probability of capacity constraints for employment prob(YC < YL) (see appendix D, proposition D.2), i.e. the optimal  $\overline{z}$  and the regime probabilities for employment do not depend on  $\overline{\varepsilon}$  and the utilization of employment. Demand uncertainty for prices and employment affects both production factors equally. The expected utilization of capacities  $U_c := E(Y)/YC$  depends on  $\overline{z}$  and on the utilization of employment (see appendix D, proposition D.3). In addition, expected prices E(p) are determined again as mark-up on costs and the average price is determined as mark-up over corrected factor costs (see appendix D, proposition D.4 and D.5),

$$\frac{\mathrm{E}(p \cdot Y)}{\mathrm{E}(Y)} = \left(\frac{w}{U_l \cdot \pi_l} + \frac{c}{U_c \cdot \pi_k}\right) / (1 + 1/\eta).$$
(38)

Finally, optimal capacities are determined as<sup>35</sup>

$$\ln YC = \eta \cdot \ln p(w) + \ln \overline{Z} + \overline{z} + \overline{\varepsilon}.$$
(39)

The whole analysis corresponds to those in section 4.1 above; the only difference is that the (under)utilization of employment must be taken into account.

The model now provides a rather detailed description of the adjustment of the firm. In the short run, the firm adjusts only output, with predetermined prices and supply. Short-run demand changes affect the utilization of the production factors, and supply constraints limit output increases. This implies a procyclical development of factor productivities and can account for prolonged delivery lags during the upswing. In the medium run, the firm adjusts prices and employment under uncertainty about demand. During the upswing, capacity constraints imply an upper bound for the increase of employment and prices rise instead; during the downswing, marginal costs and the price elasticity of demand imply a lower bound for price reductions, and the whole adjustment falls on employment. This implies an asymmetric adjustment of prices and quantities during the business cycle and with respect to demand and cost changes. In the long run, the firm adjusts capacities and the production technology. The delayed adjustment of prices and quantities implies an inefficiency, because the production factors are not always fully utilized. This inefficiency exhibits the same effect on average prices, factor inputs and output as higher factor costs.

 $<sup>^{35}</sup>$ Eq. (39) results from inserting eq. (37) into eq. (33) and solving for YC.

## 5 Conclusions

In the paper, a theoretical model of price vs. quantity adjustments of the firm is developed. The model is characterized by adjustment constraints, uncertainty about demand and imperfect competition on the product market. Capacity constraints are a reasonable assumption for the short- and medium-run adjustment of output, employment and prices and provide a microeconomic foundation of a monopolistically competitive market structure; a delayed adjustment of quantities under demand uncertainty permits an interpretation of the procyclical development of productivity in terms of optimal labour and capital hoarding during recessions; a delayed adjustment of prices introduces price stickyness and delivery lags.

The immediate adjustment of prices and quantities and perfect competition on the product market are contained as special cases. With uncertainty, prices are higher and quantities are lower due to the costs of labour hoarding and underutilization of capacities. In addition, uncertainty introduces dynamics and expectation formation into the multiplier process and can explain the persistence of disequilibria. Within the model, the short run and the long run are distinguished by the flexibility of capacities, not by the stickyness of prices as in standard Keynesian models.

The microeconomic model of the firm is complemented by aggregation. The combination of imperfect competition and adjustment constraints yields reasonable macroeconomic effects for the determination of the short-run multiplier and the price adjustment during the business cycle. The model exhibits both classical and Keynesian features without recurrence to price rigidities. The aggregate model exemplifies the prominent role of capacity utilization as business cycle indicator. The price adjustment is determined by a medium-run Phillips curve mechanism depending on production costs and capacity utilization; the medium-run demand and cost multipliers are regime dependent which implies an asymmetric price and quantity adjustment during the business cycle.

The capacity adjustment is determined through a flexible accelerator mechanism for investment which introduces a source of instability into the aggregate adjustment. However, the short-run multiplier is limited by capacity constraints. Embedding the model of the firm into a general (dis)equilibrium framework is on the agenda of future research. The model finally provides a framework to discuss the impact of demand uncertainty and competition on the adjustment. The only departures from the standard model are a delayed adjustment with uncertainty about demand and monopolistic competition on the product market.

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## Appendix A: Delayed adjustment of prices and employment

<u>Proposition A.1</u>:  $\overline{\varepsilon} = \overline{\varepsilon}(\sigma_{\varepsilon}, \eta)$ , the optimal value of  $\overline{\varepsilon}$  depend only on demand uncertainty  $\sigma_{\varepsilon}$  and the price elasticity of demand  $\eta$ .

<u>Proof</u>: Inserting the definition of expected output E(Y), eq. (15), into the first order condition w.r.t. prices, eq. (17) yields

$$(1+\eta) \cdot \int_{-\infty}^{\overline{\varepsilon}} YD \cdot f_{\varepsilon} d\varepsilon + \int_{\overline{\varepsilon}}^{\infty} YS \cdot f_{\varepsilon} d\varepsilon = 0.$$
 (A.1)

Substituting demand YD from eq. (1) and supply YS through the definition of  $\overline{\varepsilon}$  from eq. (16) yields

$$(1+\eta) \cdot \int_{-\infty}^{\overline{\varepsilon}} p^{\eta} \cdot Z \cdot \exp(\varepsilon) \cdot f_{\varepsilon} d\varepsilon + \int_{\overline{\varepsilon}}^{\infty} p^{\eta} \cdot Z \cdot \exp(\overline{\varepsilon}) \cdot f_{\varepsilon} d\varepsilon = 0.$$
(A.2)

Dividing this expression by  $p^{\eta} \cdot Z \cdot \exp(\overline{\varepsilon})$  yields

$$(1+\eta) \cdot \int_{-\infty}^{\overline{\varepsilon}} \exp(\varepsilon - \overline{\varepsilon}) \cdot f_{\varepsilon} d\varepsilon + \int_{\overline{\varepsilon}}^{\infty} f_{\varepsilon} d\varepsilon = 0.$$
 (A.3)

For the normalized random variable  $z = \varepsilon / \sigma_{\varepsilon}$ , this expression can be rewritten by changing integration variables as

$$(1+\eta) \cdot \int_{-\infty}^{\overline{\varepsilon}/\sigma_{\varepsilon}} \exp(z \cdot \sigma_{\varepsilon} - \overline{\varepsilon}) \cdot f_z dz + \int_{\overline{\varepsilon}/\sigma_{\varepsilon}}^{\infty} f_z dz = 0.$$
(A.4)

Eq. (A.4) determines  $\overline{\varepsilon}$  in terms of  $\sigma_{\varepsilon}$  and  $\eta$ .

<u>Proposition A.2</u>: The probability of demand constraints and the expected utilization of supply depend only on demand uncertainty  $\sigma_{\varepsilon}$  and the price elasticity of demand  $\eta$ .

<u>Proof</u>: The probability of demand constraints is determined as

$$\operatorname{prob}(YD < YS) = \int_{-\infty}^{\overline{\varepsilon}} f_{\varepsilon} d\varepsilon.$$
 (A.5)

The expected utiliation of supply is determined as

$$U_l := \frac{\mathrm{E}(Y)}{YS} = \int_{-\infty}^{\overline{\varepsilon}} \frac{YD}{YS} \cdot f_{\varepsilon} d\varepsilon + \int_{\overline{\varepsilon}}^{\infty} f_{\varepsilon} d\varepsilon.$$
(A.6)

Substituting demand YD from eq. (1) and supply YS through the definition of  $\overline{\varepsilon}$  from eq. (16) yields

$$U_l := \frac{\mathrm{E}(Y)}{YS} = \int_{-\infty}^{\overline{\varepsilon}} \exp(\varepsilon - \overline{\varepsilon}) \cdot f_{\varepsilon} d\varepsilon + \int_{\overline{\varepsilon}}^{\infty} f_{\varepsilon} d\varepsilon.$$
(A.7)

Since  $\overline{\varepsilon}$  depends only on  $\sigma_{\varepsilon}$  and  $\eta$ , prob(YD < YS) and  $U_l$  also depend only on  $\sigma_{\varepsilon}$  and  $\eta$ .

<u>Proposition A.3</u>: In case of sufficient capacities, the optimal price is determined by unit labour costs, the price elasticity of demand and the expected utilization of employment,  $p(w) = w/[U_l \cdot \pi_l \cdot (1+1/\eta)].$ 

<u>Proof</u>: Inserting the first order condition with respect to prices, eq. (A.3), for the first integral in eq. (A.7) above yields

$$U_{l} = \frac{1 - \text{prob}(YD < YS)}{(1 + 1/\eta)},$$
(A.8)

i.e. the expected utilization of supply can be determined from the probability of demand constraints and the price elasticity of demand. Inserting eq. (A.8) into eq. (23) yields eq. (24) in the main text.

#### Appendix B: Delayed adjustment of capacities

<u>Proposition B.1</u>:  $\overline{\varepsilon} = \overline{\varepsilon}(\sigma_{\varepsilon}, \eta, \frac{c}{\pi_k} \frac{\pi_l}{w})$ , the optimal value of  $\overline{\varepsilon}$  depend only on demand uncertainty  $\sigma_{\varepsilon}$ , the price elasticity of demand  $\eta$  and relative unit factor costs  $\frac{c}{\pi_k} \frac{\pi_l}{w}$ 

<u>Proof</u>: From eqs. (5), (8) and (9) follows

$$p(YC) = p(w) \cdot \exp[(\overline{\varepsilon} - \varepsilon)/\eta]$$
 and  $p(w) = \frac{w}{\pi_l}/(1 + 1/\eta).$  (B.1)

Inserting these expressions into the first order condition, eq. (27), yields

$$\int_{\overline{\varepsilon}}^{\infty} \left( \exp[(\overline{\varepsilon} - \varepsilon)/\eta] - 1 \right) \cdot f_{\varepsilon} d\varepsilon - \frac{c}{\pi_k} \frac{\pi_l}{w} = 0.$$
 (B.2)

For the normalized random variable  $z = \varepsilon / \sigma_{\varepsilon}$ , this expression can be rewritten by changing integration variables as

$$\int_{\overline{\varepsilon}/\sigma_{\varepsilon}}^{\infty} \{ \exp[(\overline{\varepsilon} - z \cdot \sigma_{\varepsilon})/\eta] - 1 \} \cdot f_z d_z = \frac{c}{\pi_k} \frac{\pi_l}{w}.$$
(B.3)

Eq. (B.3) determines  $\overline{\varepsilon}$  in terms of  $\sigma_{\varepsilon}$ ,  $\eta$  and  $\frac{c}{\pi_k} \frac{\pi_l}{w}$ .

<u>Proposition B.2</u>:  $\overline{\varepsilon}$  and  $\sigma_{\varepsilon}$  determine the regime probabilities and the expected utilization of capacities  $U_c$ .

<u>Proof</u>: The probability of demand constraints is defined as

$$\operatorname{prob}(YD < YC) = \int_{-\infty}^{\overline{\varepsilon}} f_{\varepsilon} d\varepsilon.$$
 (B.4)

The expected utiliation of capacities is defined as

$$U_c := \frac{\mathrm{E}(Y)}{YC} = \int_{-\infty}^{\overline{\varepsilon}} \frac{YD}{YC} \cdot f_{\varepsilon} d\varepsilon + \int_{\overline{\varepsilon}}^{\infty} f_{\varepsilon} d\varepsilon.$$
(B.5)

Substituting demand YD from eq. (1) and capacities YC through the definition of  $\overline{\varepsilon}$  from eq. (9) yields

$$U_{c} = \int_{-\infty}^{\overline{\varepsilon}} \exp(\varepsilon - \overline{\varepsilon}) \cdot f_{\varepsilon} d\varepsilon + \int_{\overline{\varepsilon}}^{\infty} f_{\varepsilon} d\varepsilon.$$
(B.6)

Proposition B.3:  $E(p) = (w/\pi_l + c/\pi_k)/(1 + 1/\eta)$ , the expected price is determined as mark-up over unit factor costs.

<u>Proof</u>: The first order condition w.r.t. the capital stock, eq. (27), can be rewritten as  $c^{\infty}$ 

$$\int_{\overline{\varepsilon}}^{\infty} p(YC) \cdot f_{\varepsilon} d\varepsilon = \int_{\overline{\varepsilon}}^{\infty} p(w) \cdot f_{\varepsilon} d\varepsilon + \frac{c}{\pi_k} / (1 + 1/\eta) = 0.$$
(B.7)

Expected prices are defined as

$$E(p) = \int_{-\infty}^{\overline{\varepsilon}} p(w) \cdot f_{\varepsilon} d\varepsilon + \int_{\overline{\varepsilon}}^{\infty} p(YC) \cdot f_{\varepsilon} d\varepsilon$$
(B.8)

Inserting eq. (B.7) for the second integral yields the requested result.

<u>Proposition B4</u>: The *average* price is determined as a mark-up over corrected factor costs.

<u>Proof</u>: Expected sales are determined as

$$E(p \cdot Y) = \int_{-\infty}^{\overline{\varepsilon}} p(w) \cdot Y(w) \cdot f_{\varepsilon} d\varepsilon + \int_{\overline{\varepsilon}}^{\infty} p(YC) \cdot YC \cdot f_{\varepsilon} d\varepsilon.$$
(B.9)

Inserting eq. (B.7) for the second integral yields

$$\mathbf{E}(p \cdot Y) = p(w) \cdot \int_{-\infty}^{\overline{\varepsilon}} Y(w) \cdot f_{\varepsilon} d\varepsilon + p(w) \cdot \int_{\overline{\varepsilon}}^{\infty} YC \cdot f_{\varepsilon} d\varepsilon + YC \cdot \frac{c}{\pi_k} / (1 + 1/\eta).$$
(B.10)

The sum of the first two integrals is equal to expected output E(Y).

$$\mathbf{E}(p \cdot Y) = p(w) \cdot \mathbf{E}(Y) + YC \cdot \frac{c}{\pi_k} / (1 + 1/\eta).$$
(B.11)

Dividing this expression by expected output yields eq. (29) in the main text. Note that expected sales are determined by expected costs and the mark-up:

$$\mathbf{E}(p \cdot Y) = \left(\mathbf{E}(Y) \cdot \frac{w}{\pi_l} + \frac{c}{\pi_k} \cdot YC\right) / (1 + 1/\eta).$$
(B.12)

The term in paranthesis is the sum of capital costs and expected labour costs.

#### Appendix C: The optimal capital-labour ratio

In case of a Cobb-Douglas production function,

$$Y = \theta \cdot L^{\alpha} \cdot K^{1-\alpha} \text{ and } \pi_l = \theta \cdot k^{1-\alpha}, \pi_k = \theta \cdot k^{-\alpha}.$$
 (C.1)

The relation of the elasticities of the factor productivities with respect to the capital-labour ratio is equal to the relative output elasticities, and the optimal capital-labour ratio is determined as

$$k = \frac{\pi_l}{\pi_k} = \frac{1 - \alpha}{\alpha} \cdot \frac{w \cdot U_c}{c}.$$
 (C.2)

i.e. k depends on the relative output elasticities of the factors and relative factor costs. In case of a CES production function,

$$Y^{-\rho} = \delta \cdot (\theta_l \cdot L)^{-\rho} + (1 - \delta) \cdot (\theta_k \cdot K)^{-\rho}.$$
 (C.3)

The elasticities of the factor productivities with respect to the capital-labour ratio are given by

$$\frac{\partial \pi_l}{\partial k} \cdot \frac{k}{\pi_l} = (1 - \delta) \cdot \theta_k^{-\rho} \cdot \pi_k^{-\rho}, \qquad \frac{\partial \pi_k}{\partial k} \cdot \frac{k}{\pi_k} = -\delta \cdot \theta_l^{-\rho} \cdot \pi_l^{-\rho}. \tag{C.4}$$

 $\rho$  is the substitution parameter,  $\delta$  is the distribution parameter and  $\theta_l, \theta_k$  are the efficiencies of labour and capital. Inserting these expressions into the first order condition with respect to the capital-labour ratio, eq. (32) in the main text, yields

$$\frac{w \cdot U_c}{c} \cdot \frac{\pi_k}{\pi_l} = \frac{\delta \cdot \theta_l^{-\rho} \cdot \pi_l^{\rho}}{(1-\delta) \cdot \theta_k^{-\rho} \cdot \pi_k^{\rho}},\tag{C.5}$$

and the optimal capital-labour ratio is determined as

$$k = \frac{\pi_l}{\pi_k} = \left(\frac{w \cdot U_c}{c}\right)^{1/(1+\rho)} \cdot \left(\frac{\delta}{1-\delta}\right)^{1/(1+\rho)} \cdot \left(\frac{\theta_l}{\theta_k}\right)^{-\rho/(1+\rho)}.$$
 (C.6)

 $\rho = 1/\sigma - 1$  and  $\sigma$  is the elasticity of substitution.

#### Appendix D: A three-step decision structure

The proofs of this model correspond largely to those above in appendix B.

Proposition D.1:  $\overline{z} = \overline{z}(\sigma_z, \eta, \frac{c}{\pi_k} \frac{\pi_l}{w})$ , the optimal value of  $\overline{z}$  depend only on uncertainty about z, the price elasticity of demand  $\eta$  and relative unit factor costs  $\frac{c}{\pi_k} \frac{\pi_l}{w}$ 

<u>Proof</u>: From eqs. (22), (24) and (33) follows

$$p(YC) = p(w) \cdot \exp[(\overline{z} - z)/\eta] \quad \text{and} \quad p(w) = \frac{w}{\pi_l \cdot U_l}/(1 + 1/\eta). \quad (D.1)$$

Inserting these expressions into the first order condition, eq. (36), yields

$$\int_{\overline{z}}^{\infty} \left( \exp[(\overline{z} - z)/\eta] - 1 \right) \cdot f_z dz - \frac{c}{\pi_k} \frac{\pi_l}{w} = 0.$$
 (D.2)

Rewriting eq. (D.2) for the normalized random variable  $z/\sigma_z$  yields an expression which determines  $\overline{z}$  in terms of  $\sigma_z, \eta$  and  $\frac{c}{\pi_k} \frac{\pi_l}{w}$ .

<u>Proposition D.2</u>:  $\overline{z}$  and  $\sigma_z$  determine the regime probabilities for employment. <u>Proof</u>: The probability of capacity constraints for employment is defined as

$$\operatorname{prob}(YC < YL(w)) = \int_{\overline{z}}^{\infty} f_z dz.$$
 (D.3)

<u>Proposition D.3</u>: The expected utilization of capacities  $U_c$  is determined by  $\overline{z}, \sigma_z$  and the expected utilization of employment  $U_l$ .

<u>Proof</u>: The expected utiliation of capacities is defined as

$$U_c := \frac{\mathrm{E}(Y)}{YC} = \left(\int_{-\infty}^{\overline{z}} \frac{YL(w)}{YC} \cdot f_z dz + \int_{\overline{z}}^{\infty} f_z dz\right) \cdot U_l.$$
(D.4)

Substituting YL(w) from eq. (25) and capacities YC through the definition of  $\overline{z}$  from eq. (33) yields

$$U_c = \left(\int_{-\infty}^{\overline{z}} \exp(z - \overline{z}) \cdot f_z dz + \int_{\overline{z}}^{\infty} f_z dz\right) \cdot U_l.$$
(D.5)

Proposition D.4:  $E(p) = (w/\pi_l + c/\pi_k)/(1 + 1/\eta)$ , the expected price is determined as mark-up over unit factor costs.

<u>Proof</u>: The first order condition w.r.t. the capital stock, eq. (36), can be rewritten as

$$\int_{\overline{z}}^{\infty} p(YC) \cdot f_z dz = \int_{\overline{z}}^{\infty} p(w) \cdot f_z dz + \frac{c}{U_l \cdot \pi_k} / (1 + 1/\eta) = 0.$$
(D.6)

Expected prices are defined as

$$E(p) = \int_{-\infty}^{\overline{z}} p(w) \cdot f_z dz + \int_{\overline{z}}^{\infty} p(YC) \cdot f_z dz$$
(D.7)

Inserting the first order condition for the second integral and p(w) from eq. (24) yields the requested result.

<u>Proposition D5</u>: The *average* price is determined as mark-up over corrected factor costs.

<u>Proof</u>: Expected sales are determined as

$$\mathbf{E}(p \cdot Y) = \left(\int_{-\infty}^{\overline{z}} p(w) \cdot YL(w) \cdot f_z dz + \int_{\overline{z}}^{\infty} p(YC) \cdot YC \cdot f_z dz\right) \cdot U_l. \quad (D.8)$$

Inserting the first order condition for the second integral yields, substituting expected output E(Y) and dividing by expected output yields eq. (38) in the main text (see appendix B, proposition B.4 above).